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THEORY OF QUASI-CATEGORIES AS A THEORETICAL BASE FOR THE CONSTRUCTION OF BRAIN LIKE COMPUTERS

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ABSTRACT

The article gives a definition of the concept of a modified category and formulates the problem of developing a theory of modified categories that opens the way to building high-performance brain-like computers of parallel action. Analyzing the structure of the category, we discovered in it some abnormality, which gave rise to a correction of the classical category concept. Having developed such an adjustment, we received a modified category, which seems to be better for the theoretical construction starting point role of the parallel action brain-like computers basis creation. We characterize the classical category, after that we will realize its predicate interpretation. As a result, we obtain a predicate category - one of the special cases of classical category. Any algebra, satisfying all the above requirements, will be regarded as an objectless classical category. It is possible to develop a theory of modified categories in parallel with the theory of classical categories. The theory of modified categories will prove to be an interesting object for theoretical research and an important tool for practical applications. It turns out that the diagrams of the theory of modified categories after their predicate interpretation coincide with the logical networks of brain-like computers. This gives us hope that the theory of modified categories will eventually become the theoretical basis for constructing of brain-like computers of parallel action.

1. Introduction

Theory of categories has formed by the 60th years of twentieth century. It develops prospective means of representation, analysis and synthesis of algebraic structures of arbitrary form. By the 1980s, the importance of the theory of categories for computerization and informatization was recognized, in particular, for the automation of programming.

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Analyzing the structure of the category, we discovered in it some abnormality, which gave rise to a correction of the classical category concept. Having developed such an adjustment, we received a modified category, which seems to be better for the theoretical construction starting point role of the parallel action brain-like computers basis creation.

2. Materials and Methods

2.1 Classical category

First, we briefly characterize the classical category [1-3], after that we will realize its predicate interpretation. As a result, we obtain a predicate category - one of the special cases of classical category.

Then we consider the most general definition of the concept of the classical category – the classical nonobjective category [4]. It is also called the classical abstract category. Example: Let M be any set. Its elements, denoted by the symbols $f,g,h,\ldots$, are called morphisms. Let, in addition to this, single-valued partial correspondence be defined as $fg=h$ with the departure area $M\times M$ and arrival area of M. It is called the multiplication of morphisms f and g. Morphism h is called intersection of morphisms f and g. The multiplication of morphisms is associative: for any $f,g,h\in M$, intersections exist $(fg)h$, $f(gh)\in M$, the equation is justly $(fg)h=f(gh)$. Let E be the set of all unit morphisms, $E\subseteq M$. Any morphism $e\in M$ is called single (or identical, or simply a unit), if it satisfies the following two conditions:

1) For each unit $e\in E$ intersection ee exists;

2) At any morphisms $f,g\in M$ and any units $e,e'\in E$, for which intersections exist $fe,e'g\in M$, congruence are met $fe=f$ and $e'g=g$.

Set of morphisms M with units, satisfying the conditions listed above, taken together with multiplication of morphisms, satisfying the above conditions, is called a classical nonobjective category K. It is written a $M=MorK$, $f\in M$, $f\in MorK$. $MorK$ is a multiplication of all K categories. If $f\in MorK$, then morphism f is K-morphisms.

This definition allows the existence in the category of many units. Namely the presence of many units and only this distinguishes the category (understood in the most general sense) from other known algebraic structures. A unit would always be one if on M multiplicity, that was accepted not as partial, but everywhere defined. The existence of many units in the category and the everywhere demand for the certainty of multiplication of morphisms are relative to each other in an irreconcilable contradiction. But if we want to weaken the need to the categorical multiplication of morphisms and accept it as partial, then only due to this a possibility of introducing many units into the categories emerges. Units e and e' are respectively called right and left for morphism correspondently, $f\in M$, if $fe=f$ and $e'f=f$. From the definition of the concept of a category it logically follows that for any $e\in E$ equation holds true $ee=e$, and that for any morphism $f\in M$ there are only one right-hand and one left-hand unit (which can differ from each other). The last assertion is called the categorical law of identity. So, for each morphism $f\in M$ there is only one right-hand unit e and the only left-hand unit e', such that $fe=e'f=f$. At the same time, for each unit $e\in E$ there are such morphisms f and g (not necessarily the only ones), that congruence are performed for them $fe=f$ and $eg=g$. For any unit $e\in E$ in the role of such morphisms $f=g=e$ can be taken.

In this way, certain category can be considered as some kind of algebra [5, 6]. In the role of its carrier stands a set of morphisms M, the role of basic elements in this algebra is performed by units, (more precisely - single-valued correspondence) is the partial multiplication of morphisms. Any algebra, satisfying all the above requirements, will be regarded as an objectless classical category.

A classical nonobjective category can be considered as one of the possible monoid generalizations concept. In it, instead of the operation (everywhere defined and unique correspondence) multiplication, appearing in the definition of a monoid. A correspondence of a more general kind is used - partial multiplication, from which the property of everywhere certainty is removed. For some pairs $f,g\in M$ intersection fg in the classical nonobjective category may not exist. The requirement of unique units is also lifted. There can be a lot of units in the category.

Units of the classical nonobjective category can be defined by the following two properties:

1) For any unit $e\in E$ $ee=e$;

2) For any $f,g\in M$ and any $e,e'\in E$, for which intersections $fe,e'g\in M$ exist, equalities $fe=f$ and $e'g=g$ are performed.

If we additionally require that the multiplication of morphisms be everywhere defined, then the category will become a monoid. Suppose that multiplication in a category is everywhere defined and, at the same time, it has two units that differ from one another e and e', $e\neq e'$. Then intersection should exist $e'e$. According to equation $fe=f$ we obtain intersection $e'e=e'$.

According to the same equation $e'f=f$ we get a different result $e'e=e$. But this is impossible, since it is assumed that multiplication has the uniqueness property for its values. We have arrived to a contradiction. This means that if there are at least two units in the category, the last cannot have an everywhere defined multiplication.

Now we concretize the concepts of category and morphism. In the process of concretization, the previously introduced concept of an objectless category receives additional details and properties and, as a result, becomes a category with objects [7]. We attach objects to K morphism non-subject category. Multiplicity of all objects of K category we write in the form Ob in K or in the form ObK. Objects are denoted by letters $A, B, C, \ldots$. If $A \in ObK$, then we say, that A is K -object. f Is said to be morphism from the object A to the object B, and is written as $f: A \to B$ or $A \xrightarrow{f} B$. Object A is the beginning of morphism f, and object B – its end. Instead of «morphism» term the word arrow is also used.

To each pair of (A,B) objects $A, B \in ObK$ some, maybe even empty multiplicity, it is set as accordance $H_k(A,B)$ morphisms of K category. It is possible that much different morphism, for example f, g, h, the same pair of objects is set as accordance (A,B), i.e. $f, g, h: A \to B$. Such morphisms are called parallel. And for some other pair of objects (C,D) in K category any f morphism, such as $f: C \to D$ may not be found. Instead of a note $H_k(A,B)$ designations $Hom_K(A,B)$, $Mor_K(A,B)$, $K(A,B)$ are also used, and if it does not lead to indeterminacy, – then it leads to more concise notes $H(A,B)$, $Hom(A,B)$, $Mor(A,B)$. Instead of the note $f \in H_K(A,B)$ it is otherwise written as $f: A \to B$ or A $\xrightarrow{f}$ B. Instead of expressions «object $A \in ObK$» and «morphism $f \in MorK$» it is written «object $A \in K$» and «morphism $f \in K$» or even easier: «K -object A» and «K -morphism f». For each morphism $f \in MorK$ there is a single pair of objects A and B, such, that $A, B \in ObK$ and $f \in H_K(A,B)$. The attribution of this property to morphisms is motivated by the fact that when interpreting them for each f function it is natural to indicate its domain of definition A and range of values B .Otherwise the function definition will be incomplete. It is written as $A = domf$ (beginning of morphism; in our interpretation - its domain of definition), $B = codf$ (end of morphism; in our interpretation – its range of values).

Let us write down the definition of the classical category with objects. Category with K objects consists of morphisms $MorK$ multiplicity and objects ObK multiplicity. It is assumed, that $MorK$ and ObK multiplicities do not intersect. Category with K objects are characterized by the following five properties:

1) Morphisms multiplicity corresponds to each pair of A, B K -objects (perhaps even empty), included to $MorK$.

2) For each $f \in MorK$ morphism the only pair of A, B K -objects, such, that $f \in H_K(A,B)$ exists.

3) In $MorK$ multiplicity definitely, a partial, single-valued correspondence is defined –morphisms multiplication; intersection fg morphisms $f: A \to B$ and $g: C \to D$ is defined only in cases, when $B = C$, in other words, when morphism f end coincides with the beginning of morphism g. In this case intersection fg is K -morphism from the A objects to the D object. Otherwise they say, that for $A, B, C \in K$ objects the reflection is defined $H_K(A,B) \times H_K(B,C) \to H_K(A,C)$. Sign $\times$ in this case denotes Cartesian intersection multiplicity of morphisms f, g of K category of $f: A \to B$ form and $g: B \to C$ are called consecutive, $f: A \to B$ and $g: A \to B$ are called– parallel.

4) Multiplication of morphisms is associative $(fg)h = f(gh)$, when morphisms $(fg)h$ and $f(gh)$ exist. In other words, associativity holds true every time, when $f: A \to B$, $g: B \to C$, $h: C \to D$. In this way, associativity is fulfilled in all those cases when it makes sense. Equation $(fg)h = f(gh)$ expresses the categorical law of associativity. In recent years, object modeling, in which the main tool is diagrams that look like private categorical diagrams had become very popular. They are also built of objects and arrows. They help in

describing the architecture of information systems. Private categorical diagrams are opposite to common ones, with their help information systems functioning patterns are described. On the fig. (1), an example of banking system diagram is provided [8].

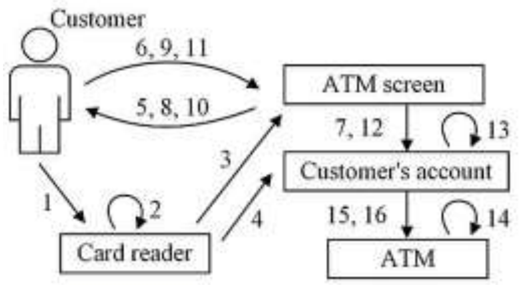

Fig. (1): Diagram of the banking system work

Arrows express the following operations: 1) receiving a card by a reading device; 2)card number reading; 3) screen initialization; 4) account opening; 5) registration number request; 6) registration number input; 7) registration number inspection; 8) transaction request and (what financial transaction should be performed?); 9) transaction selection (withdraw money); 10) request of the required money amount; 11) input of the money amount; 12) withdrawal of money from an account; 13) amount check; 14) deduction of the withdrawn amount of money from the account; 15) cash dispense; 16) issue of a check.

The theory of categories can be viewed as the doctrine of categorical algebra, which is defined on the carrier $MorK$. It introduces the basic elements in the form of identical morphisms and basic operations - multiplication of morphisms.

5) For each $B \in K$ object morphism exists $e_B : B \to B$, fig. (2), called ordinary or identical morphism of an object B, such, that $fe_B = f$ and $e_B g = g$ for any morphisms $f : A \to B$ and $g : B \to C$. Identities $fe_B = f$ and $e_B g = g$ are called categorical laws of identity. They are expressed by the following commutative diagram of the identity.

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "f"'] & B \arrow[d, "e_B"] \arrow[dr, "g"] & \\ & B \arrow[r, "g"] & C \end{tikzcd}$$

Fig. (2): commutative diagram of the identity.

For $f, g \in MorK$ morphisms fg intersection exists if and only if, when f, g – successive morphisms of K category.

2.2 Predicates

We briefly describe the algebra of predicates [5],in terms of which we will subsequently interpret the notion of category.

Predicate, given on a Cartesian product $A_1, A_2, ..., A_m$, is any function $P(x_1, x_2, ..., x_m) = \xi$, reflecting Cartesian intersection $A_1 \times A_2 \times ... \times A_m$ of $A_1, A_2, ..., A_m$ multiplicity to $\Sigma = \{0,1\}$ multiplicity.

Let L be a multiplicity of all relations on S, M multiplicity of all predicates on S. Between all relations of L multiplicity and all predicates of M multiplicity, assigned on S, there is a one-to-one correspondence. Relation of P from L and predicate P of M are

$$P(x_1, x_2, ..., x_m) = \begin{cases} 1, \text{ if } (x_1, x_2, ..., x_m) \in P, \\ 0, \text{ if } (x_1, x_2, ..., x_m) \notin P. \end{cases}$$

Correspondent, if at any $x_1 \in A_1, x_2 \in A_2, ..., x_m \in A_m$

The reverse transition from P predicate to P relation is conducted according to the rule:

if $P(x_1, x_2, ..., x_m) = 1$, then $(x_1, x_2, ..., x_m) \in P$;

if $P(x_1, x_2, ..., x_m) = 0$, then $(x_1, x_2, ..., x_m) \notin P$.

Multiplicity of all vectors $(x_1, x_2, ..., x_m)$, satisfying equation $P(x_1, x_2, ..., x_m) = 1$, forms of relation P, which is called the truth domain of the predicate P. $P \in M$ Predicate is called characteristic function of the relation $P \in L$. Any algebra, defined over a support is called algebra of M predicates. The operations of disjunction, conjunction and negation over predicates are defined by the following congruencies: $\forall x_1 \in A_1, \forall\, x_2 \in A_2, ..., \forall\, x_m \in A_m$

$$(P \vee Q)(x_1, x_2, ..., x_m) =$$
$$= P(x_1, x_2, ..., x_m) \vee Q(x_1, x_2, ..., x_m);$$
$$(P \wedge Q)(x_1, x_2, ..., x_m) =$$
$$= P(x_1, x_2, ..., x_m) \wedge Q(x_1, x_2, ..., x_m);$$
$$(\neg P)(x_1, x_2, ..., x_m) = \neg(P(x_1, x_2, ..., x_m)).$$

Symbols $\vee, \wedge, \neg$, standing to the left of the sign of congruencies, mean operations on predicates, on the right - operations on predicate values, that means, over

Boolean elements.

2.3 Predicate interpretation of the classical category

Above we have considered the concepts of the classical category and the predicate. Now let us turn to the predicate interpretation of the classical category. Such an interpretation will be called a predicate category and will be denoted as $Pred$.

We choose some universe of objects U. In the role of objects $A, B, C, \ldots$ categories $Pred$ we use arbitrary subsets of the universe U. As Ob multiplicity $Pred$ categories we take a system of all multiplicities of U universe. As morphisms of the form $f: A \to B$ $Pred$ category we use linear logic operators of $F_f(P)=Q$ type. Each such operator converts one-place predicates [9]. P in single predicates Q and is expressed as follows:

$$\exists x \in A(K_f(x,y)P(x)) = Q(y). \qquad (1)$$

In equation (1) predicates P and Q are variables. Predicate $P(x)$ are imposed on the set A, predicate $Q(y)$ on B multiplicity. Predicate $P(x)$ on A we consider as an instance of an object A, predicate $Q(y)$ on B – as an exemplar of B object. In this way, morphism $f: A \to B$ converts instances of an object A to an object B. It would be more natural to take not $A, B, C, \ldots$ multiplicities of universe elements, but multiplicities of all predicates $P(x)$, $Q(y)$, $R(z), \ldots$, given respectively on $A, B, C, \ldots$, multiplicities, but this is optional.

Predicate $K_f(x,y)$ is called the kernel of a linear logical operator [10], It completely determines the type of transformation (1). Predicate $K_f(x,y)$ is fixed, it is assumed on $A \times B$. Morphism of f type (1) is fully determined by $K_f(x,y)$ predicate. In the role of $Mor(A,B)$ multiplicity of all morphisms of $f: A \to B$ type we take the system of all possible operations of the type (1). In $Pred$ category each morphism $f \in Pred$ corresponds to $K_f(x,y)$ conversion kernel (1). Each morphism $f: A \to B$ of $Pred$ category can be assumed, indicating the predicate corresponding to it $K_f(x,y)$ on $A \times B$. Multiplicity $Mor(Pred)$ we obtain a union of all multiplicities of the form $Mor_{Pred}(A,B)$, where (A,B) – all possible pairs of multiplicities $A, B \subseteq U$, or as a multiplicity of transformations of the form (1) with all possible kernels $K(x,y)$, assumed on all possible Cartesian intersections $A \times B$ of $A, B \subseteq U$ multiplicity.

Predicate can serve as an example of a kernel morphism of $Pred$ category

$$K(x,y) = (x^a \vee y^b) y^1 \vee x^d (y^2 \vee y^3) \vee x^e y^3, \qquad (2)$$

Assumed on Cartesian intersection A×B of $A = \{a,b,c,d,e\}$ and $B = \{1,2,3,4\}$ multiplicities. On the fig. (3) $K(x,y)$ bipartite predicate graph is depicted.

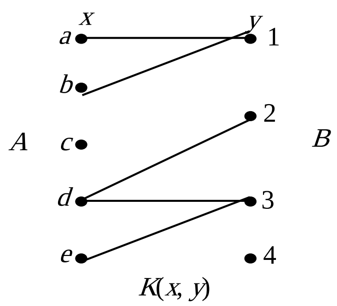

Fig. (3): bipartite predicate graph

A linear logical operator with this kernel is written in the form:

$$Q(y) = \exists x \in \{a,b,c,d,e\} \\ (((x^a \vee x^b) y^1 \vee x^d (y^2 \vee y^3) \vee x^e y^3) P(x)). \qquad (3)$$

Let's define, for example, reaction of $Q(x)$ morphism (3) for a predicate

$$P(x) = x^a \vee x^b \vee x^e. \qquad (4)$$

By the formula (3) we find:

$$Q(y) = \exists x \in \{a,b,c,d,e\} \\ (((x^a \vee x^b) y^1 \vee x^d (y^2 \vee y^3) \vee \\ \vee x^e y^3)(x^a \vee x^b \vee x^e)) = y^1 \vee y^3. \qquad (5)$$

The same result can also be graphically obtained (fig. (4)).

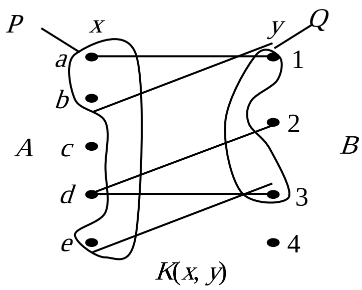

Fig. (4): Graphical representation of bipartite predicate

To obtain Q multiplicity we collect all those y elements together, which are connected by edges of the $K(x,y)$ graph with x elements, forming P multiplicity. In the result we obtain $Q=\{1,3\}$. In this way, morphism (3) converts multiplicity into $Q=\{1,3\}$ multiplicity (fig. (4)).

Now we transit to the predicate interpretation of morphism intersections. Let's define morphism $f:A\to B$ as operation (1) $F_f(P)=Q$, amorphism $g:B\to C$ – as operation $F_f(Q)=R$, defined by the equation:

$$\exists y\in B(K_g(y,z)Q(y))=R(z). \tag{6}$$

Variable predicate $R(z)$ is asset on the multiplicity C, and fixed predicate $K_g(y,z)$ – on $B\times C$. Forming the operation $F_h(P)=R$ by superposition of operations $F_f(P)=Q$ and $F_g(Q)=R$: $F_h(P)=F_g(F_f(P))=R.$ Substituting (1) in (2), we obtain an expression for the transformation F_h:

$$\exists y\in B(K_g(y,z)(\exists x\in A(K_f(x,y)P(x))))=R(z), \tag{7}$$

Which transforms the predicate $P(x)$ on A into predicate $R(z)$ on C. After the identity transformations, equation (7) acquires the form:

$$\exists x\in A((\exists y\in B(K_f(x,y)K_g(y,z)))P(x))=R(z). \tag{8}$$

Equation (7) is a linear logical operator. The role of its kernel is a predicate

$$K_h(x,z)=\exists y\in B(K_f(x,y)K_g(y,z)) \tag{9}$$

On $A\times C$ with $x\in A$ and $z\in C$ arguments. Now the transformation (7) can be written in a shorter form:

$$\exists x\in A(K_h(x,z)P(x))=R(z). \tag{10}$$

Transformation (10) will be understood as morphism $h:A\to C$ of $Pred$ category. We take it in a role of intersections fg morphisms f and i.e. in this way, $fg=h$.

Let's find, for example, intersection of two any morphisms of $Pred$ category. Find $K_h(x,z)$ graphically, fig. (5):

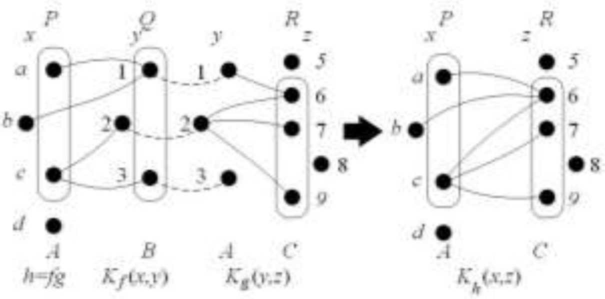

Fig. (5): Intersection morphisms f and g categories

Intersection morphisms f and g categories $Pred$ Bipartite graphs of $K_f(x,y)$ and $K_g(y,z)$ morphisms kernels f and g are on the fig. (5)on the left. They can be converted into bipartite kernel graph $K_h(x,z)$ intersection fg morphisms f and g as follows. At the first stage $K_f(x,y)$ and $K_g(y,z)$. On the second stage turn a pair of graphs $K_f(x,y)$ and $K_g(y,z)$, which we successively connected, in the equivalent one graph $K_h(x,z)$. To form the edges of a graph $K_h(x,z)$ we identify all the paths from the points of the multiplicity A to the points of the multiplicity C in the chain of graphs $K_f(x,y)$ and $K_g(y,z)$. To each of these paths we associate an edge of the graph $K_h(x,z)$. Obtained as a result of these actions graph $K_h(x,z)$ is shown on the fig. (5) on the right side.

It is possible to obtain the same $K_h(x,z)$ kernel morphism h for the explored example also analytically, making calculations using formula (9). We have: $A=\{a,b,c,d\}$; $B=\{1,2,3\}$; $C=\{5,6,7,8,9\}$;

$$K_f(x,y)=(x^3\vee x^b)y^1\vee x^c(y^2\vee y^3); \tag{11}$$

$$K_g(y,z)=y^1z^6\vee y^2(z^6\vee z^7\vee z^9). \tag{12}$$

Find K_h predicate:

$$\begin{aligned}K_h(x,z)&=\exists y\in\{1,2,3\}\\ &(((x^3\vee x^b)y^1\vee x^c(y^2\vee y^3))\wedge\\ &\wedge(y^1z^6\vee y^2(z^6\vee z^7\vee z^9)))=\\ &=(((x^3\vee x^b)z^6\vee x^c(z^6\vee z^7\vee z^9))\vee x^c\cdot 0=\\ &=(x^3\vee x^b)z^6\vee x^c(z^6\vee z^7\vee z^9).\end{aligned} \tag{13}$$

We got the same $K_h(x,z)$ kernel, which is depicted on the fig.(5), in the form of a bipartite graph.

Let's define reaction reviewed in the above example fg morphism intersections and h morphism equivalent to it. Let, for example, $P(x)=x^a\vee x^c$. First we find the reaction of fg morphism on $P(x)$ predicate. Calculate $Q(y)$ reaction of f morphism on $P(x)$ predicate by the formula (1):

$$\begin{gathered} Q(y)=\exists x\in\{a,b,c,d\}(K_f(x,y)P(x))= \\ =\exists x\in\{a,b,c,d\}(((x^a\vee x^b)y^1\vee x^c(y^2\vee y^3))\wedge \\ \wedge(x^a\vee x^c)=y^1\cdot 1\vee y^1\cdot 0\vee(y^2\vee y^3)\cdot 1\vee 0\cdot 0= \\ =y^1\vee y^2\vee y^3=Q(y). \end{gathered} \quad (14)$$

Calculate $R(z)$ morphism g on a predicate

$$Q(y)=y^1\vee y^2\vee y^3 \quad (15)$$

by the formula (7):

$$\begin{gathered} R(x)=\exists y\in\{1,2,3\}(K_g(y,z)Q(y))=\exists y\in\{1,2,3\} \\ ((y^1z^6\vee y^2(z^6\vee z^7\vee z^9))\wedge(y^1\vee y^2\vee y^3)= \\ z^6\cdot 1\vee(z^6\vee z^7\vee z^9)\cdot 1\vee 0\cdot 1=z^6\vee z^7\vee z^9. \end{gathered} \quad (16)$$

So:

$$R(x)=z^6\vee z^7\vee z^9. \quad (17)$$

Now calculate reaction $R(z)$ morphism h by the formula (7):

$$\begin{gathered} R(z)=\exists x\in\{a,b,c,d\}(K_h(x,z)P(x))= \\ =\exists x\in\{a,b,c,d\}(((x^a\vee x^b)z^6\vee x^c(z^6\vee z^7\vee z^9)\wedge \\ \wedge(x^a\vee x^c))=z^6\cdot 1\vee z^6\vee 0\vee(z^6\vee z^7\vee z^9)\cdot 1\vee 0\cdot 0= \\ =z^6\vee z^7\vee z^9. \end{gathered} \quad (18)$$

We obtained coincidence of fg and h morphisms reactions, demonstrating their identity.

Let's introduce, further, identical morphisms in $Pred$ category. In the role of the kernel of the identity morphism $e_A:A\to A$ in $Pred$ category accept predicate congruencies $D_A(x,y)$ on $A\times A$:

$$D_A(x,y)=\bigvee_{a\in A} x^a y^a. \quad (19)$$

Give an example of the identity morphism in $Pred$ category. Let $A=\{1,2,3\}$. By the formula (19) we find:

$$D_A(x,y)=x^1y^1\vee x^2y^2\vee x^3y^3. \quad (20)$$

There are many identity morphisms in predicate category. They are of the same quantity as, $D_A(x,y)$ congruencies predicates. Each A subset of U universe has its own identity morphism $e_A:A\to A$. For each morphism $f:A\to B$ of $Pred$ category single right identitymorphism e and single left identity morphism e', such, that $fe=f$ and $e'f=f$, along with this $e=e_B$ and $e'=e_A$ exist. Any identity morphism e of the predicate category has the property $ee=e$. Intersection of fg morphisms $f:A\to B$ and $g:B\to C$ in $Pred$ category always exists, along with this $domf=A$ and $codf=C$. The law of associativity for morphisms multiplication in predicate category is performed. Its fairness can be visually demonstrated on bipartite graphs. We attach to the right side the second bipartite graph to the first and then we attach the third graph to the obtained chain of graphs. As a result, we obtain a bipartite graph. We obtain exactly the same graph, if we add to the right side of second graph the third and then add obtained graph chain to the right side of the first graph.

Predicate category meets all requirements, specified to the classical category. In Congruencies (1) and (6-10) we defined fg intersection f morphisms and g in the predicate category only for cases when $f:A\to B$ and $g:B\to C$. The question is, if fg intersection ofmorphisms $f:A\to B$ and $g:C\to D$ exist or not in general, whenB$\neq$C, remained without answer. However, no matter what the answer for this question is, no one can forbid us from making a decision not to form intersections for $f:A\to B$ and $g:C\to D$ morphisms in all those cases, when $B\neq C$. If to do so, then we obtain predicate category that subjects all the requirements, for the classical category. If it turns out, that morphism intersections of such kind are impossible to form, then this decision will be forced. And if it is possible. Then the way for another, alternative definition of the predicate category that does not fit the notion of classical category will open.

Lets' try to answer the raised question. First we take two identical morphism $e_A:A\to A$ and $e_B:B\to B$ ($A\neq B$).From the definition of the classical category it follows that intersection e_Ae_B does not exist. And what about the predicate category? For example. Let $A=\{a,b,c,d\}$, $B=\{b,c,d,e\}$. Try to obtain intersection e_Ae_B morphisms e_A and e_B, graphically, as described above. (Analytical method cannot be applied, because we do not have the morphisms multiplication definitions in mathematical terms for the given case in the predicate category yet). It turns out that the graphical method successfully works and without any complications and results in a well-defined morphisms intersection. On the fig. (6)from the left side bipartite kernel graphs of morphisms e_A and e_B are depicted. We introduce horizontal links between the same points, but now not the same, but different A and B multiplicities, located next to each other, marked with symbols e_A and e_B. Next, we convert a pair of graphs e_A and e_B, which we have connected consecutively, into a single graph equivalent to them, marked with a symbol e_{AB}. In the result we obtain intersection $e_{AB}=e_A e_B$. Graph e_{AB} is depicted on the fig. (6) on the right side. It is necessary to note, that obtained morphism e_{AB} is not identical

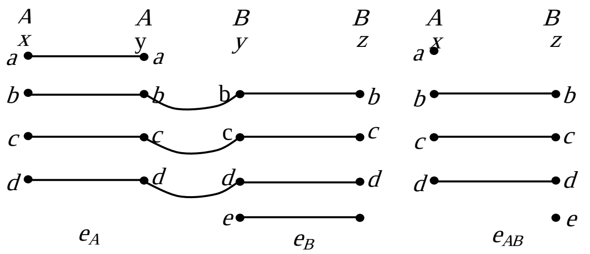

Fig. (6): Morphisms kernel graphs e_A and e_B

It is clear, that it is possible to obtain intersection of any morphisms $f:A\to B$ and $g:C\to D$ in exactly the same way in all those cases, when $B\neq C$. The corresponding example is shown on the fig. (7).

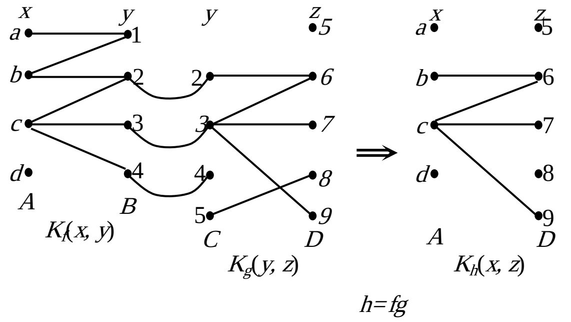

Fig. (7): Morphisms intersection f and g

So, we got a clear answer to the question. There are no limits for forming of any morphism intersections in predicate category. This means, that multiplication in it can be considered everywhere defined. This fact, on the other hand, does not prevent from considering the multiplication in the predicate category to be partial. This means, that two different definitions of a predicate category are possible. One of them uses partial morphisms multiplication; it is covered by the concept of the classical category. Such a predicate category is called classical. The second definition of a predicate category uses an everywhere defined multiplication. It is not covered by the notion of the classical category. Such predicate category is called modified. Each of the variants of the predicate category deserves attention, may be of interest for theoretical development and practical applications. We also suppose that there is a sense to summarize from the concept of a modified predicate category and as a result, as alternative to the general concept of the classical theory to form the general concept of a modified category. There is a sense to develop both the theory of classical categories and the theory of modified categories.

Now we give a mathematical definition of the multiplication of morphisms in a modified predicate category. The same result, as we obtained graphically on arbitrary graphs (fig. (6)), is obtainable also analytically by the following formula:

$$K_h(x,z)=\exists y\in B\cap C(K_f(x,y)\wedge K_g(y,z)). \qquad (21)$$

Here $h=fg$, $f:A\to B$, $g:C\to D$, $A,B,C,D-$ are arbitrarily chosen subsets of the universe U. Predicate $K_f(x,y)$ is set on $A\times B$, predicate $K_g(y,z)-$ on $C\times D$, and predicate $K_h(x,z)$ – on $A\times D$. Define intersection fg morphisms f and g in the modified predicate category for the example shown on the fig. (7). Accept $A=\{a,b,c,d\}$, $B=\{1,2,3,4\}$, $C=\{2,3,4,5\}$, $D=\{5,6,7,8,9\}$. Morphisms kernels f and g are written in the form:

$$K_f(x,y)=x^a y^1\vee x^b(y^1\vee y^2)\vee x^c(y^2\vee y^3\vee y^4); \qquad (22)$$

$$K_g(y,z)=(y^2\vee y^3)\vee z^6\vee y^3(z^7\vee z^9)\vee y^5 z^8. \qquad (23)$$

Kernel $K_h(x,z)$ of intersections $h=fg$ found by the formula (21):

$$\begin{aligned}K_h(x,z)=\exists y\in\{2,3,4\}(x^a y^1\vee x^b(y^1\vee y^2)\vee\\ \vee x^c(y^2\vee y^3\vee y^4))\wedge((y^2\vee y^3)\vee z^6\vee\\ \vee y^3(z^7\vee z^9)\vee y^5 z^8=(x^b\vee x^c)z^6\vee x^c(z^7\vee z^9).\end{aligned} \qquad (24)$$

As we see, the result of calculations by the formula exactly corresponds to the graph $K_h(x,z)$, which was obtained before by the graphic method (fig. (7)).

It is our task to compare the properties of the modified predicate category with the properties of classical category. In classical category multiplication of morphisms is partial, in modified category it is everywhere defined. In both categories for each morphism f unique identity morphism e exists, which satisfies the equation $fe=f$, and the only equal morphism e', which satisfies $e'f=f$ equation. In both categories any identical morphism satisfies the equation $ee=e$. The notion of a classical unit, as it is represented in the monoid, is defined as follows: the unit is one, and it has properties $fe = f$ and $ef = f$ at any f. Both in classical and modified predicate categories the concept of unit is somewhat deformed. In both categories instead of one, appears many units. In the classical category of identity $fe = f$ and $ef = f$ remain unchanged for any unitseand for any morphism f, but only then, when intersections fe and ef exist. In modified category intersections fe and ef exist at any e and f, but congruencies $fe = f$ and $ef = f$ are not always performed. Everything that can be detected in the classical category can be observed also in the modified category. In the modified category, the area of the observation is wider; there you can find something that does not exist in the classical category. In this way, in some sense, modified category is an extension of the classical category.

2.4 General modified category

Now passing from a particular modified predicate category to the general concept of modified category. In the beginning we give a definition to the common notion of modified non-subject category. Let M multiplicity be defined. Its elements, designated by the symbols $f, g, h, ...$, are morphisms. Let, besides, a single-valued and everywhere defined correspondence $fg = h$ with the area of departure $M \times M$ and the area of arrival M be defined. It is called multiplication of morphisms f and g. Morphism h is called morphisms intersection f and g. Morphisms multiplication is associative: at any $f, g, h \in M$ the equation $(fg)h = f(gh)$ holds true. Let, finally, E subset of M subset is defined. Its elements, called single morphisms, are defined by the condition: at any $f, g \in M$ $e, e' \in E$, for which congruencies $fe = f$ and $e'g = g$ are performed exist. Set of morphisms M with units, which meet the above condition, taken together with multiplication of morphisms, satisfying the above conditions is called K objectless modified category. As in the classical category, the identity morphism e, satisfying the condition $fe = f$ is called right identical morphism f. Identical morphism e', satisfying the condition $e'f = f$, is called left identical morphism f. Morphisms e and e' for any f and g are unique.

There are two differences of classical and modified categories-in the definition of multiplication of morphisms and in the definition of identical morphism. For the classical definition of category partial multiplication is used, for modified – everywhere defined. We know, that when using everywhere defined multiplication of morphisms, the standard definition (as for monoid) of a unit cannot be maintained due to the appearance of a contradiction in the definition of a category. And in modified category weakened definition of a unit is used. This is not a real unit, but aquasi-unit. In appearance, the definition of identical morphism remains as if it were the same: still for any morphism f single right e and single left e' identical morphisms, satisfying the conditions $fe = f$ and $e'f = f$ exist. In the definition of the classical category it is not mentioned that such identical morphisms are unique. But it does not mean anything, since this uniqueness follows logically from the definition of the classical category. Both for classical and modified categories this statement is fair for any morphism f. So where are the differences and limitations on the identical morphisms? In order to answer this question, first we will have to reformulate both definitions so that their texts differ only where semantic differences take place, and then the differences in definitions will become clearly visible, and it will be possible to understand which side has increased and what weakened.

We formulate the definition of the classical category. Let M be a set of all morphisms. Morphisms multiplication, which is unambiguous, generally speaking partial, correspondence $fg = h$ with the area of departure $M \times M$ and the area of arrival M is set. Morphisms multiplication is associative: at any $f, g, h \in M$, for which intersections $(fg)h, f(gh) \in M$ exist, equation $(fg)h = f(gh)$ holds true. Let E be a multiplicity of all single morphisms E ($E \subseteq M$). For each $e \in E$ intersection $ee \in M$ exists. At any $f, g \in M$ and any $e, e' \in E$, for which intersections $fe, e'g \in M$ exist, congruencies $fe = f$ and $e'g = g$ are performed. Multiplicity M, on which the morphisms multiplication described above is defined, all single morphisms which meet properties listed above is K objectless classical category.

Formulating the definition of the modified category. Let M be a multiplicity of all morphisms. Morphisms multiplicity, representing single-valued and everywhere defined correspondence $fg = h$ with the area of departure $M \times M$ and the area of arrival M is defined. Multiplication of morphisms is associative: at any $f, g, h \in M$ the equality $(fg)h = f(gh)$ is fair. Let E – be a subset of all single-unit morphisms E ($E \subseteq M$). At any $f, g \in M$ exist $e, e' \in E$, for which congruencies $fe = f$ and $e'g = g$ performed, existed. M Multiplicity, on which the multiplication of morphisms described above is defined, single-unit morphisms of which satisfy properties listed above is called non-objective classic K category.

The result of comparing the definitions of the classical and modified categories is as follows: at transition from a classical category to a modified one morphisms multiplicity is intensified, and units are weakened. This means, that the classical category is not a special case of a modified one, and the modified category is not a special case of a modified one. These algebras are different; none of them logically follows the other. None of them can be obtained from the other as a result of generalization or concretization.

3. Results

3.1Logical Networks

The result of a formal description of any object in the language of the predicate algebra is always a predicate $P(x_1, x_2, \ldots, x_m)$. It must express some kind of definite relation P, which is a multiplication of all objects sets $x_1, x_2, \ldots, x_m$, satisfying equation $P(x_1, x_2, \ldots, x_m) = 1$. Namely this relation that expresses the structure of the described object.

Addressing the result of a formal object description which is taken for example, we see, that it is represented differently, namely by the system of six predicates $R(i, x)$, $E_1(x, x_1)$, $E_2(x, x_2)$, $F_1(s, x_1)$, $F_2(s, x_2)$, $G(s, t)$. How to reconcile this result with just stated statement? And where is that single predicate $P(x_1, x_2, \ldots, x_m)$, which should describe the structure of our object? The answer to these questions is very simple: from predicates R, E_1, E_2, F_1, F_2, G it is possible to form not only a system, but also a conjugation which will be precisely that predicate P, which describes the structure of the object under consideration:

$$R(i, x) \wedge E_1(x, x_1) \wedge E_2(x, x_2) \wedge F_1(s, x_1) \wedge$$
$$F_2(s, x_2) \wedge G(s, t) = P(i, x, x_1, x_2, s, t).$$

In fact, for the described object space S relations R E_1, E_2, F_1, F_2, and G should be simultaneously performed, and this leads to conjugation of correspondent predicates.

To obtain P predicate of $P_1, P_2, \ldots, P_n$ predicates of the system $\{P_1, P_2, \ldots, P_n\}$ is its composition. The inverse transformation of the predicate P to the system $\{P_1, P_2, \ldots, P_n\}$ of $P_1, P_2, \ldots, P_n$ predicates is a decomposition. Composition and decomposition of predicates are inextricably linked to each other. Operation of obtaining a predicate P of predicates $P_1, P_2, \ldots, P_n$ is considered to be a composition in that case and only if there is an inverse operation, allowing restoring the same predicates by the predicate P. Similarly, the predicate transformation operation in the predicate system $P_1, P_2, \ldots, P_n$ can be called decomposition only, when there is an inverse operation, restoring predicate P at predicates $P_1, P_2, \ldots, P_n$.

Getting the predicate P in the form of a conjunction of predicates $P_1, P_2, \ldots, P_n$ is called its conjunctive composition. Decomposition of the predicate P in the conjunction of the same predicates $P_1, P_2, \ldots, P_n$ is called its conjunctive decomposition. An important special case of decomposition is the so-called binary predicate decomposition P, characterized in that each predicate in the system $\{P_1, P_2, \ldots, P_n\}$ has exactly two important arguments. Conducting a formal description of the S space object, we, unwittingly, incidentally produced a binary conjunctive decomposition of the predicate P, which led us to the concept of a logical network. Now we can give a formal definition of a logical network [11, 12], which is meaningfully understood as the device by which man thinks today, and tomorrow a brain-like computer will think in the same way. A logical network is a graphical representation of the result of a multi-place predicate binary conjunctive decomposition.

Each model has its own logical network. Any logical network consists of poles and branches. Each pole of the logical network is assigned its own model object variable, which is called the attribute of this pole. Each pole is denoted by its subject variable. With each pole we connect its domain, that is the area of the attribute change of this pole. Any pole of a logical network at any given time carries some knowledge about the meaning of its attribute. This knowledge is called the state of the pole. It is one of the subsets of the domain of the pole. When pointing the states of all poles of the network at a given time, we will get the network state at the same time. The poles of the network are divided into two classes – external and internal. Each outer pole is connected to only one branch, each internal pole to more than one branch.

Each branch of the logical network is assigned its binary relation of the model, which is called the ratio of this branch. Each branch is denoted by its relationship number. It connects two poles that correspond to those subject variables that are connected by a relation corresponding to a given branch.

Each logical network can be turned into an electronic circuit for the automatic solution of a certain class of tasks, determined by the model for which the network was built.

4. Conclusions

The concepts of classical and modified categories can be generalized; as a result we obtain an algebra that is called a quasicategory. From the classical category it uses weakened multiplication of morphisms, and from the modified one - weakened units. The definition of a quasicategory is given below. Let M be the set of all morphisms. The multiplication of morphisms, which is a single-valued, generally speaking partial correspondence $fg=h$ with the area of departure $M \times M$ and the area of arrival M is defined. Multiplication of morphisms is associative: $\forall f,g,h \in M$, for which intersections exist, the equality $(fg)h = f(gh)$ holds true. Let E be the set of all single morphisms E ($E \subseteq M$). For each $e \in E$ intersection $ee \in M$ exists. At any $f,g \in M$ exist $e,e' \in E$, for which congruences $fe=f$ and $e'g=g$ are performed. M multiplicity, on which the multiplication of morphisms described above is defined, all single morphisms of which satisfy the properties listed above, is called a quasicategory M.

The modified category is located above the monoid, but below the semigroup. It is located in a series of algebras with an everywhere defined principal operation. Classical category, as well as modified, generalizes the concept of a monoid, but in a different way. It is not a special case of a semigroup. Its introduction can be regarded as a departure to the side from the mainstream of the development of mathematics. Introduction of modified category returns the notion of a category in a family of algebras with an everywhere defined principal operation. Now it is possible to develop a theory of modified categories in parallel with the theory of classical categories. Perhaps the theory of modified categories will prove to be an interesting object for theoretical research and an important tool for practical applications. It turns out that the diagrams of the theory of modified categories after their predicate interpretation coincide with the logical networks of brain-like computers. This gives us hope that the theory of modified categories will eventually become the theoretical basis for constructing of brain-like computers of parallel action.

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نظرية شبه الفئات باعتبارها قاعدة نظرية لبناء أجهزة الكمبيوتر تشبة الدماغ

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الخلاصة :

يقدم البحث تعريفًا لمفهوم الفئة المعدلة ويصوغ مشكلة تطوير نظرية للفئات المعدلة التي تفتح الطريق لبناء أجهزة الكمبيوتر ذات الأداء المتوازي ذات الأداء العالي. عند تحليل بنية الفئة ، اكتشفنا فيها بعض الشذوذ ، مما أدى إلى تصحيح مفهوم الفئة الكلاسيكية. بعد تطوير مثل هذا التعديل ، حصلنا على فئة معدلة ، والتي يبدو أنها أفضل لدور نقطة الانطلاق النظري للبناء على أساس إنشاء الحواسيب المتوازية. تمتميز الفئة الكلاسيكية ، وحسب تعريفها الأصلي، نتيجة لذلك ، حصلنا على فئة أصلية واحدة من الحالات الخاصة للفئة الكلاسيكية. يستوفي ذلك جميع المتطلبات المذكورة أعلاه ، سيتم اعتبارها فئة كلاسيكية جديده. من الممكن تطوير نظرية الفئات المعدلة بالتوازي مع نظرية الفئات الكلاسيكية. اثبت نظرية الفئات المعدلة أنها مثيرة للاهتمام للبحث النظري وأداة مهمة للتطبيقات العملية. وضحت الرسوم البيانية لنظرية الفئات المعدلة انها تتوافق مع الشبكات المنطقية لأجهزة الكمبيوتر الشبيهة بالدماغ. وهذا يعطينا الأمل في أن تصبح نظرية الفئات المعدلة في نهاية المطاف الأساس النظري لبناء أجهزة كمبيوتر تشبه الدماغ تعمل بالتوازي.