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Weakly Semi-2-Absorbing Submodules

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ABSTRACT

In this paper we introduce and study the concept weakly semi-2-absorbing submodule as a generalization of 2-absorbing subomdule, and give some of it is basic properties and characterization of this concept.

1. Introduction

Duns in 1980 introduce the concept of semi-prime submodule , where a submodule K of a module X is called semi- prime if $b^2x \in K$, where $b \in R$, $x \in X$,it follows that $bx \in K$ [1]. In [2], they introduced the concept of semi-2-absorbing as a generalization of semi-prime. This led us to introduce the concept weakly semi-2-absorbing submodule as a generalization of 2-absorbing subomdule , where a submodule K is called 2-absorbing if $ijx \in K$, with $i,j \in R$, $x \in X$, it follows that $ix \in K$ or $jx \in K$ or $ij \in [K : X] = \{r \in R : rX \subseteq K\}$ [3].

And a submodule K of X is called semi-2-absorbing if $b^2x \in K$ with $b \in R$, $x \in X$,it follows that $bx \in K$ or $b^2 \in [K : X]$ [2].

In this work, all rings are commutative with identity and all modules are unitary R-modules.

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2. Weakly semi-2-absorbing submodules

In this section, we introduce the concept of weakly semi-2-absorbing submodule, and give some basic properties of this concept.

2.1 DEFENTION

A proper submodule K of X is called weakly semi-2-absorbing if $0 \neq b^2x \in K$ where $b \in R$, $x \in X$, it follows that $bx \in K$ or $b^2x \in [K : X]$. An ideal J of a ring R is called weakly semi-2-absorbing if $b^2r \in J$ with $b , r \in R$, it follows that $br \in J$ or $b^2 \in J$.

2.2 REMARK

Every 2-absorbing submodule us weakly semi-2-absorbing. However , the converse is not true.

Proof

Let K be a 2-absorbing submodule of a module X, and $0 \neq b^2x \in K$, where $b \in R$, $x \in X$. Then either $bx \in K$ or $b^2 \in [K : X]$, because K is 2-absorbing in X. It follows that K is a weakly semi-2-absorbing in X.

For the converse: consider X = Z ⨁ Z, R = Z and K = 10 Z ⨁ (0). It is clear that K is a weakly semi-2-absorbing, but not 2-absorbing in X.

2.3 PROPOSITION

Let E < X, where X is R-module. If E is a weakly semi 2-absorbing in X. Then [E: X] is weakly semi 2-absorbing ideal in R.

Proof

Let b , r ϵ R , with $0 \neq b^2$r ϵ [E : X]. Then (0) $\neq b^2$rx ϵ E for all nonzero x ϵ X. Assume that $b^2 \notin$ [E : X], since E is a weakly semi-2-absorbing in X, then brx ϵ E, it follows that br ϵ [E : X]. Thus [E : X] is a weakly semi-2-absorbing in R.

The converse of proposition (2.3) hold in the class of cyclic modules.

2.4 PROPSITION

Let K be a proper submodule of cyclic module X, and [K: X] is a weakly semi-2-absorbing ideal of R. Then K is a weakly semi-2-absorbing in X.

Proof

Assume that $0 \neq b^2$x ϵ K, where b ϵ R , x ϵ X, and X = R_m, then there exist l ϵ R such that x = lm. Thus $0 \neq b^2$lm ϵ K, it follows that $0 \neq b^2$lϵ [K : m] = [K : X], that is $0 \neq b^2$l [K:X] , it follows that bl ϵ [K : X] or $0 \neq b^2$ ϵ [K : X]. That is blm ϵ K or b^2 ϵ [K : X], hence bx ϵ K or b^2 ϵ [K : X].

2.5 COROLLARY

Let K be a proper submodule of cyclic module X. Then K is a weakly semi-2-absorbing submodule iff [K : X] is a weakly semi-2-absorbing ideal in R.

Proof

Direct

2.6 PROPOSITION

If a proper submodule K of a module X is a weakly semi-2-absorbing in X, then [K : y] is a weakly semi-2-absorbing ideal in R for each a nonzero y ϵ X – K.

Proof

Assume that $0 \neq b^2$ l ϵ [K : y] , where b , l ϵ R , then $0 \neq b^2$ly ϵ K, it follows that $0 \neq b^2$(ly) ϵ K, hence bly ϵ K or $0 \neq b^2$lϵ [K : X] . Thus bl ϵ [K : y] or b^2ϵ [K : y] for each nonzero y ϵ X . thus [K : y] is a weakly semi-2-absorbing in X.

2.7 PROPOSITION

A proper submodule K of a module X is a weakly semi-2-absorbing iff [K : b^2y] = [K : by] for all nonzero y ϵX , $0 \neq$ b ϵ R or b^2ϵ [K : X].

Proof

(→) let $b^2 \notin$ [K : X] , to prove that [K : b^2y] = [K : by] . We have [K : by] ⊆ [K : b^2y] . now , let s be anonzero element in [K : b^2y]. Then $0 \neq b^2$sy ϵ K, it follows that bsy ϵ K (since $b^2 \notin$ [K : X]) , it follows that s ϵ [K : by] . thus [K : b^2y] = [K : by] .

(←) Assume that [K : b^2y] = [K : by] for each a nonzero y in X and a nonzero b in R or b^2ϵ [K : X] , and $0 \neq b^2$y ϵ K , then we have [K : b^2y] = [K : by] or

b^2ϵ [K : X] . If [K : b^2y] = [K : by] , and $0 \neq b^2$y ϵ K , then we have [K : b^2y] = R , so we have [K : by] = R , and hence b y ϵ K . Thus, K is a weakly semi-2-asorbing submodule in X.

2.8 REMARK

1. The intersection of two distinct weakly semi-2-absorbing submodules of a module X need not to

be weakly semi-2-absorbing in X, as the following example: 4Z and 25Z are weakly semi-2-absorbing submodules of the z-module Z. But 4Z ∩ 25Z = 100Z is not weakly semi-2-absorbing in Z, because $0 \neq 5^2 . 4 \in 100Z$, but $5 . 4=20 \notin 100Z$ and $5^2= 25 \notin [100Z : Z] = 100Z$.

2. The intersection of two prime submodule of a module X is A weakly semi-2-absorbing in X.

Proof

Let K_1 , K_2 be two prime submodules of X. Then by [4,theo. 2.3 (1)] , where $K_1 \cap K_2$ is 2-absorbing submodule of X. Hence by Remark (2.2), $K_1 \cap K_2$ is weakly semi-2-absorbing in X.

2.9 PROPOSOTION

If K_1 is a weakly semi-2-absorbing submodule of a module X, then K_1 is a weakly semi-2-absorbing in K_2 where $K_1 \subseteq K_2$ is a submodule in X.

Proof

Assume that $0 \neq b^2 y \in K_1$, where b in R , y in $K_2 \leq X$. That is $0 \neq b^2 y \in K_1$, where b in R , y in X. Since K_1 is a weakly semi-2-absorbing in X, then b y $\in K_1$ or $b^2 \in [K_1 : X]$. If $b^2 \in [K_1 : X]$, then $b^2 X \subseteq K_1 \subseteq K_2$, it follows that

$b^2 K_2 \subseteq b^2 X \subseteq K_1$, hence , it follows that $b^2 \in [K_1 : K_2]$. It gives K_1 is a weakly semi-2-absorbing in K_2 .

2.10 REMARK

A submodule of a weakly semi-2-absorbing is not necessary weakly semi-2-absorbing . For example the submodules 36Z , 9Z of Z-module Z 9Z is weakly semi-2-absorbing in Z and $36Z \subseteq 9Z$ is not weakly semi-2-absorbing in Z , because $0 \neq 3^2 . 4 \in 36Z$, but $3 . 4 = 12 \notin 36Z$ and $3^2= 9 \notin [36Z : Z] = 36Z$.

2.11 PROPOSITION

Let K be a weakly semi-2-absorbing submodule of module X, with Kerf⊆ K, where $f : X \to X^`$ be R-epimorphism. Then f(K) is a weakly semi-2-absorbing submodule of $X^`$.

Proof

Assume that $0 \neq b^2 x^` \in f(K)$ , where $b \in R$ , $x^` \in X^`$ then $f(x) = x^`$ for some nonzero x in X. It follows that $0 \neq b^2 f(x) \in f(K)$, then $b^2 f(x) = f(k)$ for some nonzero $k \in K$. That is $f(b^2 x- k) = 0$, hence $b^2 x- k \in$ Kerf ⊆ K, it follows that

$0 \neq b^2 x \in K$, hence $bx \in K$ or $b^2 \in [K : X]$ because K is a weakly semi-2-absorbing. If $bx \in K$ then $f(bx) \in f(K)$, hence $bf(x) = bx^` \in f(K)$.

If $b^2 \in [K : X]$, then $b^2 X \subseteq K$, so that $b^2 f(X) \subseteq f(K)$, it follows that $b^2 X^` \subseteq f(K)$, then $b^2 \in [f(K) : X^`]$.

2.12 COROLLARY

If K is a weakly semi-2-absorbing submodule of a module X , then $\frac{K}{H}$ is a weakly semi-2-absorbing in $\frac{X}{H}$ for some submodule H of X , with H ⊆ K.

Proof

Since $\pi: X \to \frac{X}{H}$ defined by $\pi(x) = x + H$ for all $x \in X$ is an R-epimorphism with Kerf = H. Hence the proof follows by proposition (2.12).

2.13 PROPOSITION

The inverse image of a weakly semi-2-absorbing submodule ia a weakly semi-2-absorbing.

Proof

Let $f : X \to X`$ be R-epimorphism and F a weakly semi-2-absorbing submodule of X`. Assume that $0 \neq b^2 y f^{-1}(F)$, where b in R x in X , then

$0 \neq b^2 f(y) \in F$, it follows that $b\, f(y) \in F$ or $b^2 \in [F : X`]$, because F is weakly semi-2-absorbing in X`. It follows that $by \in f^{-1}(F)$ or $b^2 \in [F : X`]$.

If $b^2 \in [F : X`]$, then $b^2 X` \subseteq F$, then $b^2 f(X) \subseteq F$, hence $b^2 X \subseteq f^{-1}(F)$. that is $b^2 \in [f^{-1}(F) : X]$.

2.14 PROPOSITION

Let $X = X_1 \oplus X_2$ be a module, where X_1 , X_2 are modules, and K_1 is a proper submodule of X_1 . Then K_1 is a weakly semi-2-absorbing in X_1 iff $K_1 \oplus X_2$ is a weakly semi-2-absorbing in X.

Proof

($\to$) assume that $0 \neq b^2(x_1 , x_2) \in K_1 \oplus X_2$, where $b \in R$,$(x_1 , x_2) \in X_1 \oplus X_2$, where x_1 is a nonzero elements in X_1 and x_2 in X_2,then $0 \neq b^2 x_1 \in K_1$, since K_1 is weakly semi-2-absorbing in X_1, then $bx_1 \in K_1$ or $b^2 \in [K_1 : X_1]$, it follows that $b(x_1 , x_2) \in K_1 \oplus X_2$ or $b^2 \in [K_1 : X_1]$, hence $b^2 \in [K_1 \oplus X_2 : X_1 \oplus X_2]$. Thus $K_1 \oplus X_2$ is weakly semi-2-absorbing in $X_1 \oplus X_2$.

($\leftarrow$) Assume that $0 \neq b^2 x_1 \in K_1$, where b in R and x_1 in X_1, then for any nonzero $x_2 \in X_2$, we have $0 \neq b^2(x_1 , x_2) \in K_1 \oplus X_2$. It follows that

$b(x_1 , x_2) \in K_1 \oplus X_2$ or $b^2 \in [K_1 \oplus X_2 : X_1 \oplus X_2] = [K_1 : X_1]$. Hence $bx_1 \in K_1$ or $b^2 \in [K_1 : X_1]$.

2.15 PROPOSITION

Let $X = X_1 \oplus X_2$ be a module, where X_1 , X_2 are modules, and K_2 is a proper submodule of X_2 . Then K_2 is a weakly semi-2-absorbing in X_2 iff $X_1 \oplus K_2$ is a weakly semi-2-absorbing in X.

Proof

Similar as in proposition (2.14)

2.16 PROPOSITION

Let $Y \oplus Y`$ be an R-module with annY+annY` = R , and T be a weakly semi-2-absorbing submodule of $Y \oplus Y`$, Then either ( i ) $T = T_1 \oplus Y`$ and T_1 is a weakly semi-2-absorbing in Y . (ii) $T = Y \oplus T_2$ and T_2 is a weakly semi-2-absorbing in Y` ( iii ) $T = T_1 \oplus T_2$ and $T_1$ is a weakly semi-2-absorbing in Y and $T_2$ is a weakly semi-2- absorbing in Y`.

Proof

Since annY+annY` = R and T is a submodule of $Y \oplus Y`$, then by [5, theo.2.4] $T = T_1 \oplus T_2$, hence we have (i) T_1 is a submodule of Y and

$T_2 = Y`$ ( ii ) $T_1 = Y$ and $T_2$ is a submodule of Y` (iii) T_1 is a submodule of Y and T_2 is a submodule of Y`. From ( i ) we have $T = T_1 \oplus Y`$ or from (ii) we have $T = Y \oplus T_2$. Thus by proposition (2.15) we have T_1 is a weakly semi-2-absorbing in Y and T_2 is a weakly semi-2- absorbing in Y`. From ( iii ) we prove $T_1$ is a weakly semi-2-absorbing submodule in Y, let $0 \neq b^2 y \in T_1$, where $b \in R$, $y \in Y$ , then $0 \neq b^2(y , 0 ) \in T_1 \oplus T_2 = T$. Thus $0 \neq b^2(y , 0 ) \in T$ , but T is weakly semi-2-absorbing in $Y \oplus Y`$, then either $b(y,0) \in Y$ or $b^2 \in [T : Y \oplus Y`] \subseteq [T_1 : Y]$, it follows that $by \in T_1$ or $b^2 \in [T_1 : Y]$.

In the same way we can get T_2 is a weakly semi-2-absorbing in Y`.

2.17 PROPOSITIOIN

Let $Y \oplus Y`$ be an R-module, and T_1 , T_2 are weakly semi-2-absorbing submodules in Y and

Y` respectively , with [ $T_1$ : Y ] = [$T_2$ : Y`] . Then T = $T_1 \oplus T_2$ is a weakly semi -2-absoring of Y ⊕ Y`.

Proof

Let $0 \neq b^2$(y , y`) ϵ $T_1 \oplus T_2$ for b ϵ R , (y , y`) ϵ Y ⊕ Y`, where y is a nonzero element in Y and y` is a nonzero element in Y`. Then

$0 \neq b^2$y ϵ T_1 and $0 \neq b^2$y` ϵ $T_2$. But $T_1$ and $T_2$ are weakly semi-2-absorbing in Y and Y` respectively , then by ϵ T_1 or b^2ϵ [T_1 : Y] and by` ϵ T_2 or

b^2ϵ [T_2 : Y`] = [$T_1$ : Y] , so by ϵ $T_1$ and by` ϵ T_2 or b^2ϵ [T_1 : Y]. thus

b (y , y`) ϵ $T_1 \oplus T_2$ or $b^2$ϵ [ T : Y ⊕ Y`] .

2.18 PROPOSITION

Let Y be an R-module and T be a weakly semi-2-absorbing submodule of Y . Then $S^{-1}T$ is a weakly semi-2-absorbing submodule of $S^{-1}Y$.

Proof

Let $0 \neq (\bar{x})^2\bar{y}$ ϵ $S^{-1}T$, where $\bar{x}= \frac{x}{s_1}$ ϵ $S^{-1}R$, $\bar{y}= \frac{y}{s_2}$ ϵ $S^{-1}Y$ and x ϵ R , s_1 , s_2ϵ S. Then $0 \neq (\frac{x}{s_1})^2$. $(\frac{y}{s_2})$ ϵ $S^{-1}T$, then $\frac{x^2 y}{{s_1}^2 s_2}$ ϵ $S^{-1}T$. That is $\frac{x^2 y}{t}$ ϵ $S^{-1}T$, where ${s_1}^2 s_2 = t$ ϵ S . Then tere exist t in S with $0 \neq tx^2$ y ϵ T , it follows that x^2t y ϵ T or x^2ϵ [T : Y] . Hence $\frac{xty}{s_1 s_2 t} = \frac{x}{s_1} \frac{y}{s_2}$ ϵ $S^{-1}T$, or $(\frac{x}{s_1})^2$ϵ [$S^{-1}T$: $S^{-1}Y$] .

Thus $S^{-1}T$ is a weakly semi-2-absorbing in $S^{-1}Y$.

2.19 PROPOSITION

Every weakly semi-2-absorbing submodule of an R-module Y is weakly semi-2-absorbing in Y .

Proof

Let T be a weakly semi-2-absorbing submodule of Y. And $0 \neq b^2$yϵ T , b ϵ R , y ϵ Y . That is $0 \neq$ b b y ϵ T. Then either by ϵ T or b^2ϵ [T : Y] .

2.20 PROPOSOTION

Every semi-2-absorbing submodule is weakly semi-2-absorbing

Proof

Clear

2.21 PROPOSITION

Every semi-prime submodule is a weakly semi-2-absorbing

Proof

Since every prime submodule is a semi-prime [6], we have the following corollary .

2.22 COROLLARY

Every prime submodule is a weakly semi-2-absorbing .

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المقاسات الجزئية الشبة المستحوذة من النمط – 2 الضعيفة

هيبة كريم محمدعلي **خلف حسن الحبيب**

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