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Supra semi preopen sets in supra topological spaces

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ABSTRACT

The purpose of this article is to provide a new set which is supra semi preopen set via supra
topological space $(\mathcal{X}, \mathcal{M}_{\mathcal{X}})$. with the help of some examples and properties. Also, some of their properties have been investigate. With we illustrate the relationships between this concept and supra preopen set (respectively, supra open set).

Also, we define two spaces which are supra semi pre compact and supra semi pre Lindelöf spaces with the relationships between them. At the end of this work, we provide some examples, properties to support this work.

1-Introduction

In 1983**,** Mashhour **[1]** introduced the concept of supra topological space, which is, for any set $\mathcal{X} \neq \emptyset$, and the collection $M_{\mathcal{X}}$ of subset of $\mathcal{X}$ that $\emptyset$ and X belong to $M_{\mathcal{X}}$, also arbitrary union of elements of $M_{\mathcal{X}}$ is an element in $M_{\mathcal{X}}$. The pair $(\mathcal{X}, M_{\mathcal{X}})$is called supra topological space (briefly, su.top.sp), and the elements of $M_{\mathcal{X}}$ are said to be supra open (briefly, su.o) sets and its complements are supra closed (briefly, su.c) sets. Also, he presented the relation between this concept and topological space (for short top. sp.) which is (Every top.sp is su.top.sp). The auther in [2], the concept of supra interior was defined, the supra interior of a subset $\mathcal{A}$ of supra space $(\mathcal{X}, M_{\mathcal{X}})$, which is su.int $(\mathcal{A})$ $=\cup\{U$: $U \subseteq \mathcal{A}$, where $U \in \mathcal{M}_{\mathcal{X}}\}$, and supra closure of $\mathcal{A}$,that is su.cl $(\mathcal{A}) = \cap\{F$: $\mathcal{A} \subseteq F$, where $F^c \in M_{\mathcal{X}}\}$,

After then in 2010, Sayed [3] provided a new concept which is supra preopen (respectively, supra pre closed) set, briefly, su. pr.o (respectively, su. pr. c) set, such that $\mathcal{A}$ is su.pr.o if $\mathcal{A} \subseteq$ su.int (su.cl($\mathcal{A}$)).

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The complement of a su. pr. o set is said to be su.pr.c, The collection of all su. pr. o (respectively, su.pr.c) sets of $\mathcal{X}$ is denoted by su.pr.o $(\mathcal{X})$ (respectively, su. pr.c ($\mathcal{X}$)). And this author defined, The union of all su.pr.o $(\mathcal{X})$ set contained in a subset $\mathcal{A}$ of su.sp. $(\mathcal{X}, M_{\mathcal{X}})$ is su.pr.interior of $\mathcal{A}$ and we denotel by su.int_p $(\mathcal{A})$. Also, The intersection of all su.pr.c $(\mathcal{X})$ sets containing $\mathcal{A}$ is said to be su.pr closure of $\mathcal{A}$ and we denoted by su.cl_p $(\mathcal{A})$. It is well known that top. sp and su. top. sp have been generalized and studied in many ways (see for example, [4], [5], [6]).

In 2020, Al-Shami and others [7], presented the concept of supra pre compact and supra pre Lindelöf spaces with many properties, examples and theorem with the relationships between these two spaces.

In this work, we provide a new concept which is supra semi* preopen (briefly, su. s^* pr.o) set in su.top.sp and the fact that is ″every su.o set is su. s^* pr.o set ″, but the convers is not true, also the relation between su. pr.o set and su. s^* pr.o set, see (Remark 2.2, part(2)) in this research, Finally, we provide anew spaces namely su. s^* pr. compact and su. s^* pr. Lindelöf spaces (briefly, su. s^* pr.com and su. s^* pr. Lin) with some properties, examples for these spaces.

2- supra. s^*pre. open set

At 2002 ,Veera Kumar [8], introduce the concept semi- pre-open set. By the same context via supra topology, we define the following: -

Definition 2. 1: - A subset $\mathcal{A}$ of su. sp $(\mathcal{X}, M_{\mathcal{X}})$ is said to be su. s^*pr. o, if there exists su. pr.o set. $\mathcal{G}$, such that $\mathcal{G} \subseteq \mathcal{A} \subseteq$ su.cl$_p(\mathcal{A})$

Remarks 2. 2: -

1-Every su. o set. is su. s^*pr. o set. but the converse may be not true.

2- Every su. pr.o set is su.s^*pr.o set , but the convers is not true.

Example 2. 3: - In su. indiscrete sp. $(\mathcal{R}, M_{ind})$, the set of all rational numbers is su. s^*pr. o set. but not su.o set.

Example 2. 4: - Let $\mathcal{X} = \{1,2,3\}, \mathcal{M}_{\mathcal{X}} = \{\emptyset, \mathcal{X}, \{1\}, \{3\}, \{1,3\}, \{2,3\}\}$.

Then, $\mathcal{M}_{\mathcal{X}}{}^c = \{\emptyset, \mathcal{X}, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$

the su. pr. o$(\mathcal{X}) = \{\emptyset, X, \{1\}, \{3\}, \{1,3\}, \{2,3]\}$

and su.s^*pr. o $(\mathcal{X}) = \{\emptyset, X, \{1\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2\}\}$,

we note that $\{1,2\}$ is su.s^*pr. o set but not su. pr. o set.

Proposition 2. 5: - The union of any family of su. s^*pr.o $(\mathcal{X})$ is su. s^*pr. o $(\mathcal{X})$

Proof: - Let $\{A_\alpha\}_{\alpha\in\Omega}$ be a family of su su. s^*pr. o set of $\mathcal{X}$, so there exists pr.o subset $\{U_\alpha\}_{\alpha\in\Omega}$ such that $U_\alpha \subseteq A_\alpha \subseteq$cl$_p$ (U_α) , for each $\alpha\in\Omega$.

Also, from (theorem2 [3]) $\bigcup_{\alpha\in\Omega} U_\alpha \subseteq \bigcup_{\alpha\in\Omega} A_\alpha \subseteq \bigcup_{\alpha\in\Omega} su.cl\,(U_\alpha) \subseteq$ su.cl$(\bigcup_{\alpha\in\Omega} U_\alpha)$

Then, $\bigcup_{\alpha\in\Omega}\{A_\alpha\}$ is su. s^*pr. o set.

Corollary 2. 6: - The intersection of any family of su. s^*pr.c $(\mathcal{X})$ is su. s^*pr.c $(\mathcal{X})$.

Proof:- Let $\{F_\alpha\}_{\alpha\in\Omega}$ be su.s^*pr.o sets, by (theorem2 [3]), we get $\bigcup_{\alpha\in\Omega}\{\mathcal{X} - F_\alpha\}$ be su.s^*pr.o set, but $\bigcup_{\alpha\in\Omega}\{\mathcal{X} - F_\alpha\} = \mathcal{X} - \bigcap_{\alpha\in\Omega}\{F_\alpha\}$,

so $\mathcal{X} - (\mathcal{X} - \bigcap_{\alpha\in\Omega}\{F_\alpha\}) = \bigcap_{\alpha\in\Omega}\{F_\alpha\}$ is su. s^*pr.cl $(\mathcal{X})$.

Remark 2. 7: - The intersection of two su. s^*pr.o sets is may be not su. s^*pr.o set.

Example 2. 8: I n (Example2.4) the intersection of two su.s^*pr.o sets $\{1,2\} \cap \{2,3\} = \{2\}$ is not su.s^*pr.o set.

Proof: - Suppose $\mathcal{A}$ is su. pr.o set and $\mathcal{A} \subseteq \mathcal{A} \subseteq$ su.cl$_p(\mathcal{A})$, then it is su.s^*pr.o set .

Proposition 2. 9: - A subset F of su.sp $(\mathcal{X}, M_{\mathcal{X}})$ is su.s^*pr.c set if and only if F =su.s^*pr.cl(F).

Proof: - Let F be su.s^*pr.c set, the smallest intersection of all su.s^*pr.c sets which contain F is equal to F. Therefore, F = su. s^* pr.cl (F).

Conversely, since su.s^*pr.cl(F) always su.s^*pr.c set, by corollary (2.6), Then, F is also su.s^*pr.c set.

Corollary 2. 10: - For a subset F of su.sp $(\mathcal{X}, M_{\mathcal{X}})$, we have su.$s^*$pr.cl (su.$s^*$pr.cl(F)) = su.$s^*$pr.cl(F)

Proof: - Since by corollary (2.6), su.s^*pr.cl(F)) is su.s^*pr.c set, then by (proposition 2.9), we get the result.

Definition 2. 11: - A point $x \in \mathcal{X}$, is said to be su.s^*pr. interior point to a subset $\mathcal{A}$ of su.sp $(\mathcal{X}, M_{\mathcal{X}})$, if there is su.$s^*$pr.o set U with $x \in U \subseteq \mathcal{A}$.

Definition 2. 12: - A point $x \in \mathcal{X}$, is said to be su.s^*pr-adherent point to a subset F of su.sp $(\mathcal{X}, M_{\mathcal{X}})$, if $U \cap$ F $\neq \emptyset$, where U is su.s^*pr.o set containing x.

Proposition 2. 13: -For a subset G of su.sp $(\mathcal{X}, M_{\mathcal{X}})$we have su.$s^*$pr.int$(\mathcal{X} - G) = \mathcal{X} - ($su.$s^*$pr.cl$(G))$.

Proof:- Suppose $x \notin \mathcal{X} - ($su.s^*pr.cl$(G))$,so $x \in$ su.s^*pr.cl$(G))$, then for each su.s^*pr.o set U containing x, we get $U \cap G \neq \emptyset$, that is x is not su.s^*pr. interior point to $\mathcal{X} - G$, that is $x \notin$ su.s^*pr.int$(\mathcal{X} - G)$, then su.s^*pr.int$(\mathcal{X} - G) \subseteq \mathcal{X} - ($su.$s^*$pr.cl$(G)))$

Conversely, Let $x \notin$ su.s^*pr.int$(\mathcal{X} - G)$, then there is su.s^*pr.o set U containing x with $U \nsubseteq \mathcal{X} - G$, then $U \cap G \neq \emptyset$**,** so x is su.s^*pr-adherent point to G, that is $x \in$su.s^*pr.cl(G) .Hence, $x \notin \mathcal{X} - ($su.s^*pr.cl$(G))$, then $\mathcal{X} - ($su.s^*pr.cl$(G)) \subseteq$ su.s^*pr.int(G). Hence, su.s^*pr.int$(\mathcal{X} - G) = \mathcal{X} - ($su.$s^*$pr.cl $(G))$.

Proposition 2. 14: - A subset G of $su.sp$ $(\mathcal{X}, M_{\mathcal{X}})$is su.$s^*$pr.o set if and only if su.s^*pr. int$(G) = G$.

Proof: - Let su.s^*pr. int$(G) = G$, but the union of su.s^*pr.o set is also su.s^*pr.o set, then su.s^*pr. int$(G)$is su.s^*pr.o set, which equal to G, then G is su.s^*pr.o set.

Conversely, suppose G is su.s^*pr.o set, so it is su.s^*pr-neighborhood of each its point, so su.s^*pr. int$(G) = G$.

3- su.s^*pr - Lindelöf space

In this section, we introduce new concepts namely su.s^*pr-compact space (briefly, su.s^*pr-com. sp) and su.s^*pr-Lindelöf space (briefly, su.s^*pr-Lind. sp).

Definition 3. 1: - A su.sp $(\mathcal{X}, M_{\mathcal{X}})$is said to be su.$s^*$pr-com.sp , if for each su.s^*pr.o cover to $\mathcal{X}$ has a finite subcover.

Definition 3. 2: - A su.sp $(\mathcal{X}, M_{\mathcal{X}})$is said to be su.$s^*$pr-Lind. sp, if for each $su.s*po$ cover to $\mathcal{X}$ has a countable subcover.

Remark 3. 3: - Every su.s^*pr-com sp is su.s^*pr-Lind. sp.

Proposition 3. 4: - Every su.s^*pr.c subset of su.s^*pr-Lind. sp is su.s^*pr-Lind.

Proof: - Let K be su.s^*pr.c subset of su.s^*pr-Lind. sp $(\mathcal{X}, M_{\mathcal{X}})$ and $C = \{V_\alpha : \alpha \in \Omega\}$be su.$s^*$pr.o cover to K, that is $K \subseteq \bigcup_{\alpha\in\Omega} V_\alpha$, so $\mathcal{X} \subseteq (\bigcup_{\alpha\in\Omega} V_\alpha) \cup K^c$, where K^c. is su.s^*pr. o set, but $\mathcal{X}$ is su.s^*pr-Lind., then $\mathcal{X} \subseteq (\bigcup_{i=1}^{\infty} V_{\alpha i}) \cup K^c$. Hence, $\mathcal{X} \subseteq \bigcup_{i=1}^{\infty} V_{\alpha i}$, then K is su.s^*pr-Lind.

Proposition 3.5: -The union of two su.s^*pr-Lind. subsets of su.sp $(\mathcal{X}, M_{\mathcal{X}})$ is su.s^*pr-Lind

Proof: - Let H and K be two su.s^*pr-Lind. subsets and $C = \{V_\alpha : \alpha\in\Omega\}$be su.$s^*$pr. o cover to $H \cup K$, that is, $H \cup K \subseteq \bigcup_{\alpha\in\Omega} V_\alpha$, then C is su.$s^*$po cover to H and K, then H$\subseteq \bigcup_{i=1}^{\infty} V_{\alpha i}$ and K$\subseteq \bigcup_{j=1}^{\infty} V_{\alpha j}$, since H and K are su.$s^*$p- Lind. set. So, H∪K $\subseteq \bigcup_{i,j=1}^{\infty} V_{\alpha ij}$.
Hence, H∪K is su. s^*p- Lind. set.

Proposition 3. 6: -A su. sp $(\mathcal{X}, M_{\mathcal{X}})$is su. s^* p- comp. if and only if every collection of su.s^*pc set of $\mathcal{X}$ with FIP has non empty intersection.

Proof:- Let $\mathcal{X}$ be su.s^*p- comp. and$\{G_\alpha : \alpha\in\Omega\}$be su.$s^*$pc subsets of $\mathcal{X}$ with assume that $\bigcap_{\alpha\in\Omega} G_\alpha = \emptyset$, then $(\bigcap_{\alpha\in\Omega} G_\alpha)^c = \emptyset^c$, then $\bigcup_{\alpha\in\Omega} G_\alpha^c = \mathcal{X}$, but for each $\alpha\in\Omega$, we have G_α is su.s^*po set, so $\{G_\alpha^c : \alpha\in\Omega\}$ be a cover of su.s^*po sets of $\mathcal{X}$, which is su.s^*p- comp., then $\mathcal{X} \subseteq \bigcup_{i=1}^{n} G_{\alpha i}^c$, so $\mathcal{X}^c = (\bigcup_{i=1}^{n} G_{\alpha i}^c)^c$. Hence, $\emptyset = \bigcap_{i=1}^{n} G_{\alpha i}$, which is a contradiction with FIP, therefore $\bigcap_{\alpha\in\Omega} G_\alpha \neq \emptyset$.

Proposition 3. 7: - Let $(\mathcal{X}, M_{\mathcal{X}})$ be su. sp, then every subspace of $\mathcal{X}$ is su. s^*p- Lind., if every su.s^*po of $\mathcal{X}$ is su.s^*pr- Lind.

Proof: - Let H be any set in $\mathcal{X}$ and $\{S_\alpha : \alpha\in\Omega\}$be su. s^*po cover to H. So, H$\subseteq \bigcup_{\alpha\in\Omega} S_\alpha$, by proposition 2.5, $\bigcup_{\alpha\in\Omega} S_\alpha$ is su. s^*po, hence it is su. s^*p- Lind. , then H$\subseteq \bigcup_{\alpha\in\Omega} S_\alpha \subseteq \bigcup_{i=1}^{\infty} S_{\alpha i}$. Therefore, H is su. s^*p- Lind..

Conclusion

In this work, we presented a type of a set, which is Supra semi preopen sets in supra topological spaces, with some characteristics, examples, and theories associated with that sets. Whoever reads this work should consider other groups in the topological space such as Supra ω- preopen set in supra topological spaces or Supra θ- preopen set in supra topological spaces with proceed with the same research method to reach what is desired.

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المجاميع المفتوحة من النمط شبه الفوقية في الفضاءات الطبولوجية الفوقية

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الخلاصة:

الغرض من هذا البحث هو تقديم مجموعة جديدة هي المجاميع المفتوحة من النمط شبه الفوقية في الفضاءات الطوبولوجية الفوقيـة بمسـاعدة بعـض الأمثلة والخصائص. كما تم التحقيق في بعض تلك الخصائص. مع توضيح العلاقات بين هذا المفهوم والمجموعة المفتوحة أعلاه (على التوالي، المجموعة المفتوحة من النمط شبة الفوقية). أيضًا، قمنا بتعريف فضاءين هما الفضاء شبه المرصوص الاعلى وفضاء شبه اللاليندلوف الاعلى مع توضيح العلاقـات بينهما. وفي نهاية هذا العمل، قدمنا بعض الأمثلة والخصائص لدعم هذا العمل.

الكلمات المفتاحية: – الفضاء الطوبولوجي الفوقي. المجموعة قبل المفتوحة. المجموعة قبل المفتوحة الفوقية. المجموعة قبل المفتوحة شبه الفوقية. الفضاء المرصوص قبل الفوقية.