P- ISSN 1991-8941 E-ISSN 2706-6703
2023,(17), (2):317– 325

Journal of University of Anbar for Pure Science (JUAPS) Open Access

On Free Resolution of Weyl Module and Zero Characteristic Resolution in the Case of Partition (7,5,3)

Ghsoon Rabeea Raheem[1], Haytham Razooki Hassan[2*]

[1)] Ministry of Education, Directorate General of Education in Baghdad, karkh 1, Baghdad, Iraq;
[2)] Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, Iraq;

ARTICLE INFO
Received: 09 / 07 /2023
Accepted: 06 / 08/ 2023
Available online: 19 / 12 / 2023

DOI: 10.37652/juaps.2023.141655.1096

Keywords:
characteristic zero resolution ,
weyl module ,
free resolution ,
characteristic-free resolution ,
resolution.

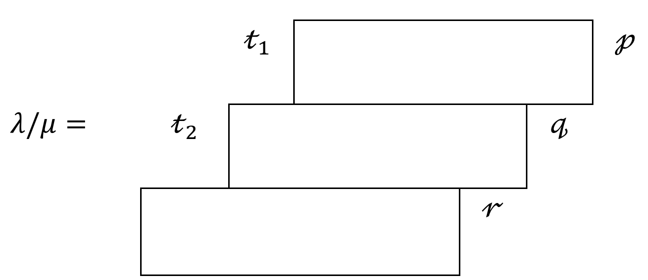
ABSTRACT

The characteristic-free resolution of $\mathcal{K}_{(7,5,3)}$ is applied to the Lascoux resolution of $\mathcal{K}_{(7,5,3)}$F (characteristic zero resolution) in this paper. This research teaches us about the relationship between the resolution of the weyl module $\mathcal{K}_{(7,5,3)}$F in the characteristic-free mode and the Lascoux mode.

In this work, let R be a commutative ring with 1, Ƒ be a free R-module and $Ḍ_{ị}$ be "the divided power algebra" of degree ị. Ṃ is a left-graded module with for Ɯ $=$ $Ƶ^{\text{ĸ}}_{21}$ Ė Å and Ʋ Ė $Ḍ_{ʙ_1} \otimes Ḍ_{ʙ_2}$. We have Ɯ(Ʋ) $=$ $Ƶ^{\text{ĸ}}_{21}$(Ʋ) $=$ $\partial^{\text{ĸ}}_{21}$(Ʋ). where the separator ẋ vanishes between $Ƶ^{(ț)}_{ạƀ}$ and $\partial^{(ț)}_{ạƀ}$. Also by using Capelli identities we prove the sequences and the subsequences of the terms of characteristic zero satisfy the commutative for each diagram in these sequences and subsequences. Finally we get the reduction of the terms of the resolution of the Weyl module for characteristic free to the terms of the resolution of the Weyl module for characteristic 0.

1-INTRODUCTION

Assume that R is a commutative ring with 1 and F is a free R -module. The divided power algebra $\mathcal{D}F = \sum_{i\geq 0} \mathcal{D}_i F$ is known as the graded abelian algebra formed by $\boldsymbol{x^i}$ where $x \in$ F and i are non-negative integers, and $\mathcal{D}_{\text{i}}$F is the divided power algebra of degree i.The partition resolution ($\wp$ +t_1 +t_2, q + t_2, r) is illustrated by the figure below, and in our instance $t_1 = t_2 = 0$.



The authors of [1] and [2] explain Lascoux's description of the characteristic zero skeleton in the resolution of skew-shapes. While the writers in [3], [4], and [5] show the development of the provisions of Lascoux resolution. The authors in [6] demonstrate the terms and precision of the Weyl resolution in the situation of partition (7,5). In addition, in [7], they analyze the terms of the complex of characteristic zero in the case of the partition (7,5,3) and the diagram for the complex of characteristic zero in the case of the partition (7,5,3). The next section surveys the terms of characteristic free resolution of Weyl module in the case of the partition (7,5,3), which is the generalization of the partition (3,3,3), whilst in the previous part we stratified the resolution acquire it in the next section to the Lascoux resolution by itself track of authors in [8] and [9] with Capelli identities as in [10].

2-THE CHARACTERISTIC-FREE RESOLUTION IN THE CASE OF SKEW-PARTITION (7,5,3)

We divided the following formula for the case of partition $(\wp_1, \wp_2, \wp_3)$ to obtain the terms of the resolution for the partition (7,5,3), [15]

*Corresponding: author at: Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, Iraq;
ORCID:https://orcid.org/0000-0000-00000-;Tel:+964
E-mail address: haythamhassaan@uomustansiriyah.edu.iq

$Res([p_1, p_2; 0]) \otimes D_{p_3} \oplus \sum_{\mathcal{J}\geq 0} \mathcal{Z}_{32}^{(\mathcal{J}+1)} y Res([p_1, p_2 + \mathcal{J} + 1; \mathcal{J} + 1]) \otimes D_{p_3-\mathcal{J}-1} \oplus \sum_{\mathcal{J}_1\geq 0, \mathcal{J}_2 \geq \mathcal{J}_1} \mathcal{Z}_{32}^{(\mathcal{J}_2+1)} y \mathcal{Z}_{31}^{(\mathcal{J}_1+1)} z Res([p_1 + \mathcal{J}_1 + 1, p_2 + \mathcal{J}_2 + 1; \mathcal{J}_2 - \mathcal{J}_1]) \otimes D_{p_{3-(\mathcal{J}_1+\mathcal{J}_2+2)}}$

where $\mathcal{Z}_{a'b}^{(m)}$ is the Bar complex

$$0 \to \underbrace{\mathcal{Z}_{a'b} w \mathcal{Z}_{a'b} w \ldots \mathcal{Z}_{a'b}}_{m-times} \to \sum_{k_i\geq 1, \sum k_i = m} \mathcal{Z}_{a'b}^{(k_1)} w \mathcal{Z}_{a'b}^{(k_2)} w \ldots \mathcal{Z}_{a'b}^{(k_{m-1})} \to \cdots \to \mathcal{Z}_{a'b}^{(m)} \to 0$$

Hence the terms of the resolution for the case for the partition (7,5,3) is

$$Res([7,5;0]) \otimes D_3 \oplus \sum_{\mathcal{J}\geq 0} \mathcal{Z}_{32}^{(\mathcal{J}+1)} y Res([7,5 + \mathcal{J} + 1; \mathcal{J} + 1]) \otimes D_{3-(\mathcal{J}+1)} \oplus \sum_{\mathcal{J}_1\geq 0, \mathcal{J}_2\geq \mathcal{J}_1} \mathcal{Z}_{32}^{(\mathcal{J}_2+1)} y \mathcal{Z}_{31}^{(\mathcal{J}_1+1)} z Res([7 + \mathcal{J}_1 + 1, 5 + \mathcal{J}_2 + 1; \mathcal{J}_2 - \mathcal{J}_1]) \otimes D_{p_{3-(\mathcal{J}_1+\mathcal{J}_2+2)}} \quad \ldots(1)$$

So

$$\sum_{\mathcal{J}\geq 0} \mathcal{Z}_{32}^{(\mathcal{J}+1)} y Res([7,5 + \mathcal{J} + 1; \mathcal{J} + 1]) \otimes D_{3-\mathcal{J}-1} = \mathcal{Z}_{32} y Res([7,6;1]) \otimes D_2 \oplus \mathcal{Z}_{32}^{(2)} y\, Res([7,7;2]) \otimes D_1 \oplus \mathcal{Z}_{32}^{(3)} y\, Res([7,8;3]) \otimes D_0$$

And

$$\sum_{\mathcal{J}_1\geq 0, \mathcal{J}_2\geq \mathcal{J}_1} \mathcal{Z}_{32}^{(\mathcal{J}_2+1)} y \mathcal{Z}_{31}^{(\mathcal{J}_1+1)} z Res([7 + \mathcal{J}_1 + 1, 5 + \mathcal{J}_2 + 1; \mathcal{J}_2 - \mathcal{J}_1]) \otimes D_{3-(\mathcal{J}_1+\mathcal{J}_2+2)} = \mathcal{Z}_{32} y \mathcal{Z}_{31} z Res([8,6,0]) \otimes D_1 \oplus \mathcal{Z}_{32}^{(2)} y\, \mathcal{Z}_{31} z Res([9,8;1]) \otimes D_0$$

Where $\mathcal{Z}_{32} y$ *is the Bar complex*

$$0 \longrightarrow \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32} \longrightarrow 0$$

And

$\mathcal{Z}_{32}^{(2)} y$ *is the Bar complex*

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} \longrightarrow 0$$

And

$\mathcal{Z}_{32}^{(3)} y$ *is the Bar complex*

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} y\, \mathcal{Z}_{32} y \oplus \mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} \longrightarrow 0$$

And $\mathcal{Z}_{31} z$ *is the Bar complex*

$$0 \longrightarrow \mathcal{Z}_{31} z \xrightarrow{\partial_z} \mathcal{Z}_{31} \longrightarrow 0$$

where x, y *and* z stand for separator variables, and the boundary map is $\partial_x + \partial_y + \partial_z$.

Assume Bar$(M, \mathcal{A}; S)$ is the free module of bar in the set $S = \{x, y, z\}$,and A is the free associative (non-commutative) algebra created by *by* $\mathcal{Z}_{21}, \mathcal{Z}_{32}$ *and* $\mathcal{Z}_{31}$ and their split power's with the following relations: $\mathcal{Z}_{32}^{(a)} \mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)} \mathcal{Z}_{32}^{(a)}$ *and* $\mathcal{Z}_{21}^{(a)} \mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)} \mathcal{Z}_{21}^{(a)}$

And the module M is the direct sum of the tensor product of the divided power modules $D_{p_1} \otimes D_{p_2} \otimes D_{p_3}$ for appropriate p_1, p_2 *and* p_3 with the action of $\mathcal{Z}_{21}, \mathcal{Z}_{32}$ *and* $\mathcal{Z}_{31}$ and their divided powers.

The characteristics-free resolution's terms (1);

where $'b, 'b_1, 'b_2, 'b_3, 'b_4, 'b_5, 'b_6, 'b_7, \mathcal{U}_1, \mathcal{U}_2 \in \mathbb{Z}^+$ are:

- In dimension zero (χ_0) we have
  - $D_7 \oplus D_5 \oplus D_3$.
- In dimension one (χ_1) we have
  - $\mathcal{Z}_{21}^{('b)} x\, D_{7+'b} \oplus D_{5-'b} \oplus D_3$, $1 \leq 'b \leq 5$.
  - $\mathcal{Z}_{32}^{('b)} y\, D_7 \oplus D_{5+'b} \oplus D_{3-'b}$, $1 \leq 'b \leq 3$.
- In dimension two (χ_2) we have
  - $\mathcal{Z}_{21}^{('b_1)} x\, \mathcal{Z}_{21}^{('b_2)} x\;\; D_{7+|'b|} \oplus D_{5-|'b|} \oplus D_3$, $2 \leq |'b| \leq 5$, where $|'b| = 'b_1 + 'b_2$ and $'b_1 \geq 2$.
  - $\mathcal{Z}_{32}\, y\, \mathcal{Z}_{21}^{('b)} x\;\; D_{7+'b} \oplus D_{6-'b} \oplus D_2$, $2 \leq 'b \leq 6$.
  - $\mathcal{Z}_{32}^{(2)}\, y\, \mathcal{Z}_{21}^{('b)} x\;\; D_{7+'b} \oplus D_{7-'b} \oplus D_1$, $3 \leq 'b \leq 7$.
  - $\mathcal{Z}_{32}^{(3)}\, y\, \mathcal{Z}_{21}^{('b)} x\;\; D_{7+'b} \oplus D_{8-'b} \oplus D_0$, $4 \leq 'b \leq 8$.
  - $\mathcal{Z}_{32}^{('b_1)}\, y\, \mathcal{Z}_{32}^{('b_2)} y\;\; D_7 \oplus D_{5+|'b|} \oplus D_{3-|'b|}$, $2 \leq |'b| \leq 3$. where $|'b| = 'b_1 + 'b_2$.
  - $\mathcal{Z}_{32}^{('b_1)}\, y\, \mathcal{Z}_{31} z\;\; D_8 \oplus D_{5+|'b|} \oplus D_{2-|'b|}$, $1 \leq |'b| \leq 2$. *where* $|'b| = 'b_1 + 'b_2$.
- In dimension three (χ_3) we have
  - $\mathcal{Z}_{21}^{('b_1)} x\, \mathcal{Z}_{21}^{('b_2)} x\;\, \mathcal{Z}_{21}^{('b_3)} x\;\; D_{7+|'b|} \oplus D_{5-|'b|} \oplus D_3$, $3 \leq |'b| \leq 5$,where $|'b| = \sum_i^3 'b_i, 'b_1 \geq 3$
  - $\mathcal{Z}_{32}\, y\, \mathcal{Z}_{21}^{('b_1)} x\, \mathcal{Z}_{21}^{('b_2)} x\;\; D_{7+|'b|} \oplus D_{6-|'b|} \oplus D_2$, $3 \leq |'b| \leq 6$, where $|'b| = 'b_1 + 'b_2, 'b_1 \geq 3$
  - $\mathcal{Z}_{32}^{(2)}\, y\, \mathcal{Z}_{21}^{('b_1)} x\, \mathcal{Z}_{21}^{('b_2)} x\;\; D_{7+|'b|} \oplus D_{7-|'b|} \oplus D_1$, $4 \leq |'b| \leq 7$, *where* $|'b| = 'b_1 + 'b_2, 'b_1 \geq 4$
  - $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{('b_1)} x \mathcal{Z}_{21}^{('b_2)} x D_{7+|'b|} \oplus D_{8-|'b|} \oplus D_0$, $6 \leq |'b| \leq 8$,where $|'b| = 'b_1 + 'b_2, 'b_1 \geq 5$
  - $\mathcal{Z}_{32}\, y\, \mathcal{Z}_{32} y\, \mathcal{Z}_{21}^{('b)} x\;\; D_{7+'b} \oplus D_{7-'b} \oplus D_1$, $3 \leq 'b \leq 7$.
  - $\mathcal{Z}_{32}^{(\mathcal{U}_1)}\, y \mathcal{Z}_{32}^{(\mathcal{U}_2)}\, y\;\, \mathcal{Z}_{21}^{('b)} x\;\; D_{7+'b} \oplus D_{8-'b} \oplus D_0$, where $\mathcal{U}_1 + \mathcal{U}_2 = 3$, $4 \leq 'b \leq 8$.
  - $\mathcal{Z}_{32}\, y\, \mathcal{Z}_{32} y\;\; \mathcal{Z}_{32}\, y\, D_7 \oplus D_8 \oplus D_0$.

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{31}z\ \mathcal{Z}_{21}^{(ƀ)}x\ D_{8+'ƀ} \oplus D_{6-'ƀ} \oplus D_1,\ 1 \le\ 'ƀ \le 6$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}\ y\mathcal{Z}_{31}z\ D_8 \oplus D_7 \oplus D_0$

⋄ $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ)}x\ D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$,$2 \le |\ 'ƀ\ | \le 7$, *where* $|'ƀ| = ƀ_1 \ge 2$

♦ In dimension four (χ_4) we have

⋄ $\mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\quad D_{7+|'ƀ|} \oplus D_{5-|'ƀ|} \oplus D_3$, $4 \le |\ 'ƀ\ | \le 5$, where $|'ƀ| = \sum_i^4 'ƀ_i, 'ƀ_1 \ge 2$

⋄ $\mathcal{Z}_{32}\ y\mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\quad D_{7+|'ƀ|} \oplus D_{6-|'ƀ|} \oplus D_2$, $4 \le |\ 'ƀ\ | \le 6$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 3$.

⋄ $\mathcal{Z}_{32}^{(2)}\ y\ \mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\quad D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_1$, $5 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 4$

⋄ $\mathcal{Z}_{32}^{(3)}\ y\ \mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\quad D_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$, $7 \le |'ƀ| \le 8$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 5$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_1$, $4 \le |'ƀ| \le 7$, where $|'ƀ| = 'ƀ_1 + 'ƀ_2,\ 'ƀ_1 \ge 3$

⋄ $\mathcal{Z}_{32}^{(\mathcal{U}_1)}y\mathcal{Z}_{32}^{(\mathcal{U}_2)}y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \ D_{7+'ƀ} \oplus D_{8\ -'ƀ} \oplus D_0$, where $\mathcal{U}_1 + \mathcal{U}_2 = 3{,}5 \le\ 'ƀ\ \le 8$, $|'ƀ| = 'ƀ_1 + 'ƀ_2,\ 'ƀ_1 \ge 4$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}y\ \ \mathcal{Z}_{32}\ y\mathcal{Z}_{21}^{('ƀ)}x\ \ D_{7+'ƀ} \oplus D_{8-'ƀ} \oplus D_0$ $4 \le\ 'ƀ\ \le 8$

⋄ $\mathcal{Z}_{32}\ y\mathcal{Z}_{31}z\ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}xD_{8+|'ƀ|} \oplus D_{6-|'ƀ|} \oplus D_1$,$2 \le |'ƀ| \le 6$, where $|'ƀ| = 'ƀ_1 + 'ƀ_2$,and $'ƀ_1 \ge 1$

⋄ $\mathcal{Z}_{32}^{(2)}\ y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \ D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $4 \le |'ƀ| \le 7$, where $|'ƀ| = 'ƀ_1 + 'ƀ_2,\ 'ƀ_1 \ge 3$

♦ In dimension five (χ_5) we have

⋄ $\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ \ \mathcal{Z}_{21}^{('ƀ_5)}x\ D_{12} \oplus D_0 \oplus D_3$,

⋄ $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ D_{7+|'ƀ|} \oplus D_{6-|'ƀ|} \oplus D_2$,$6 = |\ 'ƀ\ |$; where $|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 2$.

⋄ $\mathcal{Z}_{32}^{(2)}\ y\ \mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_1$, $6 \le |'ƀ| \le 7$, *where*$|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 3$.

⋄ $\mathcal{Z}_{32}^{(3)}y\ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}xD_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$,$7 \le |'ƀ| \le 8$, where $|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 4$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_1$, $5 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 3$

⋄ $\mathcal{Z}_{32}^{(\mathcal{U}_1)}y\mathcal{Z}_{32}^{(\mathcal{U}_2)}y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\ D_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$, where $\mathcal{U}_1 + \mathcal{U}_2 = 3$, $6 \le\ 'ƀ\ \le 8$ $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 4$.

⋄ $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ D_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$, $5 \le |'ƀ| \le 8$, where $|'ƀ| = 'ƀ_1 + 'ƀ_2$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{31}z\ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \ D_{8+|'ƀ|} \oplus D_{6-|'ƀ|} \oplus D_1$, $3 \le |'ƀ| \le 5$, where $|'ƀ| = \sum_i^3 'ƀ_i$, and $'ƀ_1 \ge 1$

⋄ $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $3 \le |'ƀ| \le 7$, where $|'ƀ| = 'ƀ_1 + 'ƀ_2$

⋄ $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}xD_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$,$4 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 2$

♦ In dimension six (χ_6) we have

⋄ $\mathcal{Z}_{32}\ y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ D_{13} \oplus D_0 \oplus D_0$

⋄ $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$,$7 \le |'ƀ|$,*where* $|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 4$

⋄ $\mathcal{Z}_{32}^{(2)}\ y\ \mathcal{Z}_{21}^{(3)}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ D_{14} \oplus D_0 \oplus D_1$

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\ D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $4 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 2$

⋄ $\mathcal{Z}_{32}^{(\mathcal{U}_1)}\ y\mathcal{Z}_{32}^{(\mathcal{U}_2)}\ y\ \ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\mathcal{Z}_{21}^{('ƀ_4)}x\ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, where $\mathcal{U}_1 + \mathcal{U}_2 = 3$, $7 \le\ 'ƀ\ \le 8$, $|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 4$

⋄ $\mathcal{Z}_{32}\ y\mathcal{Z}_{32}\ y\mathcal{Z}_{32}\ y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ D_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$,$4 \le |'ƀ| \le 8$, where$|'ƀ| = \sum_i^3 'ƀ_i,\ 'ƀ_1 \ge 4$.

⋄ $\mathcal{Z}_{32}^{(2)}\ y\ \mathcal{Z}_{31}z\ \mathcal{Z}_{21}^{('ƀ_1)}x\mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ \ D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $6 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^4 'ƀ_i$,and $'ƀ_1 \ge 2$.

♦ In dimension seven (χ_7) we have

⋄ $\mathcal{Z}_{32}\ y\mathcal{Z}_{32}\ y\ \mathcal{Z}_{21}^{(3)}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ \mathcal{Z}_{21}x\ D_{14} \oplus D_0 \oplus D_1$.

⋄ $\mathcal{Z}_{32}y\ \mathcal{Z}_{31}z\ \mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ \mathcal{Z}_{21}^{('ƀ_5)}x\ D_{8+|'ƀ|} \oplus D_{6-|'ƀ|} \oplus D_1$, $5 \le |'ƀ| \le 6$, where $|'ƀ| = \sum_i^5 'ƀ_i,\ 'ƀ_1 \ge 1$.

⋄ $\mathcal{Z}_{32}^{(\mathcal{U}_1)}\ y\mathcal{Z}_{32}^{(\mathcal{U}_2)}\ y\ \ \mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\mathcal{Z}_{21}^{('ƀ_3)}x\mathcal{Z}_{21}^{('ƀ_4)}x\ \mathcal{Z}_{21}^{('ƀ_5)}x\ D_{7+|'ƀ|} \oplus D_{8-|'ƀ|} \oplus D_0$, where $\mathcal{U}_1 + \mathcal{U}_2 = 3$, $9 \le\ 'ƀ\ \le 10$ $|'ƀ| = \sum_i^5 'ƀ_i,\ 'ƀ_1 \ge 4$.

⋄ $\mathcal{Z}_{32}\ y\ \mathcal{Z}_{32}y\ \ \mathcal{Z}_{32}\ y\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\ \mathcal{Z}_{21}^{('ƀ_4)}x\ D_{7+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $7 \le |'ƀ| \le 8$, where$|'ƀ| = \sum_i^4 'ƀ_i,\ 'ƀ_1 \ge 4$.

⋄ $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{('ƀ_1)}x\ \mathcal{Z}_{21}^{('ƀ_2)}x\ \mathcal{Z}_{21}^{('ƀ_3)}x\mathcal{Z}_{21}^{('ƀ_4)}x$

$Z_{21}^{('ƀ_5)} x D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $5 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^5 'ƀ_i$ and $'ƀ_1 \ge 2$.

♦ In dimension eight (χ_8) we have

⋄ $Z_{32}\, y\, Z_{31} z\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{14} \oplus D_0 \oplus D_1$

⋄ $Z_{32}^{(\mathcal{U}_1)}\, y Z_{32}^{(\mathcal{U}_2)}\, y\, Z_{31} z\, Z_{21}^{('ƀ_1)} x\, Z_{21}^{('ƀ_2)} x Z_{21}^{('ƀ_3)} x\, Z_{21}^{('ƀ_4)} x\, Z_{21}^{('ƀ_5)} x$, $D_{8+|'ƀ|} \oplus D_{7-|'ƀ|} \oplus D_0$, $6 \le |'ƀ| \le 7$, where $|'ƀ| = \sum_i^5 'ƀ_i, 'ƀ_1 \ge 2, and\ \mathcal{U}_1 + \mathcal{U}_2 = 2$

⋄ $Z_{32}\, y Z_{32} y Z_{32}\, y Z_{21}^{(4)} x\, Z_{21} x\, Z_{21} x Z_{21} x\, Z_{21} x\, D_{15} \oplus D_0 \oplus D_0$

3-LASCOUX RESOLUTION OF THE PARTITION (7,5,3)

The Lascoux Resolution of The weyl module associated to the partition (7,5,3):

$$D_9F \otimes D_5F \otimes D_1F \longrightarrow \begin{matrix} D_9F \otimes D_4F \otimes D_2F \\ \oplus \\ D_8F \otimes D_6F \otimes D_1F \end{matrix} \longrightarrow \begin{matrix} D_8F \otimes D_4F \otimes D_3F \\ \oplus \\ D_7F \otimes D_6F \otimes D_2F \end{matrix} \longrightarrow D_7F \otimes D_5F \otimes D_3F$$

where the position of the terms of the complex determined by the length of the permutations to which they correspond.

We currently have the following matrix with the division (7,5,3):

$$\begin{bmatrix} D_7F & D_4F & D_1F \\ D_8F & D_5F & D_2F \\ D_9F & D_6F & D_3F \end{bmatrix}$$

Then the Lascoux complex has the correspondence between it,s terms as follows:

$D_7F \otimes D_5F \otimes D_3F \longleftrightarrow$ identity
$D_8F \otimes D_4F \otimes D_3F \longleftrightarrow (1\ 2)$
$D_7F \otimes D_6F \otimes D_2F \longleftrightarrow (2\ 3)$
$D_9F \otimes D_4F \otimes D_2F \longleftrightarrow (1\ 2\ 3)$
$D_8F \otimes D_6F \otimes D_1F \longleftrightarrow (1\ 3\ 2)$
$D_9F \otimes D_5F \otimes D_1F \longleftrightarrow (1\ 3)$

As in (8)the terms can exhibit as pursue

$x_0 = ₤_0 = \mathfrak{M}_0$
$x_1 = ₤_1 \oplus \mathfrak{M}_1$
$x_2 = ₤_2 \oplus \mathfrak{M}_2$
$x_3 = ₤_3 \oplus \mathfrak{M}_3$
$x_J = \mathfrak{M}_J$ for j = 4,5,…….,11

Where $₤_e$ are the sum of the Lascous terms and $\mathfrak{M}_e$ are the sum of the others .

Now , we acquaint the map $\sigma_1 : \mathfrak{M}_1 \longrightarrow ₤_1$ as pursue

•$Z_{21}^{(2)} x(\nu) \mapsto \frac{1}{2} Z_{21} x \partial_{21}(\nu)$; where $\nu \in D_9F \otimes D_3F \otimes D_3F$

•$Z_{21}^{(3)} x(\nu) \mapsto \frac{1}{3} Z_{21} x\, \partial_{21}^{(2)}(\nu)$; where $\nu \in D_{10}F \otimes D_2F \otimes D_3F$

•$Z_{21}^{(4)} x(\nu) \mapsto \frac{1}{4} Z_{21} x\, \partial_{21}^{(3)}(\nu)$; where $\nu \in D_{11}F \otimes D_1F \otimes D_3F$

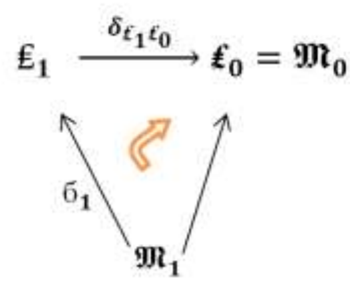
•$Z_{21}^{(5)} x(\nu) \mapsto \frac{1}{5} Z_{21} x\, \partial_{21}^{(4)}(\nu)$; where $\nu \in D_{12}F \otimes D_0F \otimes D_3F$

•$Z_{32}^{(2)} y(\nu) \mapsto \frac{1}{2} Z_{32} y \partial_{32}(\nu)$; where $\nu \in D_7F \otimes D_7F \otimes D_1F$

•$Z_{32}^{(3)} y(\nu) \mapsto \frac{1}{3} Z_{32} y\, \partial_{32}^{(2)}(\nu)$; where $\nu \in D_7F \otimes D_8F \otimes D_0F$

We ought to indicate that the boundary of the map σ_1 implement the identity

$$\delta_{₤_1 ₤_0} \sigma_1 = \delta_{\mathfrak{M}_1 \mathfrak{M}_0} \quad \ldots(2)$$



Where $\delta_{₤_1 ₤_0}$ the component of the boundary of the fat complex which conveys $₤_1$ to $₤_0$. we employ the registration $\delta_{₤_{t+1} ₤_t}$, $\delta_{₤_{t+1} \mathfrak{M}_1}$ etc.

thus we can acquaint $\partial_1 : ₤_1 \to ₤_0$

$$\partial_1 = \delta_{₤_1 ₤_0}$$

It is plainsman to exhibit that ∂_1 implement 2, for example we adopt one of them :

$(\delta_{₤_1 ₤_0} \circ \sigma_1)(Z_{21}^{(3)} x(\nu)) = \delta_{₤_1 ₤_0} \circ \sigma_1 (\frac{1}{3} Z_{21} x\, \partial_{21}^{(2)}(\nu))$
$= \frac{1}{3}(\partial_{21} \partial_{21}^{(2)}(\nu)) = \partial_{21}^{(2)}(\nu) = \delta_{\mathfrak{M}_1 \mathfrak{M}_0}(Z_{21}^{(3)} x(\nu))$

as long as we can acquaint $\partial_2 : ₤_2 \to ₤_1$ by

$\partial_2 = \delta_1 \circ \sigma_1 \delta_{₤_2 \mathfrak{M}_1}$

Proposition 3.1 :

(4) ,(5) and (8) the composition $\partial_1 \circ \partial_2$ equal to zero .

proof :

$(\partial_1 \circ \partial_2)(g) = \delta_{₤_1 ₤_0} \circ (\delta_{₤_2 ₤_1}(g) + \sigma_1 \delta_{₤_2 \mathfrak{M}_1}(g)$

P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
2023,(17), (2):317– 325

$=\delta_{\text{₤}_1\text{₤}_0}\circ\delta_{\text{₤}_2\text{₤}_1}(\mathfrak{g}) + \delta_{\text{₤}_1\text{₤}_0}\circ б_1\delta_{\text{₤}_2\mathfrak{M}_1}(\mathfrak{g})$

But $\delta_{\text{₤}_1\text{₤}_0}\, б_1 = \delta_{\mathfrak{M}_1\mathfrak{M}_0}$.then we attain

$\partial_1 \circ \partial_2(\mathfrak{g}) = \delta_{\text{₤}_1\text{₤}_0}\circ\delta_{\text{₤}_2\text{₤}_1}(\mathfrak{g}) + \delta_{\mathfrak{M}_1\mathfrak{M}_0}\delta_{\text{₤}_2\mathfrak{M}_1}(\mathfrak{g})$ which equal to zero , because of the properties of the boundary mapδ, so we attain $\partial_1\circ\partial_2 = 0$

Now we have to acquaint $\sigma_2:\mathfrak{M}_2 \to \text{₤}_2$ such that

$\delta_{\mathfrak{M}_2\text{₤}_1} + б_1{}^\circ\delta_{\mathfrak{M}_2\mathfrak{M}_1} = (\delta_{\text{₤}_2\text{₤}_1} + б_1{}^\circ\delta_{\text{₤}_2\mathfrak{M}_1})^\circ б_2$ ...(3)

we acquaint this map as pursue :

.$Z_{21}xZ_{21}x(\nu) \mapsto 0$; where $\nu \in D_9F\otimes D_3F\otimes D_3F$

.$Z_{21}^{(2)}xZ_{21}x(\nu) \mapsto 0$; where $\nu \in D_{10}F\otimes D_2F\otimes D_3F$

• $Z_{21}^{(1)}xZ_{21}^{(2)}x(\nu) \mapsto 0$; where $\nu \in D_{10}F\otimes D_2\otimes D_3F$

.$Z_{21}^{(3)}xZ_{21}x(\nu) \mapsto 0$; where $\nu \in D_{11}F\otimes D_1F\otimes D_3F$

•$Z_{21}^{(2)}xZ_{21}^{(2)}x(\nu) \mapsto 0$; where $\nu \in D_{11}F\otimes D_1F\otimes D_3F$

•$Z_{21}xZ_{21}^{(3)}x(\nu) \mapsto 0$; where $\nu \in D_{11}F\otimes D_1F\otimes D_3F$

•$Z_{21}^{(4)}xZ_{21}x(\nu) \mapsto 0$; where $\nu \in D_{12}F\otimes D_0F\otimes D_3F$

•$Z_{21}xZ_{21}^{(4)}x(\nu) \mapsto 0$; where $\nu \in D_{12}F\otimes D_0F\otimes D_3F$

•$Z_{21}^{(3)}xZ_{21}^{(2)}x(\nu) \mapsto 0$; where $\nu \in D_{12}F\otimes D_0F\otimes D_3F$

•$Z_{21}^{(2)}xZ_{21}^{(3)}x(\nu) \mapsto 0$; where $\nu \in D_{12}F\otimes D_0F\otimes D_3F$

•$Z_{32}y\,Z_{21}^{(2)}x(\nu) \mapsto \frac{1}{2}Z_{32}yZ_{21}x\,\partial_{21}(\nu)$;

where $\nu \in D_9F\otimes D_4F\otimes D_2$

•$Z_{32}y\,Z_{21}^{(3)}x(\nu) \mapsto \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}(\nu)$;

where $\nu \in D_{10}F\otimes D_3F\otimes D_2F$

•$Z_{32}yZ_{21}^{(4)}x(\nu) \mapsto \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(2)}(\nu)$;

where $\nu \in D_{11}F\otimes D_3F\otimes D_2F$

.$Z_{32}yZ_{21}^{(5)}x(\nu) \mapsto \frac{1}{10}Z_{32y}Z_{21}^{(2)}x\,\partial_{21}^{(3)}(\nu)$;

where $\nu \in D_{12}F\otimes D_1F\otimes D_2F$

•$Z_{32}yZ_{21}^{(6)}x(\nu) \mapsto \frac{1}{15}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(4)}(\nu)$;

where $\nu \in D_{13}F\otimes D_0F\otimes D_2F$

. $Z_{32}^{(2)}yZ_{21}^{(3)}x(\nu) \mapsto \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}\,\partial_{32}(\nu) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\,\partial_{31}(\nu)$; where $\nu \in D_{10}F\otimes D_4F\otimes D_1F$

• $Z_{32}^{(2)}yZ_{21}^{(4)}x(\nu) \mapsto \frac{1}{12}Z_{32y}Z_{21}^{(2)}x\,\partial_{21}^{(2)}\,\partial_{32}(\nu) + \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\,\partial_{31}(\nu)$;

where $\nu \in D_{11}F\otimes D_3F\otimes D_1F$

• $Z_{32}^{(2)}yZ_{21}^{(5)}x(\nu) \mapsto \frac{1}{30}Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}^{(2)}\,\partial_{31}(\nu)$;

where $\nu \in D_{12}F\otimes D_2F\otimes D_1F$

• $Z_{32}^{(2)}yZ_{21}^{(6)}x(\nu) \mapsto \frac{1}{60}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(3)}\,\partial_{31}(\nu) - \frac{1}{6}Z_{32}yZ_{31}z\,\partial_{21}^{(5)}(\nu)$;

where $\nu \in D_{13}F\otimes D_1F\otimes D_1F$

• $Z_{32}^{(2)}yZ_{21}^{(7)}x(\nu) \mapsto \frac{1}{105}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(4)}\,\partial_{31}(\nu) - \frac{1}{7}Z_{32}yZ_{31}z\,\partial_{21}^{(6)}(\nu)$; where $V \in D_{14}F\otimes D_{0F}\otimes D_1F$

• $Z_{32}^{(3)}yZ_{21}^{(4)}x(\nu) \mapsto \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(2)}(\nu) - \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(2)}\,\partial_{32}^{(2)}(\nu)\frac{1}{3}Z_{32}yZ_{31}z\,\partial_{21}^{(3)}\,\partial_{32}(\nu)$;

where $\nu \in D_{11}F\otimes D_4F\otimes D_0F$

• $Z_{32}^{(3)}yZ_{21}^{(5)}x(\nu) \mapsto \frac{1}{30}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(3)}\,\partial_{32}^{(3)}(\nu) - \frac{1}{6}Z_{32}yZ_{31}z\,\partial_{21}^{(3)}\,\partial_{31}(\nu)$;where $\nu \in D_{12}F\otimes D_3F\otimes D_0F$

• $Z_{32}^{(3)}yZ_{21}^{(7)}x(\nu) \mapsto \frac{1}{210}Z_{32y}Z_{32}^{(3)}y\,Z_{21}^{(6)}x(\nu) - \frac{1}{90}Z_{32}yZ_{21}^{(2)}x\,\partial_{32}^{(2)}\,\partial_{21}^{(4)}(\nu)$

$-\frac{2}{9}Z_{32}yZ_{31}z\,\partial_{32}\,\partial_{21}^{(5)}(\nu)$;where $\nu \in D_{13}F\otimes D_2F\otimes D_0F$

$Z_{21}^{(2)}x\,\partial_{21}^{(4)}\,\partial_{32}\,\partial_{31}(\nu) - \frac{1}{70}$

$Z_{32}yZ_{21}^{(2)}\,\partial_{21}^{(2)}\,\partial_{21}^{(3)}\,\partial_{31}^{(2)}(\nu) - \frac{1}{21}Z_{32}yZ_{31}z\,\partial_{32}\,\partial_{21}^{(6)}(\nu)$;

where $\nu \in D_{14}F\otimes D_1F\otimes D_0F$

• $Z_{32}^{(3)}yZ_{21}^{(8)}x(\nu) \mapsto \frac{1}{45}$

$Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(4)}\,\partial_{31}^{(2)}(\nu) - \frac{1}{9}Z_{32}yZ_{31}z\,\partial_{21}^{(7)}\,\partial_{32}(\nu)$;

where $\nu \in D_{15}F\otimes D_0F\otimes D_0F$

• $Z_{32}yZ_{32}y(\nu) \mapsto 0$; where $\nu \in D_7F\otimes D_7F\otimes D_1F$

• $Z_{32}yZ_{32}^{(2)}y(\nu) \mapsto 0$; where $\nu \in D_7F\otimes D_8F\otimes D_0F$

• $Z_{32}^{(2)}yZ_{32}y(\nu) \mapsto 0$; where $\nu \in D_7F\otimes D_8F\otimes D_0F$

• $Z_{32}^{(2)}yZ_{31}z(\nu) \mapsto \frac{1}{3}Z_{32}y\,Z_{31}z\,\partial_{32}(\nu)$;

where $\nu \in D_7F\otimes D_8F\otimes D_0F$

It is plainsman to exhibit that∂_2 implement 3, for example we adopt one of them :

• $(\delta_{B_2A_1} + б_1\delta_{B_2B_1})(Z_{32}yZ_{21}^{(3)}x(\nu))$;

where $\nu \in D_{10}F\otimes D_3F\otimes D_2F$

$= б_1(Z_{21}^{(3)}x\partial_{32}(\nu)) + б_1(Z_{21}^{(2)}x\partial_{31}(\nu)) - Z_{32}y\partial_{21}^{(3)}(\nu)$

$= \frac{1}{3}(Z_{21}x\partial_{21}^{(2)}\partial_{32}(\nu)) + \frac{1}{2}Z_{21}x\partial_{21}\partial_{31}(\nu) - Z_{32}y\partial_{21}^{(3)}(\nu)$

$= \frac{1}{3}(Z_{21}x\partial_{32}\partial_{21}^{(2)}(\nu)) - \frac{1}{3}Z_{21}x\partial_{21}\partial_{31}(\nu) + \frac{1}{2}Z_{21}x\partial_{21}\partial_{31}(\nu) - Z_{32}y\partial_{21}^{(3)}(\nu)$

$= \frac{1}{3}(Z_{21}x\partial_{32}\partial_{21}^{(2)}(\nu)) + \frac{1}{6}Z_{21}x\partial_{21}\partial_{31}(\nu) - Z_{32}y\partial_{21}^{(3)}(\nu)$

And

$(\delta_{A_2A_1} + б_1\delta_{A_2B_1})(\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}(\nu))$

$= б_1(\frac{1}{3}Z_{21}^{(2)}x\,\partial_{32}\,\partial_{21}(\nu)) + (\frac{1}{3}Z_{21}x\,\partial_{32}\,\partial_{21}(\nu)) - Z_{32}y\,\partial_{21}^{(3)}(\nu))$

$= \frac{1}{6}Z_{21}x\,\partial_{21}\,\partial_{32}\,\partial_{21}(\nu) + \frac{1}{3}Z_{21}x\,\partial_{21}\,\partial_{31}(\nu) - Z_{32}y\,\partial_{21}^{(3)}(\nu)$

P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
2023,(17), (2):317– 325

$=\frac{1}{6}Z_{21}x\,\partial_{32}\,\partial_{21}\,\partial_{21}(\nu)-$
$\frac{1}{6}Z_{21}x\,\partial_{21}\,\partial_{31}(\nu)+\frac{1}{3}Z_{21}x\,\partial_{21}\,\partial_{31}(\nu)-Z_{32}y\,\partial_{21}^{(3)}(\nu)$
$=\frac{1}{3}Z_{21}x\,\partial_{32}\,\partial_{21}^{(2)}(\nu)+\frac{1}{6}Z_{21}x\,\partial_{21}\,\partial_{31}(\nu)-Z_{32}y\,\partial_{21}^{(3)}(\nu)$
As long as we can acquaint $\partial_2:\pounds_2 \to \pounds_1$ by
$\partial_2=\delta_1\circ \sigma_1\delta_{\pounds_2\mathfrak{M}_1}$.

Proposition 3.2:

We have exactness at $\pounds_i$, i=1,2,3.

proof: see (4) ,(5) and (8) .

now by employ σ_2 we can also acquaint $\partial_3:\pounds_3 \to \pounds_2$by
$\partial_3 = \delta_{\pounds_3\pounds_2}+ \sigma_2\circ \delta_{\pounds_3\mathfrak{M}_2}$.

Proposition 3.3 :

The composition $\partial_1 \circ \partial_2$ equal to zero.

proof: The oneself track employ in proposition 3.2.

we requirement to acquaint

$\sigma_3 : \mathfrak{M}_3 \to \pounds_3$ which implement $\delta_{\mathfrak{M}_3\pounds_2}+$
$\sigma_2\circ \delta_{\mathfrak{M}_3\mathfrak{M}_2}=(\delta_{\pounds_3\pounds_2}+ \sigma_2\circ \delta_{\pounds_3\mathfrak{M}_2})\circ\sigma_3$ …(4)

As pursue :

- $Z_{21}xZ_{21}xZ_{21}x(\nu)\mapsto 0$;where v∈ $D_{10}F\otimes D_1F\otimes D_3F$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}x(\nu)\mapsto 0$;where v ∈ $D_{11}F\otimes D_1F\otimes D_3F$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}x(\nu)\mapsto 0$; where v ∈ $D_{11}F\otimes D_1F\otimes D_3F$
- $Z_{21}xZ_{21}xZ_{21}^{(2)}x(\nu)\mapsto 0$;where v ∈ $D_{11}F\otimes D_1F\otimes D_3F$
- $Z_{21}^{(3)}xZ_{21}xZ_{21}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{21}xZ_{21}^{(3)}xZ_{21}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{21}xZ_{21}xZ_{21}^{(3)}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{21}^{(2)}xZ_{21}^{(2)}xZ_{21}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}^{(2)}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}^{(2)}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_0F\otimes D_3F$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(1)}x(\nu)\mapsto 0$; where v ∈ $D_{10}F\otimes D_3F\otimes D_2F$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}x(\nu)\mapsto 0$; where v ∈ $D_{11}F\otimes D_2F\otimes D_2F$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(2)}x(\nu)\mapsto 0$; where v ∈ $D_{11}F\otimes D_2F\otimes D_2F$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}^{(1)}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_1F\otimes D_2F$
- $Z_{32}yZ_{21}^{(1)}xZ_{21}^{(4)}x(\nu)\mapsto 0$; where v ∈ $D_{12}F\otimes D_1F\otimes D_2F$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(2)}x(\nu)\mapsto 0$;where v∈ $D_{12}F\otimes D_1F\otimes D_2F$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(3)}x(\nu)\mapsto 0$; where v∈ $D_{12}F\otimes D_1F\otimes D_2F$
- $Z_{32}yZ_{21}^{(5)}x\ Z_{21}^{(1)}x(\nu)\mapsto 0$; where v∈ $D_{13}F\otimes D_0F\otimes D_2F$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}^{(2)}x(\nu)\mapsto 0$; where v∈ $D_{13}F\otimes D_0F\otimes D_2F$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(3)}x(\nu)\mapsto 0$;where v∈ $D_{13}F\otimes D_0F\otimes D_2F$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(4)}x(\nu)\mapsto 0$;where v∈ $D_{13}F\otimes D_0F\otimes D_2F$
- $Z_{32}yZ_{32}yZ_{21}^{(3)}x(\nu)\mapsto 0$;where v∈ $D_{10}F\otimes D_4F\otimes D_1F$
- $Z_{32}yZ_{32}yZ_{21}^{(4)}x(\nu)\mapsto 0$;where v∈ $D_{11}F\otimes D_3F\otimes D_1F$
- $Z_{32}yZ_{32}yZ_{21}^{(5)}x(\nu)\mapsto -\frac{1}{10}(Z_{32}yZ_{31}ZZ_{21}x\,\partial_{21}^{(3)}(\nu))$
;where v∈ $D_{12}\otimes D_2\otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(6)}x(\nu)\mapsto -\frac{1}{15}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(\nu))$
;where v ∈ $D_{13}\otimes D_1\otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(7)}x(\nu)\mapsto -\frac{1}{21}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}(\nu))$
;where v ∈ $D_{14}\otimes D_0\otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}x(\nu)\mapsto 0$
- $\frac{1}{210}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(\nu))$;where v ∈
$D_{14}\otimes D_1\otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(8)}x(\nu)\mapsto \frac{-1}{42}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(\nu))-$
$\frac{1}{21}(Z_{32}yZ_{31}zZ_{21}x\partial_{32}(\nu))$
;where v∈ $D_{15}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(4)}yZ_{21}x(\nu)\mapsto$
$\frac{1}{6}(Z_{32}yZ_{31}zZ_{21}x\,\partial_{21}^{(2)}\partial_{31}(V))$;where v ∈ $D_{12}\otimes D_3\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(5)}yZ_{21}x(\nu)\mapsto$
$\frac{4}{15}(Z_{32}yZ_{31}zZ_{21}x\,\partial_{32}\partial_{21}^{(4)}(\nu))-$
$\frac{1}{10}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(\nu))$
where v ∈ $D_{13}\otimes D_2\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(4)}xZ_{21}^{(2)}x(\nu)\mapsto \frac{1}{2}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(V))$
; where v∈ $D_{13}\otimes D_2\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(6)}xZ_{21}x(\nu)\mapsto \frac{1}{30}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(\nu))-$
$\frac{1}{6}(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(5)}(\nu))$;where v ∈ $D_{14}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(5)}xZ_{21}^{(2)}x(\nu)\mapsto$
$\frac{1}{6}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(\nu)$
)) $\frac{2}{15}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(\nu)\right)$; where v ∈
$D_{14}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}yZ_{32}^{(4)}xZ_{21}^{(3)}(\nu)\mapsto$
$-\frac{1}{3}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(\nu))-$
$\frac{5}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(\nu)\right)$;
where v ∈ $D_{14}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(6)}xZ_{21}^{(2)}x(\nu)\mapsto$
$-\frac{2}{9}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(\nu)$

) $\frac{7}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)\right)$; *where* $v \in D_{15}\otimes D_0\otimes D_0$

- $Z_{32}^{(3)}yZ_{32}^{(5)}yZ_{21}^{(3)}x(v) \mapsto \frac{8}{9}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)) - \frac{5}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)\right)$;*where* $v \in D_{15}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}yZ_{31}^{(4)}zZ_{21}^{(4)}x(v) \mapsto -\frac{5}{9}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)) - \frac{5}{9}(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(6)}(v))$;*where* $v \in D_{15}\otimes D_0\otimes D_0$
- $Z_{32}yZ_{32}yZ_{32}y(v) \mapsto 0$;*where* $v \in D_7\otimes D_8\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(2)}x(v) \mapsto \frac{1}{6}(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}(v)) + \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{31}(v)\right)$; *where* $v \in D_{10}\otimes D_5\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(3)}x(v) \mapsto \frac{1}{6}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}\partial_{31}(v)) + \frac{1}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$;*where* $v \in D_{11}\otimes D_4\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(4)}x(v) \mapsto \frac{1}{30}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v))$;*where* $v \in D_{12}\otimes D_3\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(5)}x(v) \mapsto -\frac{1}{60}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)) \frac{1}{12}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$; ;*where* $v \in D_{13}\otimes D_2\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(6)}x(v) \mapsto \frac{1}{35}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v))$;*where* $v \in D_{14}\otimes D_1\otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(7)}x(v) \mapsto \frac{1}{18}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)) - \frac{2}{63}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)\right)$;*where* $v \in D_{15}\otimes D_0\otimes D_0$

Again we can exhibit that б$_3$, which is realized above implement 4 , and we adopt one of them as an example:

- $(\delta_{B_3A_2}+б_2\delta_{B_3B_2})\left(Z_{32}^{(3)}yZ_{21}^{(5)}xZ_{21}x(v)\right)$; where $v \in D_{13}\otimes D_2\otimes D_0$

$$=б_2\left(Z_{21}^{(5)}xZ_{21}x\partial_{32}^{(3)}(v) + Z_{21}^{(4)}xZ_{21}x\partial_{32}^{(2)}\partial_{31}(v) + Z_{21}^{(3)}xZ_{21}x\partial_{32}\partial_{31}^{(2)}(v) + Z_{21}^{(2)}xZ_{21}x\partial_{31}^{(3)}(v) - 6Z_{32}^{(3)}yZ_{21}^{(6)}x(v) + Z_{32}^{(3)}yZ_{21}^{(5)}x\partial_{21}(v)\right)$$

$$= \frac{-1}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v) + \frac{4}{15}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) + Z_{32}yZ_{31}z\partial_{21}^{(5)}\partial_{32}(v) - \frac{28}{90}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{7}{90}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}\partial_{31}(v) - \frac{10}{9}Z_{32}yZ_{31}z\partial_{21}^{(5)}\partial_{32}(v) - \frac{2}{9}Z_{32}yZ_{31}z\partial_{21}^{(4)}\partial_{31}(v)$$

$$= \frac{-1}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v) - \frac{2}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{9}Z_{32}yZ_{31}z\,\partial_{21}^{(5)}\partial_{32}(v) - \frac{7}{90}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(3)}\partial_{32}\partial_{31}(v) - \frac{2}{9}Z_{32}yZ_{31}z\,\partial_{21}^{(4)}\partial_{31}(v)$$

And

$$(\delta_{A_3A_2}+б_2\delta_{A_3B_2})\left(\frac{-1}{18}Z_{32}yZ_{31}zZ_{21}x\,\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{45}Z_{32}yZ_{31}zZ_{21}x\,\partial_{21}^{(4)}\partial_{32}(v)\right)$$

$$= б_2\left(\frac{1}{18}Z_{21}xZ_{21}x\,\partial_{32}^{(2)}\partial_{21}^{(3)}\partial_{31}(v)\right) - \frac{1}{18}Z_{32}yZ_{21}^{(2)}x\,\partial_{32}\partial_{21}^{(3)}\partial_{31}(v) + б_2\left(\frac{1}{18}Z_{32}yZ_{32}y\,\partial_{21}^{(2)}\partial_{21}^{(3)}\partial_{31}(v)\right) - \frac{1}{18}Z_{32}yZ_{31}z\,\partial_{21}\partial_{21}^{(3)}\partial_{31}(v) + б_2\left(\frac{1}{45}Z_{21}xZ_{21}x\,\partial_{32}^{(2)}\partial_{21}^{(4)}\partial_{32}(v)\right) - \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\,\partial_{32}\partial_{21}^{(4)}\partial_{32}(v) + б_2\left(\frac{1}{45}Z_{32}yZ_{32}y\,\partial_{21}^{(2)}\partial_{21}^{(4)}\partial_{32}(v)\right) - \frac{1}{45}Z_{32}yZ_{31}z\,\partial_{21}\partial_{21}^{(4)}\partial_{32}(v)$$

$$= \frac{-1}{18}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(3)}\partial_{32}\partial_{31}(v) - \frac{2}{18}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(2)}\partial_{31}^{(2)}(v) - \frac{4}{18}Z_{32}yZ_{31}z\,\partial_{21}^{(4)}\partial_{31}(v) - \frac{2}{45}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\,\partial_{21}^{(3)}\partial_{32}\partial_{31}(v) - \frac{5}{45}Z_{32}yZ_{31}z\,\partial_{21}^{(5)}\partial_{32}(v)$$

$=$
$-\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\,\partial_{21}^{(2)}\,\partial_{31}^{(2)}(\text{v})-$
$\frac{2}{45}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\,\partial_{21}^{(4)}\,\partial_{32}^{(2)}(\text{v})-$
$\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\,\partial_{21}^{(5)}\,\partial_{32}(\text{v})-$
$\frac{7}{90}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\,\partial_{21}^{(3)}\,\partial_{32}\,\partial_{31}(\text{v})-$
$\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\,\partial_{21}^{(4)}\,\partial_{31}(\text{v})$
so from all , we have done above we have complex

$$0 \longrightarrow ₤_3 \xrightarrow{\partial_3} ₤_2 \xrightarrow{\partial_2} ₤_1 \xrightarrow{\partial_1} ₤_0$$

Where ∂_1 is the operation of indicated polarization operators, ∂_2 acquaint as pursue

- $\partial_2(\mathcal{Z}_{21}\text{x}(\text{v})) = \partial_{21}(\text{v})$;Where v∈ $D_8 \otimes D_4 \otimes D_3$
- $\partial_2(\mathcal{Z}_{32}\text{y}(\text{v})) = \partial_{32}(\text{v})$;where v ∈ $D_7 \otimes D_6 \otimes D_2$
- $\partial_2(Z_{32}\text{y}\, Z_{21}^{(2)}\text{x}(\nu)) = \frac{1}{2}Z_{21}\text{x}\,\partial_{21}\partial_{32}(\nu) + Z_{21}\text{x}\,\partial_{31}(V)$ - $Z_{32}\text{y}\,\partial_{21}^{(2)}(\nu)$;

where $\nu \in D_9F \otimes D_4F \otimes D_2$

$\partial_2(Z_{32}\text{y}Z_{31}\text{z}\,(\nu)) = \frac{1}{2}Z_{32}\text{y}\,\partial_{32}\,\partial_{21}(\text{v}) + Z_{21}\text{x}\,\partial_{32}^{(2)}(\text{v})$ - $Z_{32}y\,\partial_{32}^{(2)}(\nu)$;

where $\nu \in D_8F \otimes D_6F \otimes D_1 F$

And the map ∂_3 acquaint as

- $\partial_2(Z_{32}\text{y}\, Z_{31}\text{z}Z_{21}\text{x}(\nu)) =$ $Z_{32}\text{y}Z_{21}^{(2)}x\,\partial_{32}(\text{v}) + Z_{32}\text{y}Z_{31}\text{z}\,\partial_{21}(v)$;

where v∈ $D_9F \otimes D_5F \otimes D_1F$

Proposition 3.4 :

The complex

$$0 \longrightarrow ₤_3 \xrightarrow{\partial_3} ₤_2 \xrightarrow{\partial_2} ₤_1 \xrightarrow{\partial_1} ₤_0 \longrightarrow \mathcal{K}_{(7,5,3)}$$

Is exact.

Proof:

See (4) ,(5) and (8) .

REFERENCES:

[1].David AB, Taylor B.D. Homotopies for Resolution of Skew-Hook Shapes. Adv Appl Math.2003; 30(0196-8858):26-43.

[2]. Haytham RH. The Reduction of Resolution of Weyl Module From Characteristic-Free Resolution in Case (4,4,3). IHJPAS. 2012; 25(3)(2521-3407):341-355.

[3].Giandomenico B, David AB. Threading Homology Through Algebra: Selected Patterns. Clorendon Press. Oxford; 2006.

[4].Haytham RH, Mays MM. Application of the resolution of the characteristic-free resolution of Weyl module to Lascoux resolution in case (6,6,3). Int J Eng Appl Sci. 2016; 3(2394-3661):57-63.

[5]. Najah MM. Resolution of Wely Module in the Case of the Partition (7,6,3). M.Sc. Thesis: College of Science, Al-Mustansiriyah University, Iraq; 2017.

[6].Haytham RH, Niran SJ. Application of Weyl Module in the Case of Two Rows. J.Phys.:Conf.Ser. 2018; 1003 (012051)(1742 – 6588, 742 – 6596): 1-15, doi :10.1088/1742-6596/1003/1/012051.

[7].Haytham RH, Niran SJ. A Complex of Characteristic Zero in the Case of the Partition (8,7,3). Sci. Int. (Lahore). 2018; 30(4)(1013-5316):639-641.

[8].David AB. A Characteristic Free Example of Lascoux Resolution, and Letter Place Methods for Intertwining Numbers. Eur J Comb.2004; 25(0195-6698):1169-1179.

[9].Haytham RH. Application of the characteristicfree resolution of Weyl Module to the Lascoux resolution in the case (3,3,3). Ph.D. Thesis: Universita di Roma "Tor Vergata", Italian (Roma); 2006.

[10].Haytham RH, Najah MM. Complex of Lascoux in the Case of Partition (7,6,3). Aust J Basic Appl Sci. 2016; 10(18)(1991-8178):89-93.

P- ISSN 1991-8941 E-ISSN 2706-6703 2023,(17), (2):317– 325

Journal of University of Anbar for Pure Science (JUAPS)

Open Access

حول التحلل الحر لمقاس وايل وتحلل المميز الصفري في حالة التجزئة*(7,5,3)*

غصون ربيع رحيم [1] ، هيثم رزوقي حسن [2]

[1] وزارة التربية، المديرية العامة لتربية بغداد الكرخ الاولى، بغداد، العراق
[2] قسم الرياضيات، كلية العلوم، الجامعة المستنصرية، بغداد، العراق

الخلاصة

تحلل المميز الحرلـ $\mathcal{K}_{(7,5,3)}$ طبق بتحلل لاسكو لـ $\mathcal{K}_{(7,5,3)}$ (تحلل المميز الصفري) في هذا البحث. هذا البحث يدرس العلاقة بين تحلل مقاس وايل لـ $\mathcal{K}_{(7,5,3)}$ *في حالة المميز الحر وحالة المميز الصفري.*

في هذا العمل، نفرض R حلقة ابدالية ذات 1، Ƒ مقاس R حر و $Ḍ_{į}$ جبر تقسيم القوى من الدرجة i . M مقاس ايسر متدرج مع $Ꮃ = Ƶ_{21}^{ĸ} \dot{\in}$ $Å$ و $Ḍ_{Ƀ_1} \otimes Ḍ_{Ƀ_2} \dot{\in} V$. لدينا $Ꮃ(V) = Ƶ_{21}^{ĸ}(V) = \partial_{21}^{ĸ}(V)$ حيث $\dot{x}$ متغير يقع بين $Ƶ_{ạƀ}^{(ţ)}$ و $\partial_{ạƀ}^{(ţ)}$. ايضا باستخدام احاديات كابيلي نبرهن السلاسل والسلاسل الجزئية لعناصر المميز الصفري تحقق التبادل لكل شكل في هذه السلاسل والسلاسل الجزئية. واخيرا نجد اختزال عناصر تحلل مقاس وايل ذات المميز الحر الى عناصر تحلل مقاس وايل ذات المميز الصفري.