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# A Survey on Riemannian Curvature Tensor for Certain Classes of Almost Contact Metric Manifolds

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**ABSTRACT**

This paper surveyed the components of Riemannian curvature tensor over the associated space of G-structure for certain classes of almost contact metric manifolds. These classes under consideration are only twelve and known as cosymplectic manifolds, Sasakian manifolds, Kenmotsu manifolds, $C_9$-manifolds, $C_{12}$-manifolds, normal manifolds of Killing type (CNK-manifold), nearly Kenmotsu manifolds, locally conformal almost cosymplectic manifolds (LCAC-manifolds), quasi-Sasakian manifolds, almost $C(\lambda)$-manifolds, nearly cosymplectic manifolds, and Kenmotsu type manifolds.



## Introduction

The Riemannian curvature tensor (RC-tensor) is one of the interesting fields in the studying differential geometry. The Riemannian manifold of flat RC-tensor is locally isometric to the Euclidean space. Also, RC-tensor win its importance in the gravity theory and general relativity theory because its contraction is the Ricci tensor that a central mathematical tool in Einstein's theory. Based on the above, many authors studied RC-tensor of the manifolds and specially the almost contact metric manifolds that classified by D. Chinea and C. Gonzalez [1]. Especially among them, E. S. Volkova [2] determined the components of RC-tensor of CNK-manifolds. S. V. Umnova [3] established the components of RC-tensor of Kenmotsu manifolds and generalized Kenmotsu manifolds (nearly Kenmotsu manifolds). V. F. Kirichenko and A. R. Rustanov [4] deduced the components of RC-tensor of quasi-Sasakian manifolds. N. N. Dondukova [5]. found the components of RC-tensor of cosymplectic manifolds and Sasakian manifolds. S. V. Kharitonova [6].

Concluded the components of RC-tensor of LCAC-manifolds. V. F. Kirichenko and E. V. Kusova [7] studied the components of RC-tensor of weakly cosymplectic manifolds (nearly cosymplectic manifolds).

So, according to the previous, we summarize these results in this paper and more than ones to have a survey about the RC-tensor of almost contact metric manifolds.

## Preliminaries

**Definition 1**. [8] A topological space $M$ is said to be a smooth manifold of dimension $n$ and denoted by $M^n$, if $M$ is $T_2$ −space, second countable, locally homeomorphic to $\mathbb{R}^n$, and has a smooth structure.

The symbol $\mathcal{X}(M)$ denotes to the module of whole vector fields on $M^n$.

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**Definition 2**. [8] A bilinear map $g : \mathcal{X}(M) \times \mathcal{X}(M) \longrightarrow \mathbb{R}$ is said to be a metric tensor on $M^n$, if $g$ is symmetric and positive definite.

**Definition 3**. [1] If a Riemannian manifold $(M^{2n+1}, g)$ is provided by triple of a structure tensor $(\Phi, \eta, \xi)$, where $\eta, \xi, \Phi$ are tensors over $M$ of types (1, 0), (0, 1), and (1, 1) respectively, such that $\forall\, Z_1, Z_2 \in \mathcal{X}(M)$, the following achieved:

$$\eta(\xi) = 1; \quad \eta \circ \Phi = 0; \quad \Phi(\xi) = 0; \quad id + \Phi^2 = \eta \otimes \xi;$$

$g(\Phi Z_1, \Phi Z_2) + \eta(Z_1)\eta(Z_2) = g(Z_1, Z_2)$, then it is known an almost contact metric (ACM-) manifold and denoted by $(M^{2n+1}, \xi, \eta, \Phi, g)$.

**Definition 4**. [8] A connection on a smooth manifold $M$ is a mapping $\nabla : \mathcal{X}(M) \times \mathcal{X}(M) \to \mathcal{X}(M)$ defined by $\nabla(Z_1, Z_2) = \nabla_{Z_1} Z_2$ and it attains the subsequent properties:

(1) $\boldsymbol{\nabla_{f_1 Z_1 + f_2 Z_2} Z_3 = f_1 \nabla_{Z_1} Z_3 + f_2 \nabla_{Z_2} Z_3}$;

(2) $\boldsymbol{\nabla_{Z_3}(f_1 Z_1 + f_2 Z_2) = f_1 \nabla_{Z_3} Z_1 + f_2 \nabla_{Z_3} Z_2 + Z_3(f_1) Z_1 + Z_3(f_2) Z_2}$,

for all $f_1, f_2 \in C^\infty(M)$ and $Z_1, Z_2, Z_3 \in \mathcal{X}(M)$.

**Lemma 1**. [8] Suppose that $\nabla$ is a connection over $M$ and $U, V \in \mathcal{X}(M)$. If $U = 0$, or $V = 0$ then $\nabla_U V = 0$.

**Definition 5**. [8] A Riemannian connection over the Riemannian manifold $(M, g)$ is a connection $\nabla$ on $M$ that possess the following properties:

(i) $\boldsymbol{\nabla_{Z_1} Z_2 - \nabla_{Z_2} Z_1 = [Z_1, Z_2]}$, where $\boldsymbol{[Z_1, Z_2] = Z_1 \circ Z_2 - Z_2 \circ Z_1}$;

(ii) $\boldsymbol{Z_1\big(g(Z_2, Z_3)\big) = g\big(\nabla_{Z_1} Z_2, Z_3\big) + g\big(Z_2, \nabla_{Z_1} Z_3\big)}$,

for all $Z_1, Z_2, Z_3 \in \mathcal{X}(M)$.

There are several classes of ACM-manifolds $(M^{2n+1}, \xi, \eta, \Phi, g)$. We define some of these classes according to their Riemannian connection as the following:

**Table 1.** Some defining classes

| Classes | Defining conditions |
|---|---|
| **Cosymplectic [9]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 = 0}$ |
| **Nearly cosymplectic [10]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 + \nabla_{Z_2}(\Phi) Z_1 = 0}$ |
| **Kenmotsu [11]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 + g(Z_1, \Phi Z_2)\xi = -\eta(Z_2)\Phi Z_1}$ |
| **Sasakian [12]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 + \eta(Z_2) Z_1 = g(Z_1, Z_2)\xi}$ |
| **C₉ [13]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 = \eta(Z_2)\nabla_{\Phi Z_1}\xi - g(\Phi Z_1, \nabla_{Z_2}\xi)\xi}$ |
| **C₁₂ [14]** | $\boldsymbol{-\eta(Z_1)\{\eta(Z_2)\Phi(\nabla_\xi \xi) + g(\nabla_\xi \xi, \Phi Z_2)\xi\} = \nabla_{Z_1}(\Phi) Z_2}$ |
| **CNK [2]** | **Normal and** $\boldsymbol{\nabla_{Z_1}(\eta) Z_2 + \nabla_{Z_2}(\eta) Z_1 = 0}$ |
| **Nearly Kenmotsu [15]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 + \nabla_{Z_2}(\Phi) Z_1 = -\eta(Z_2)\Phi Z_1 - \eta(Z_1)\Phi Z_2}$ |
| **Kenmotsu type [16]** | $\boldsymbol{\nabla_{Z_1}(\Phi) Z_2 + \eta(Z_2)\Phi Z_1 = \nabla_{\Phi Z_1}(\Phi)\Phi Z_2}$ |

for all $Z_1, Z_2 \in \mathcal{X}(M)$, where $\nabla$ refer to Riemannian connection. Moreover, an ACM-manifold $(M^{2n+1}, \xi, \eta, \Phi, g)$ is called normal if $2N + \xi \otimes d\eta = 0$, where for all $\mathcal{U}, \mathcal{V} \in \mathcal{X}(M)$:

$$N(\mathcal{U}, \mathcal{V}) = \frac{1}{4}\big([\Phi\mathcal{U}, \Phi\mathcal{V}] + \Phi^2[\mathcal{U}, \mathcal{V}] - \Phi[\Phi\mathcal{U}, \mathcal{V}] - \Phi[\mathcal{U}, \Phi\mathcal{V}]\big),$$

is the Nijenhuis tensor of the structure tensor $\Phi$ (see [2]).

**Definition 6**. [6] An ACM-manifold $(M^{2n+1}, \xi, \eta, \Phi, g)$ is bearing an almost cosymplectic manifold if $d\Omega = 0$ and $d\eta = 0$, where

$$\Omega(Z_1, Z_2) = g(Z_1, \Phi Z_2) \quad \text{and}$$
$$2d\eta(Z_1, Z_2) = \nabla_{Z_1}(\eta) Z_2 - \nabla_{Z_2}(\eta) Z_1;$$
$$3d\Omega(Z_1, Z_2, Z_3) = \nabla_{Z_1}(\Omega)(Z_2, Z_3) + \nabla_{Z_2}(\Omega)(Z_3, Z_1) + \nabla_{Z_3}(\Omega)(Z_1, Z_2), \text{ for all } Z_1, Z_2, Z_3 \in \mathcal{X}(M).$$

**Definition 7**. [6] An ACM-manifold $(M^{2n+1}, \xi, \eta, \Phi, g)$ is bearing a LCAC-manifold if the ACM-manifold $\big(M^{2n+1}, \tilde{\xi}, \tilde{\eta}, \Phi, \tilde{g}\big)$ is an almost cosymplectic manifold, where $\tilde{\xi} = exp(\alpha)\xi$; $\tilde{\eta} = exp(-\alpha)\eta$; $\tilde{g} = exp(-2\alpha) g$, and $\alpha$ is a smooth function.

**Definition 8**. [17] An ACM-manifold $M^{2n+1}$ is known as quasi-Sasakian manifold if $d\Omega = 0$ and $M$ is normal.

**Definition 9**. [8] An RC-tensor of type (3, 1) on a Riemannian manifold $(N, g)$ is a tensor $R : \mathcal{X}(N) \times$

$\mathcal{X}(N)\times\mathcal{X}(N)\to\mathcal{X}(N)$ that defined by $R(Z_1,Z_2)Z_3=\left([\nabla_{Z_1},\nabla_{Z_2}]-\nabla_{[Z_1,Z_2]}\right)Z_3$, for all $Z_1,Z_2,Z_3\in\mathcal{X}(N)$, where $\nabla$ is Riemannian connection over $N$. Furthermore, the RC-tensor $R$ of type (4, 0) is given by the formula $R(Z_1,Z_2,Z_3,Z_4)=g(R(Z_3,Z_4)Z_2,Z_1)$, with $Z_4\in\mathcal{X}(N)$.

**Definition 10**. [18] The associated space of G-structure for an ACM-manifold $(M^{2n+1},\xi,\eta,\Phi,g)$ is a set of all A-frame $(x;\Upsilon_0=\xi,\Upsilon_1,\dots,\Upsilon_n,\Upsilon_{\hat{1}},\dots,\Upsilon_{\hat{n}})$, where $x\in M$, $\Upsilon_a=\frac{1}{\sqrt{2}}\left(\chi_a-\sqrt{-1}\Phi(\chi_a)\right)$, $\Upsilon_{\hat{a}}=\frac{1}{\sqrt{2}}\left(\chi_a+\sqrt{-1}\Phi(\chi_a)\right)$, $a=1,2,\dots,n$, $\hat{a}=a+n$ and $\{\chi_0=\xi,\chi_1,\dots,\chi_n,\chi_{\hat{1}},\dots,\chi_{\hat{n}}\}$ is a basis of $\mathcal{X}(M)$ which satisfies $g(\chi_p,\chi_q)=\delta_{pq}$, for all $p,q=0,1,\dots,2n$.

**Lemma 2**. [19] Suppose that $(M^{2n+1},\xi,\eta,\Phi,g)$ is an ACM-manifold and $R$ its RC-tensor of kind (4, 0) with components $R_{pqrs}$ on the associated space of G-structure. Then the subsequent relations are satisfied:

(1) $\boldsymbol{R_{pqrs}=-R_{qprs}}$;
(2) $\boldsymbol{R_{pqrs}=-R_{pqsr}}$;
(3) $\boldsymbol{R_{pqrs}=R_{rspq}}$;
(4) $\boldsymbol{R_{pqrs}+R_{psqr}+R_{prsq}=0}$,

where $p,q,r,s=0,1,\dots,2n$.

**Definition 11**. [20] An ACM-manifold $(M^{2n+1},\xi,\eta,\Phi,g)$ is said to be an almost $C(\lambda)$-manifold if its RC-tensor $R$ fulfill the following identity:

$$\begin{aligned}&g(R(Z_3,Z_4)Z_2,Z_1)\\&=g(R(\Phi Z_3,\Phi Z_4)Z_2,Z_1)\\&-\lambda\{g(Z_1,Z_4)g(Z_2,Z_3)\\&-g(Z_1,Z_3)g(Z_2,Z_4)\\&-g(Z_1,\Phi Z_4)g(Z_2,\Phi Z_3)\\&+g(Z_1,\Phi Z_3)g(Z_2,\Phi Z_4)\},\end{aligned}$$

where $Z_1,Z_2,Z_3,Z_4\in\mathcal{X}(M)$, and $\lambda\in\mathbb{R}$. Moreover, a normal almost $C(\lambda)$-manifold is said to be $C(\lambda)$-manifold.

## The Components of Riemannian Curvature Tensor on the Associated Space of G-Structure

In this section, we review the ingredients of RC-tensor on the associated space of G-structure for certain classes of ACM-manifolds.

**Theorem 1**. [5] The components of RC-tensor of cosymplectic manifolds are given by: $R_{\hat{a}bc\hat{d}}=A^{ad}_{bc}$, and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc}=A^{ad}_{[bc]}=0$.

**Theorem 2**. [5] The components of RC-tensor of Sasakian manifolds are given by:

1. $R_{\hat{a}bc\hat{d}}=A^{ad}_{bc}-2\delta^a_b\delta^d_c-\delta^a_c\delta^d_b$;
2. $R_{\hat{a}\hat{b}cd}=\delta^{ab}_{cd}=\delta^a_c\delta^b_d-\delta^a_d\delta^b_c$;
3. $R_{\hat{a}0b0}=\delta^a_b$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc}=A^{ad}_{[bc]}=0$.

**Theorem 3**. [5] The components of RC-tensor of Kenmotsu manifolds are given by:

1. $R_{\hat{a}bc\hat{d}}=A^{ad}_{bc}-\delta^a_c\delta^d_b$;
2. $R_{\hat{a}\hat{b}cd}=\delta^{ab}_{dc}=\delta^a_d\delta^b_c-\delta^a_c\delta^b_d$;
3. $R_{\hat{a}0b0}=-\delta^a_b$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc}=A^{ad}_{[bc]}=0$.

**Theorem 4**. [20] The components of RC-tensor of almost $C(\lambda)$-manifolds are given by:

1. $R_{\hat{a}\hat{b}cd}=\lambda\delta^{ab}_{cd}$;
2. $R_{\hat{a}0b0}=\lambda\delta^a_b$;
3. $R_{\hat{a}bc\hat{d}}-R_{\hat{a}cb\hat{d}}=-\lambda\delta^{ad}_{bc}$,

and the other components are vanish or given by Lemma 2, or their conjugates.

**Theorem 5**. [13] The components of RC-tensor of $C_9$-manifolds are given by:

1. $R_{0a\hat{b}0}=F_{ac}F^{cb}$;
2. $R_{0ab0}=-F_{ab0}$;
3. $R_{0ab\hat{c}}=-F_{ab}{}^{c}$;
4. $R_{\hat{a}bc\hat{d}}=A^{ad}_{bc}+F^{ad}F_{bc}$;
5. $R_{abcd}=-2F_{a[c}F_{|b|d]}$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc}=A^{ad}_{[bc]}=0$, and $F^{ab}$, $F_{ab}$, $F_{ab0}$, $F_{ab}{}^{c}$ are components of Kirichenko's fifth structure tensor $F$ (see [16]) and their covariant derivatives respectively.

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**Theorem 6**. [14] The components of RC-tensor of $C_{12}$-manifolds are given by:

1. $C^a_b - C^a C_b = R_{\hat{a}0b0}$;
2. $C^{ab} - C^a C^b = R_{\hat{a}0\hat{b}0}$;
3. $A^{ad}_{bc} = R_{\hat{a}bc\hat{d}}$,

and the disappeared components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc} = A^{ad}_{[bc]} = 0$, and $C^a$, $C_a$, $C^{ab}$, $C^a_b$ are components of Kirichenko's sixth structure tensor $G$ (see [16]) and their covariant derivatives respectively.

**Theorem 7**. [16] The components of RC-tensor over the manifolds of Kenmotsu type are seemed as follow:

1. $-\delta^a_c = R_{\hat{a}0c0}$;
2. $2A^a_{bcd} = R_{\hat{a}bcd}$;
3. $A^{ad}_{bc} - \delta^a_c\,\delta^d_b - B^{ah}{}_c\,B_{bh}{}^d = R_{\hat{a}bc\hat{d}}$;
4. $2\left(-\delta^a_{[c}\,\delta^b_{d]} + B^{ab}{}_{[cd]}\right) = R_{\hat{a}\hat{b}cd}$ ;
5. $-B^{ab}{}_h\,B^{hd}{}_c + B^{ab}{}_c{}^d = R_{\hat{a}\hat{b}c\hat{d}}$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ and $A^a_{bcd}$ are suitable smooth functions and $B^{ab}{}_c$, $B_{ab}{}^c$, $B^{ab}{}_{cd}$, $B^{ab}{}_c{}^d$ are components of Kirichenko's first structure tensor $B$ (see [16]) and their covariant derivatives respectively.

**Theorem 8**. [6] The components of RC-tensor of LCAC-manifolds are appeared as follow:

1. $2\left(A^a_{bcd} - \alpha_0 B_{b[d}\delta^a_{c]} + 4\alpha^{[a}\delta^{h]}_{[c}B_{d]hb}\right) = R_{\hat{a}bcd}$;
2. $2\left(2\delta^{[b}_{[c}\alpha^{a]}_{d]} - \delta^a_{[c}\delta^b_{d]}\alpha_0^2 + 2B^{hab}B_{hdc}\right) = R_{\hat{a}\hat{b}cd}$;
3. $A^{ad}_{bc} - 4B^{dah}B_{chb} + 4\alpha^{[a}\delta^{h]}_c\alpha_{[h}\delta^d_{b]} - \delta^a_c\delta^d_b\alpha_0^2 + B^{ad}B_{bc} = R_{\hat{a}bc\hat{d}}$;
4. $2\left(2B_{[c|ab|d]} + B_{a[c}B_{d]b} - 2\alpha_{[a}B_{b]cd}\right) = R_{abcd}$;
5. $2\left(\alpha_{0[c}\delta^a_{d]} - 2\alpha^{[a}\delta^{h]}_{[c}B_{d]h} + B^{ab}B_{bcd}\right) = R_{\hat{a}0cd}$;
6. $A^{ac0}_b - \delta^c_b\alpha_0\alpha^a + \alpha_b B^{ac} = R_{\hat{a}b\hat{c}0}$;
7. $2B_{cab}\alpha_0 + 2B_{cab0} = R_{abc0}$;
8. $-\delta^a_b\alpha_{00} - B_{cb}B^{ac} - \delta^a_b\alpha_0^2 - \alpha^a\alpha_b - \alpha^a_b + 2\alpha^{[a}\delta^{c]}_b\alpha_c = R_{\hat{a}0b0}$;
9. $2\alpha_0 B^{ab} + 2B^{bac}\alpha_c - D^{ab0} - \alpha^{ab} - \alpha^a\alpha^b = R_{\hat{a}0\hat{b}0}$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$, $A^{ac0}_b$ and $A^a_{bcd}$ are suitable smooth functions, $B^{abc}$, $B_{abc}$, $B_{abcd}$, $B_{abc0}$ are components of Kirichenko's second structure tensor $C$ (see [16]) and their covariant derivatives respectively, $B^{ab}$, $B_{ab}$, $D^{ab0}$ are the components of Kirichenko's third structure tensor $D$ (see [21]) and their covariant derivatives respectively, $\alpha^a$, $\alpha_a$, $\alpha_0$ are the components of $d\alpha$, $\alpha^a_b$, $\alpha^{ab}$ are the components of $d\alpha^a$, and $\alpha_{00}$, $\alpha_{0a}$ are the components of $d\alpha_0$.

**Theorem 9**. [4] The components of RC-tensor of quasi-Sasakian manifolds are given by:

1. $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - 2B^a_b B^d_c - B^a_c B^d_b$;
2. $R_{\hat{a}b0c} = B^a_{bc}$;
3. $R_{\hat{a}b0\hat{c}} = B^{ac}_b$;
4. $R_{\hat{a}0b0} = B^a_c B^c_b$;
5. $R_{\hat{a}\hat{b}cd} = 2B^a_{[c}B^b_{d]}$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are smooth functions satisfy $A^{[ad]}_{bc} = A^{ad}_{[bc]} = 0$, and $B^a_b$, $B^{ac}_b$, $B^a_{bc}$, are components of Kirichenko's fourth structure tensor $E$ (see [16]) and their covariant derivatives respectively.

**Theorem 10**. [2] The components of RC-tensor of CNK-manifolds are given by:

1. $R_{\hat{a}bcd} = 2A^a_{bcd}$;
2. $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - 2B_b{}^a B_c{}^d - B_c{}^a B_b{}^d + B^{ah}{}_c B_{hb}{}^d$;
3. $R_{\hat{a}bc0} = -C^a_{bc} - B_{[b}{}^h B_{c]h}{}^a$;
4. $R_{\hat{a}0b0} = B_b{}^h B_h{}^a$;
5. $R_{\hat{a}\hat{b}cd} = 2\left(B^{ab}{}_{[dc]} + B^a_{[c}B^b_{d]}\right)$;
6. $R_{\hat{a}\hat{b}c0} = 2B_h{}^{[a}B^{b]h}{}_c$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$, $A^a_{bcd}$ are suitable smooth functions, $B_b{}^a$, $C^a_{bc}$ are the components of Kirichenko's fourth structure tensor $E$ and their covariant derivatives respectively, and $B^{ab}{}_c$, $B_{ab}{}^c$, $B^{ab}{}_{cd}$ are the components of Kirichenko's first structure tensor $B$ and their covariant derivatives respectively.

**Theorem 11**. [15] The components of RC-tensor of nearly Kenmotsu manifolds are given by:

1. $R_{\hat{a}bcd} = -\frac{2}{3}\delta^a_b F_{cd} + \frac{1}{3}\delta^a_c F_{db} + \frac{1}{3}\delta^a_d F_{bc}$;
2. $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - C^{adh}C_{hbc} - \frac{1}{2}F^{ad}F_{bc} - \delta^a_c\delta^d_b$;
3. $R_{\hat{a}\hat{b}cd} = 2C^{abh}C_{hcd} + F^{ab}F_{cd} - 2\delta^a_{[c}\delta^b_{d]}$;
4. $R_{\hat{a}\hat{b}\hat{c}\hat{d}} = C^{acdb} - \frac{1}{2}\left(F^{ab}F^{cd} + F^{ac}F^{db} + F^{ad}F^{bc}\right)$;
5. $R_{\hat{a}00b} = F^{ac}F_{cb} + \delta^a_b$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are suitable smooth functions, $F^{ab}$, $F_{ab}$ are the components of Kirichenko's fifth structure tensor $F$, and $C^{abc}$, $C_{abc}$, $C^{abcd}$ are the components of Kirichenko's second structure tensor $C$ and their covariant derivatives respectively.

**Theorem 12**. [7] The components of RC-tensor of nearly cosymplectic manifolds are given by:

1. $R_{abcd} = -2B_{ab[cd]}$;
2. $R_{\hat{a}\hat{b}cd} = -2B^{abh}B_{hcd}$;
3. $R_{\hat{a}0b0} = C^{ac}C_{bc}$;
4. $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - B^{adh}B_{hbc} - \frac{5}{3}C^{ad}C_{bc}$,

and the other components are vanish or given by Lemma 2, or their conjugates, where $A^{ad}_{bc}$ are suitable smooth functions, $C^{ab}$, $C_{ab}$ are the components of Kirichenko's third structure tensor $D$, and $B^{abc}$, $B_{abc}$, $B_{abcd}$ are the components of Kirichenko's second structure tensor $C$ and their covariant derivatives respectively.

## Conclusions

This paper collected the theories that determined the components of RC-tensors for 12 different classes of ACM-manifolds. So, the readers can be recognized the difference among these classes from the theorems in this paper. Then we concluded that the RC-tensor distinct according to its class.

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## Conflict of Interest

The author declares no conflict of interest.

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# مراجعة حول تنسر الانحناء الريماني لبعض فئات المنطويات المترية التلامسية تقريباً

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**الخلاصة:**

استعرض هذا البحث مركبات تنسر الانحناء الريماني على الفضاء المتعلق بالبنية G – لبعض فئات المنطويات المترية التلامسية تقريباً. الفئات التي تم دراستنا هي اثنتا عشرة فئة فقط والمعروفة بالاسماء منطويات كوسمبلكتك ومنطويات ساساكي ومنطويات كينموتسو ومنطويات – $C_9$ ومنطويات – $C_{12}$ والمنطويات الطبيعية من النوع المعدوم ومنطويات كينموتسو التقريبي ومنطويات التحويل الكونفورمي المحلي للكوسمبلكتك تقريباً ومنطويات شبه ساساكي ومنطويات – $C(\lambda)$ تقريباً ومنطويات كوسمبلكتك التقريبي والمنطويات من نوع كينموتسو.

**الكلمات المفتاحية** : منطويات كينموتسو، منطويات – $C(\lambda)$ ، منطويات ساساكي، منطويات كوسمبلكتك، منطويات اينشتاين.