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Characteristic 0 Resolution of the Weyl Module in the Event of Partitioning (4, 4, 3)

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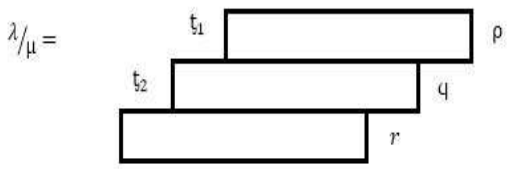
ABSTRACT

Let R be a commutative ring with 1, Ƒ be a free R-module and $Ḍ_ị$ be the divided power algebra of degree ị. Ṃ is a left-graded module with for Ш $= Ƶ^{\kappa}_{21} \dot{\in}$ Å and $V \dot{\in} Ḍ_{ɞ_1} \otimes Ḍ_{ɞ_2}$. We have $Ш(V) = Ƶ^{\kappa}_{21}(V) = \partial^{\kappa}_{21}(V)$. where the separator ẋ vanishes between $Ƶ^{(ţ)}_{ạḃ}$ and $\partial^{(ţ)}_{ạḃ}$. We depend on the definition of the mapping Cone and applying that for the partition (4, 4, 3) to find the resolution of the Weyl module for characteristic 0 in the situation of partition (4, 4, 3) without depends on the resolution of the Weyl module for characteristic free. Also by using Capelli identities we prove the sequences and the subsequences of the terms of characteristic zero satisfy the mapping Cone. Finally by the commutative of each diagram in these sequences and subsequences we get the reduction of the terms of the resolution of the Weyl module for characteristic free to the terms of the resolution of the Weyl module for characteristic 0.

1-INTRODUCTION

Let R be a commutative ring with 1, Ƒ be a free R-module and $Ḍ_ị$ be the divided power algebra of degree ị.

The partition resolution ($ρ + ţ_1 + ţ_2, q + ţ_2, r$) this as depicted in the diagram below:



The publications of [1-3] studied Weyl module resolution for partitions (4, 4, 4), (7, 6, 3) and (8, 7, 3), correspondingly. In [4] Haytham R.H. and Niran S.J demonstrate the scope and precision of the Weyl resolution in the situation of division (8,7) They also demonstrate the concepts of characteristic-free resolution and Lascoux partition resolution in [5] (4, 4, 3). in [6] D.A.Buchsbaum and Rota G.C.

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Describe the Capelli identities as follows:

Let ị, ĵ, ḥ, $l \in p^{+}$,the divided powers of the place polarizations meet the following identities:

1- *If* ḥ ≠ ĵ

$$\dot{\partial}^{(ŗ)}_{ịĵ}\dot{\partial}^{(Ş)}_{ịḥ} = \sum_{\alpha\geq 0}\dot{\partial}^{(Ş-\alpha)}_{ịḥ}\dot{\partial}^{(ŗ-\alpha)}_{ịĵ}\dot{\partial}^{(\alpha)}_{ịḥ}$$

$$\dot{\partial}^{(Ş)}_{ịḥ}\dot{\partial}^{(ŗ)}_{ịĵ} = \sum_{\alpha\geq 0}(-1)^{\alpha}\dot{\partial}^{(Ş-\alpha)}_{ịḥ}\dot{\partial}^{(ŗ-\alpha)}_{ịĵ}\dot{\partial}^{(\alpha)}_{ịḥ}$$

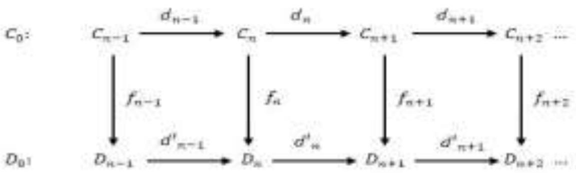
2- *if* ị≠ḥ and ĵ ≠ l then $\dot{\partial}^{(Ş)}_{ịḥ}\dot{\partial}^{(ŗ)}_{ịl} = \dot{\partial}^{(ŗ)}_{ịl}\dot{\partial}^{(Ş)}_{ịḥ}$

In this article, we use mapping Cone to assess the resolution of the Weyl module for characteristic 0 in the situation of partition (4, 4, 3) ,without relying on" the resolution of the Weyl module for characteristic free for a similar partitioning.

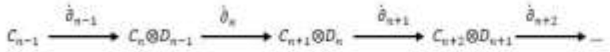
2-CHARACTERISTIC (0) REESOLUTION OF WEYL MODULE WITH MAPPING CONE IN THE EVENT OF (4, 4, 3)

We need to examine the concept of mapping Cone as in [7] prior to analyze the resolution of Weyl module for characteristic 0 in isolation of characteristic-free.

Consider the commute diagram below



If a row sequencing must be exact,
$\dot{\partial}_{n-1}: C_n \otimes D_{n-1} \rightarrow C_{n+1} \otimes D_n$ *is known by*
$(\alpha, b) \mapsto (-dn(\alpha), d'_{n-1}(b)t + f_n(\alpha))$
Such that $\dot{\partial}_{n-1} \circ \dot{\partial}_n = 0$; $\forall n \in Z^+$



Is exact.

Consider Lascoux complex on your partitioning (4, 4, 3) like the diagram below:

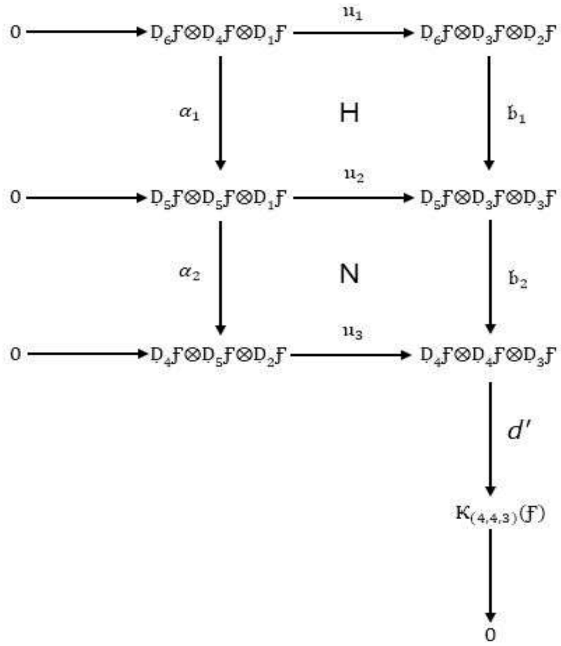


Figure (2, 1)

Where

$u_1(v) = \dot{\partial}_{32}(v)$; $v \in D_6F \otimes D_4F \otimes D_1F$

$\alpha_1(v) = \dot{\partial}_{21}(v)$; $v \in D_6F \otimes D_4F \otimes D_1F$

$u_2(v) = \dot{\partial}_{32}^{(2)}(v)$; $v \in D_5F \otimes D_5F \otimes D_1F$

$u_3(v) = \dot{\partial}_{32}(v)$; $v \in D_4F \otimes D_5F \otimes D_2F$

And

$b_2(v) = \dot{\partial}_{21}(v)$; $v \in D_5F \otimes D_3F \otimes D_3F$

We define b_1 by $b_1(v) = ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32} + \dot{\partial}_{31})(v)$; $v \in D_6 \otimes D_3 \otimes D_2$

Proposition (2.1): The diagram H is commute

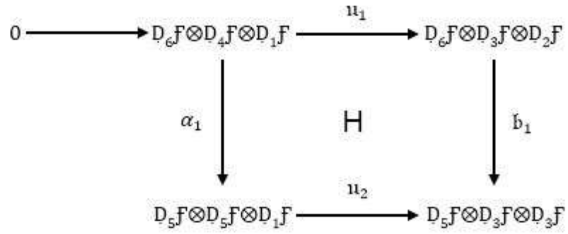
Proof:

$(u_2 \circ \alpha_1)(v) = (b_1 \circ u_1)(v)$
$(\dot{\partial}_{32}^{(2)}\, \dot{\partial}_{21})(v) = (b_1 \circ u_1)(v)$
From Capelli identities, we know that

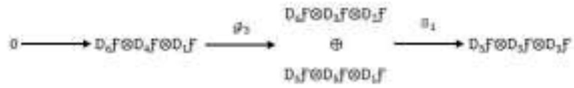
$$\dot{\partial}_{32}^{(2)}\, \dot{\partial}_{21} = \dot{\partial}_{21}\dot{\partial}_{32}^{(2)} + \dot{\partial}_{32}\dot{\partial}_{31} = ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32} + \dot{\partial}_{31})\dot{\partial}_{32}$$

$b_1(v) = ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32} + \dot{\partial}_{31})(v)$; $v \in D_6 \otimes D_3 \otimes D_2$

Consider the following diagram:



Getting the sub complex:
Where



$g_3(\dot{x}) = (\dot{\partial}_{32}(\dot{x}), \dot{\partial}_{21}(\dot{x}))$
And
$s_1(\dot{x}_1, \dot{x}_2) = \dot{\partial}_{32}^{(2)}(\dot{x}_2) + ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32} + \dot{\partial}_{31})(\dot{x}_1)$

Proposition (2.2): In the above diagram we have $Im(g_3) \subseteq \ker(s_1)$

Proof:

$$(s_1 \circ g_3)(b) = s_1(-\dot{\partial}_{32}(b), \dot{\partial}_{21}(b)) = \dot{\partial}_{32}^{(2)}(\dot{\partial}_{21}(b)) + ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32} + \dot{\partial}_{31})(-\dot{\partial}_{32}(b))$$
$$= \left(\dot{\partial}_{32}^{(2)}\dot{\partial}_{21}\right)(b) - ({}^1\!/_2\, \dot{\partial}_{21}\dot{\partial}_{32}\dot{\partial}_{32})(b) - (\dot{\partial}_{32}\dot{\partial}_{31})(b)$$
$$= \left(\dot{\partial}_{32}^{(2)}\dot{\partial}_{21}\right)(b) - \left(\dot{\partial}_{21}\dot{\partial}_{32}^{(2)}\right)(b) - (\dot{\partial}_{32}\dot{\partial}_{31})(b)$$

But from Capelli identities;

$$(s_1 \circ g_3)(b) = \left(\dot{\partial}_{32}^{(2)}\dot{\partial}_{21}\right)(b) + (\dot{\partial}_{32}\dot{\partial}_{31})(b) - \left(\dot{\partial}_{21}\dot{\partial}_{32}^{(2)}\right)(b) - (\dot{\partial}_{32}\dot{\partial}_{31})(b) = 0$$

Consider this diagram:

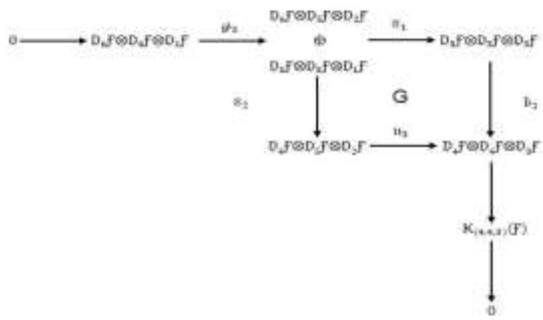
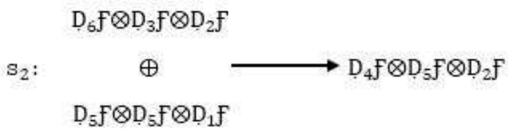


Figure (2, 2)

Now we define



$$\mathrm{s}_2(\mathrm{\dot a},\mathrm{b}) = \dot\partial_{21}^{(2)}(\mathrm{\dot a}) + ({}^{1}/_{2}\,\dot\partial_{32}\dot\partial_{21} - \dot\partial_{31})(\mathrm{b})$$

Proposition (2.3): G in figure (2.2) is commute.
Proof:

$(\mathrm{b}_2\circ \mathrm{s}_1)(\mathrm{\dot a},\mathrm{b}) = (\mathrm{u}_3\circ \mathrm{s}_2)(\mathrm{\dot a},\mathrm{b})$

$(\mathrm{b}_2\circ \mathrm{s}_1)(\mathrm{\dot a},\mathrm{b}) = \mathrm{b}_2\,((\ \dot\partial_{32}^{(2)}(\mathrm{b})\ ) + ({}^{1}/_{2}\,\dot\partial_{21}\dot\partial_{32} + \dot\partial_{31})(\mathrm{\dot a}))$

$= \dot\partial_{21}((\ \dot\partial_{32}^{(2)}(\mathrm{b})\ ) + ({}^{1}/_{2}\,\dot\partial_{21}\dot\partial_{32} + \dot\partial_{31})(\mathrm{\dot a}))$

$= ((\,\dot\partial_{21}\,\dot\partial_{32}^{(2)}(\mathrm{b})\ ) + ({}^{1}/_{2}\,\dot\partial_{21}\dot\partial_{21}\dot\partial_{32} + \dot\partial_{21}\dot\partial_{31})(\mathrm{\dot a}))$

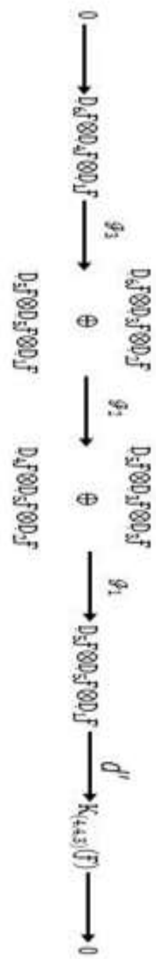
$= (\dot\partial_{32}^{(2)}\dot\partial_{21} - \dot\partial_{32}\dot\partial_{31})\ (\mathrm{b})\ + (\dot\partial_{32}\dot\partial_{21}^{(2)} - \dot\partial_{21}\dot\partial_{31} + \dot\partial_{21}\dot\partial_{31})(\mathrm{\dot a})$

But from Capelli identities ;

$(\mathrm{b}_2\circ \mathrm{s}_1)(\mathrm{\dot a},\mathrm{b}) = \dot\partial_{32}(({}^{1}/_{2}\,\dot\partial_{32}\dot\partial_{21} - \dot\partial_{31})(\mathrm{b}) + \dot\partial_{21}^{(2)}(\mathrm{\dot a}))$

$= (\mathrm{u}_3\circ \mathrm{s}_2)(\mathrm{\dot a},\mathrm{b})$

The following complex:



Where

$g_2(\mathrm{\dot a},\mathrm{b}) = \left(-\mathrm{s}_1(\mathrm{\dot a},\mathrm{b}), \mathrm{s}_2(\mathrm{\dot a},\mathrm{b})\right)$

$= \left(\ -\dot\partial_{32}^{(2)}(\mathrm{b}) - ({}^{1}/_{2}\,\dot\partial_{21}\dot\partial_{32} - \dot\partial_{31})(\mathrm{\dot a}), \dot\partial_{21}^{(2)}(\mathrm{\dot a}) + ({}^{1}/_{2}\,\dot\partial_{32}\dot\partial_{21} - \dot\partial_{31})(\mathrm{b})\ \right)$

$g_1(\mathrm{\dot a},\mathrm{b}) = \dot\partial_{21}(\mathrm{\dot a}) + \dot\partial_{32}(\mathrm{b})$

Proposition (2.4): In the above diagram we have $Im(g_3) \subseteq \ker(g_2)$
Proof:

$(g_2\circ g_3)(\mathrm{\dot a}) = g_2\left(-\dot\partial_{32}(\mathrm{\dot a}), \dot\partial_{21}(\mathrm{\dot a})\right)\quad ;\ \mathrm{\dot a} \in \mathrm{D}_6\otimes\mathrm{D}_4\otimes\mathrm{D}_1$

$= (\left(-\dot\partial_{32}^{(2)}\dot\partial_{21}\right)(\mathrm{\dot a}) + ({}^{1}/_{2}\,\dot\partial_{21}\dot\partial_{32}\dot\partial_{32} - \dot\partial_{31}\dot\partial_{32})(\mathrm{\dot a}), \left(-\dot\partial_{21}^{(2)}\dot\partial_{32}\right)(\mathrm{\dot a}) + ({}^{1}/_{2}\,\dot\partial_{32}\dot\partial_{21}\dot\partial_{21} + \dot\partial_{21}\dot\partial_{31})(\mathrm{\dot a}))$

$$= \Big(\left(-\dot\partial_{32}^{(2)}\dot\partial_{21}\right)(\mathrm{\dot a}) + \left(\dot\partial_{21}\dot\partial_{32}^{(2)} - \dot\partial_{31}\dot\partial_{32}\right)(\mathrm{\dot a}), \left(-\dot\partial_{21}^{(2)}\dot\partial_{32}\right)(\mathrm{\dot a}) + \left(\dot\partial_{32}\dot\partial_{21}^{(2)} + \dot\partial_{21}\dot\partial_{31}\right)(\mathrm{\dot a})\Big)$$

But from Capelli identities;

$(g_2 \circ g_3)(\dot{a}) = \left(\left(-\dot{\partial}_{32}^{(2)}\partial_{21}\right)(\dot{a}) + \left(\partial_{21}\dot{\partial}_{32}^{(2)}\right)(\dot{a}) + \left(\dot{\partial}_{32}\dot{\partial}_{31}\right)(\dot{a}) - \left(\dot{\partial}_{31}\dot{\partial}_{32}\right)(\dot{a}),\right.$
$\left.\left(-\dot{\partial}_{21}^{(2)}\dot{\partial}_{32}\right)(\dot{a}) + \left(\dot{\partial}_{32}\dot{\partial}_{21}^{(2)}\right)(\dot{a}) - \left(\dot{\partial}_{21}\dot{\partial}_{31}\right)(\dot{a}) + \left(\dot{\partial}_{21}\dot{\partial}_{31}\right)(\dot{a})\right)$
$= (0,0)$

Proposition (2.5): In the above diagram we have $Im(g_2) \subseteq \ker(g_1)$

Proof:

$$(g_1 \circ g_2)(\dot{a}, \mathfrak{b}) = g_1\left(-\dot{\partial}_{32}^{(2)}(\mathfrak{b}) - \left({}^1\!/_2\,\dot{\partial}_{21}\dot{\partial}_{32} - \dot{\partial}_{31}\right)(\dot{a}), \dot{\partial}_{21}^{(2)}(\dot{a}) + \left({}^1\!/_2\,\dot{\partial}_{32}\dot{\partial}_{21} - \dot{\partial}_{31}\right)(\mathfrak{b})\right)$$

$= (-\dot{\partial}_{21}\dot{\partial}_{32}^{(2)})(\mathfrak{b}) - ({}^1\!/_2\,\dot{\partial}_{21}\dot{\partial}_{21}\dot{\partial}_{32})(\dot{a}) - (\dot{\partial}_{21}\dot{\partial}_{32})(\dot{a}) + (\dot{\partial}_{32}\,\dot{\partial}_{21}^{(2)})(\dot{a}) + ({}^1\!/_2\,\dot{\partial}_{32}\dot{\partial}_{32}\dot{\partial}_{21})(\mathfrak{b}) - (\dot{\partial}_{32}\dot{\partial}_{31})(\mathfrak{b})$

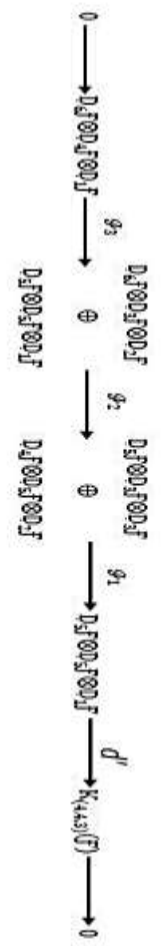
$= (-\dot{\partial}_{21}\dot{\partial}_{32}^{(2)})(\mathfrak{b}) - (\dot{\partial}_{21}^{(2)}\dot{\partial}_{32})(\dot{a}) - (\dot{\partial}_{21}\dot{\partial}_{31})(\dot{a}) + \left(\dot{\partial}_{32}\,\dot{\partial}_{21}^{(2)}\right)(\dot{a}) + (\dot{\partial}_{32}^{(2)}\dot{\partial}_{21})(\mathfrak{b}) - (\dot{\partial}_{32}\dot{\partial}_{31})(\mathfrak{b})$

But from Capelli identities;

$(g_1 \circ g_2)(\dot{a}, \mathfrak{b}) = (-\dot{\partial}_{21}\dot{\partial}_{32}^{(2)})(\mathfrak{b}) - (\dot{\partial}_{21}^{(2)}\dot{\partial}_{32})(\dot{a}) + (\dot{\partial}_{21}\dot{\partial}_{31})(\dot{a}) - (\dot{\partial}_{21}\dot{\partial}_{31})(\dot{a}) + \left(\dot{\partial}_{32}\,\dot{\partial}_{21}^{(2)}\right)(\dot{a}) + (\dot{\partial}_{32}^{(2)}\dot{\partial}_{21})(\mathfrak{b}) + (\dot{\partial}_{32}\dot{\partial}_{31})(\mathfrak{b}) - (\dot{\partial}_{32}\dot{\partial}_{31})(\mathfrak{b})$
$= 0$

Finally, we provide the following theorem, which demonstrates that "the complex of Lascoux" is exact in the situation of partition (4,4,3).

Theorem (2.6): The complex:



Is exact.

Proof :

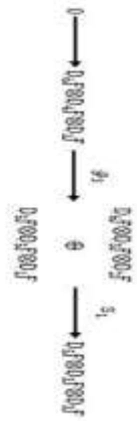
H and N in a figure (2.1) are commutes:

u_1: $D_6F \otimes D_4F \otimes D_1F \to D_6F \otimes D_3F \otimes D_2F$; *where* $u_1(v) = \dot{\partial}_{32}(v)$

And

u_2: $D_5F \otimes D_5F \otimes D_1F \to D_5F \otimes D_3F \otimes D_3F$; *where* $u_2(v) = \dot{\partial}_{32}(v)$

If [6] is injective, we get a commute diagram with an exact row. Proposition (2.2) $(s_1 \circ g_3) = 0$ this signifies that "the mapping Cone" criteria have been met implying that complex:

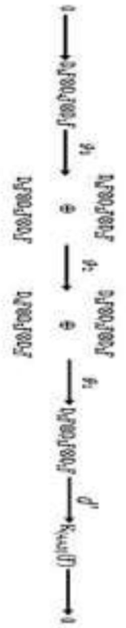


Is exact.

consider figure (2.2), G is commute

$u_3 : D_4F \otimes D_5F \otimes D_2F$
$\rightarrow D_4F \otimes D_4F \otimes D_3F \quad ; where \;\; u_3(v) = \dot{\partial}_{32}(v)$

Injective [6] ,so figure (2.2) commute with exact rows. $(g_2 \circ g_3) = 0$ and $(g_1 \circ g_2) = 0$ " the mapping Cone" conditions are
satisfied, implies that the complex:



Is exact.

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تحلل المميز الصفري لمقاس وايل في حالة التجزئة(3 ,4 ,4)

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الخلاصة

لنفرض R حلقة ابدالية ذات 1، $\mathcal{F}$ مقاس R حر و $\mathcal{D}_i$ جبر تقسيم القوى من الدرجة i . M مقاس ايسر متدرج مع $W = Z_{21}^{\kappa} \in \mathring{A}$ و $\mathcal{D}_{B_2} \otimes \mathcal{D}_{B_1} \in V$. لدينا $W(V) = Z_{21}^{\kappa}(V) = \partial_{21}^{\kappa}(V)$ حيث $\dot{x}$ متغير يقع بين $Z_{ab}^{(t)}$ و $\partial_{ab}^{(t)}$. سنعتمد على تعريف تطبيق كون ونطبقه على التجزئة (3 ,4 ,4) لايجاد تحلل مقاس وايل للميز الصفري للتجزئة (3 ,4 ,4) دون الاعتماد على تحلل مقاس وايل للميز الحرز ايضا باستخدام احاديات كابيلي نبرهن السلاسل والسلاسل الجزئية لعناصر المميز الصفري تحقق تطبيق كون. واخيرا بالتبادل لكل شكل في هذه السلاسل والسلاسل الجزئية نجد اختزال عناصر تحلل مقاس وايل ذات المميز الحر الى عناصر تحلل مقاس وايل ذات المميز الصفري.