P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
2023,(17), (2):326– 337

Bipolar Valued Fuzzy SA-subalgebras and Fuzzy SA-ideals of SA- algebra.

Alaa Salih Abed[1*] **, Areej Tawfeeq Hameed**[2]
[1,2]Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq;

ARTICLE INFO
Received: 08/ 08 /2023
Accepted: 28 / 08 / 2023
Available online: 19 / 12 / 2023

DOI: 10.37652/juaps.2023.181582

Keywords:
SA-algebras, fuzzy SA-subalgebra, bipolar valued fuzzy SA-subalgebra, fuzzy SA-ideal, bipolar valued fuzzy SA-ideal.

ABSTRACT

In this paper, the notions of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals on SA-algebras with their properties are familiarized. Several theorems are stated and proved with their examples. After that we introduced new notion which is negative anti-fuzzy SA-subalgebra(SA-ideal) of SA-algebra . The image and inverse image of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals are defined and how the homomorphic images and inverse images of bipolar valued become bipolar valued fuzzy on SA-algebras is studied as well.
.

Introduction:

Areej Tawfeeq Hameed and et al ([2]) presented a different algebraic building, named SA-algebra, they have calculated a few belongings of these algebras, the conception of SA-ideals on SA-algebras was conveyed and some of its properties are scrutinized. The conception of a fuzzy set, was familiarized by L.A. Zadeh [10]. In [9], S.M. Mostafa and A.T. Hameed made an extension of the conception of fuzzy set by an interval-valued fuzzy set (i.e., a fuzzy set with an interval-valued membership function).

This interval- valued fuzzy KUS-ideals on KUS-algebras is referred to as an i-v fuzzy KUS-ideals on KUS-algebras. they created a way of estimated inference using his i-v fuzzy KUS-ideals on KUS-algebras. In this paper, using the conception of bipolar valued fuzzy subset, we familiarize the conception of a bipolar valued fuzzy SA-ideals (briefly**,** BVFSAI) of a SA-algebra, and reading some of their properties.

*Corresponding author at: Department of Mathematics, College of Education for Girl, University of Kufa , Najaf, Iraq;
ORCID:https://orcid.org/0000-0002-0525-9172;Tel:+9647812639334
E-mail address: alaas.abed@uokufa.edu.iq

Using a bipolar valued level set of a bipolar valued fuzzy set, we public a characterization of a bipolar valued fuzzy SA-ideals. We evidence that every SA-ideals of a SA-algebra Ӗ can be appreciated as a bipolar valued level SA-ideals of a bipolar valued fuzzy SA ideals of Ӗ. In connection with the idea of homomorphism, we educat.

how the images and inverse images of bipolar valued fuzzy SA-ideals develop bipolar valued fuzzy SA-ideals.

2. PRELIMINARIES

Now, we offer some definitions and preliminary results wanted in the later sections.

Definition (2.1)[2]. Let (Ӗ ; $+,-,0$) be an algebra with two binary operations $(+)$ and $(-)$ and constant (0). Ӗ is named an SA-algebra if it fulfills the next identities: for any $ч, \xi, \varsigma \in$ Ӗ ,

(SA_1) $ч - ч = 0$,

(SA_2) $ч - 0 = ч$,

(SA_3) $(ч - \xi) - \varsigma = ч - (\varsigma + \xi)$,

(SA_4) $(ч + \xi) - (ч + \varsigma) = \xi - \varsigma$.

In Ѥ we can describe a binary relation ($\le$) by : $ч \le \xi$
if and only if $ч + \xi = 0$ and $ч - \xi = 0$, $ч, \xi \in$ Ѥ. And we will symbolize it by Ǣ

Example (2.2)[2]. Let Ѥ ={ 0 , 1 , 2 , 3 } be a set with the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

–	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then(Ѥ ; $+, -, 0$) is Ǣ

Lemma (2.3)[2]. In Ǣ .For any $ч, \xi \in$ Ѥ,

(L_1) $ч + \xi = ч - (-\xi)$.

(L_2) $ч - \xi = ч + (-\xi)$,

(L_3) $ч - \xi = -\xi + ч$.

Proposition (2.4)[2]. In Ǣ .The next holds: for any $ч, \xi, \varsigma \in$ Ѥ,

$(a_1)(ч - \xi) - \varsigma = (ч - \varsigma) - \xi$,

(a_2) $0 - (ч - \xi) = (\xi - ч)$,

(a_3) $ч - \xi \le \varsigma$ imply $ч - \varsigma \le \xi$,

$(a_4) ч \le \xi$ imply $\varsigma + \xi \le \varsigma + ч$,

$(a_5)(ч - \xi) - (\varsigma - \xi) \le ч - \varsigma$ and $(ч - \xi) - (ч - \varsigma) \le \varsigma - \xi$,

(a_6) $ч \le \xi$ and $\xi \le \varsigma$ imply $ч \le \varsigma$.

Definition (2.5)[2]. In Ǣ, let S be a nonempty set of Ѥ. S is named a *SA*-subalgebra of Ѥ if $ч + \xi \in S, ч - \xi \in S$, whenever $ч, \xi \in$ S.And we will symboliz it by SAS- Ǣ

Definition (2.6)[2]. A nonempty subset I of a Ǣ is named a *SA*-ideal of Ѥ if it fulfills: for $ч, \xi, \varsigma \in$Ѥ,

(1) $0 \in I$,

(2) $(ч + \varsigma) \in I$ *and* $(\xi - \varsigma) \in I$ *imply* $(ч + \xi) \in I$. And we will symboliz it by SAI- Ǣ

Proposition (2.7)[2]. Every *SA*- Ǣ of Ǣ is a *SAS*- Ǣ of Ѥ and the converse is not true .

Lemma (2.8)[2]. An *SAI* of Ǣ has the following property:

1- If for any $ч \in$ Ѥ, for all $\xi \in$ I, $ч \le \xi$ implies $ч \in$ I.

2- If for any $ч \in$ I implies $-ч \in I$.

Definition (2.9)[10]. Suppose Ѥ be a nonempty set, a fuzzy subset ρ of Ѥ is a function $\rho :$ Ѥ $\to$ [0,1].

Definition (2.10)[2]. Suppose Ѥ be a nonempty set and ρ be a fuzzy subset of Ѥ, for t $\in$ [0,1], the set $\rho_t = \{ч \in Ѥ | \rho(ч) \ge t\}$ is named a level subset of ρ .

Definition (2.11)[2]. IN Ǣ, a fuzzy subset ρ of Ѥ is called a **fuzzy *SA*-subalgebra of Ѥ** if for all $ч, \xi \in$ Ѥ, $\rho(ч + \xi) \ge min\{\rho(ч), \rho(\xi)\}$ and $\rho(ч - \xi) \ge min\{\rho(ч), \rho(\xi)\}$.And we will symbliz it by FSAS- Ǣ

Theorem (2.12)[2]. Suppose ρ be a fuzzy subset of Ǣ, then

1- If ρ is a FSAS- Ǣ of Ѥ , then for any $t \in [0,1]$, ρ_t is a SAS- Ǣ of Ѥ, when $\rho_t \ne \emptyset$.

2- If for all $t \in [0,1]$, ρ_t is a SAS- Ǣ of Ѥ, then ρ is a FSAS- Ǣ of Ѥ.

Definition (2.13)[4]. In Ǣ , a fuzzy subset ρ of Ѥ is called a **fuzzy *SA*-ideal of Ѥ** if for all $ч, \xi, \varsigma \in$Ѥ, $\rho(0) \ge \rho(ч)$ and $\rho(ч + \xi) \ge min\{\rho(ч + \varsigma), \rho(\xi - \varsigma)\}$.

And we will symbliz it by FSAI- Ǣ

Theorem (2.14)[4]. suppose ρ be a fuzzy subset of Ǣ, then

1- If ρ is a FSAS- Ǣ of Ѥ , then for any t $\in$ [0,1] , ρ_t is a SAS- Ǣ of Ѥ, when $\rho_t \ne \emptyset$.

2- If for all $t \in [0,1]$, ρ_t is a SAS- Ǣ of Ѥ, then ρ is a FSAS- Ǣ of Ѥ.

Theorem (2.15)[4]. Suppose ρ be a fuzzy subset of Ǣ. ρ is a FSAI- Ǣ of Ѥ if and only if, for every t $\in$ [0,1], ρ_t is a SAI- Ǣ of Ѥ, when $\rho_t \ne \emptyset$.

Proposition (2.16)[4]. FSAI- Ǣ of Ǣ is a FSAS- Ǣ of Ѥ and the converse is not true.

Definition (2.17)[2]. Suppose(Ѥ; $+, -, 0$)and $(\mathfrak{Q}; +', -', 0')$ be *SA*-algebras, the mapping $\mathfrak{A}$: (Ѥ; $+, -, 0) \to (\mathfrak{Q}; +', -', 0')$ is named **a homomorphism** if it fulfills:

$\mathfrak{A}(ч + \xi) = \mathfrak{A}(ч) +' \mathfrak{A}(\xi)$, $\mathfrak{A}(ч - \xi) = \mathfrak{A}(ч) -' \mathfrak{A}(\xi)$, for all $ч, \xi \in$ Ѥ.

Definition (2.18)[7,8]. Suppose $\mathfrak{A}$: (Ѥ; $+, -, 0) \to (\mathfrak{Q}; +', -', 0')$ be a mapping nonempty sets Ѥ and $\mathfrak{Q}$ respectively. If ρ is a fuzzy subset of Ѥ, then the fuzzy subset β of $\mathfrak{Q}$ defined by:

$$\mathfrak{A}(\rho)(\xi) = \begin{cases} sup\{\rho(ч) : ч \in \mathfrak{A}^{-1}(\xi)\} & if \ \mathfrak{A}^{-1}(\xi) = \{ч \in Ѥ, \mathfrak{A}(ч) = \xi\} \ne \emptyset \\ 0 & otherwise \end{cases}$$

is known as the image of ρ under $\mathfrak{A}$.

Similarly if β is a fuzzy subset of $\mathfrak{Q}$, then the fuzzy subset $\rho = (\beta \circ \mathfrak{A})$ of Ꞓ
(i.e the fuzzy subset defined by ρ (ч) $= \beta$ ($\mathfrak{A}$ (ч)) for all ч $\in$ Ꞓ) is named the pre-image of β under $\mathfrak{A}$.

Theorem (2.19)[2]. 1- An onto homomorphic pre-image of a FSAS- Ǣ is also a FSAS- Ǣ.

2- An onto homomorphic pre-image of a FSAI- Ǣ is also a FSAI- Ǣ.

Definition (2.20)[7,8]. A fuzzy subset ρ of a set Ꞓ **has sup property** if for any subset T of Ꞓ, there exist $t_0 \in T$ such that $\rho(t_0) = sup\{\rho(t) | t \in T\}$.

Theorem (2.21)[1]. Let $\mathfrak{A}$: (Ꞓ; $+,-,0$) $\rightarrow$ ($\mathfrak{Q}$; $+',-',0'$) be a homomorphism between SA-algebras Ꞓ and $\mathfrak{Q}$ respectively.

1- For every FSAS- Ǣ , ρ of Ꞓ and with sup property, $\mathfrak{A}$ (ρ) is a FSAS- Ǣ of $\mathfrak{Q}$.

2- For every FSAI- Ǣ, ρ of Ꞓ and with sup property, $\mathfrak{A}$ (ρ) is a FSAI- Ǣ of $\mathfrak{Q}$.

Definition (2.22)[9]. Assume (Ꞓ; $+,-,0$) be Ǣ, a fuzzy subset ρ of Ꞓ is named **an anti-fuzzy SA-subalgebra of** Ꞓ if for all ч, $\xi \in$ Ꞓ,
$AFSAS_1$) ρ(ч $+ \xi) \leq max\{\rho$ (ч)$, \rho(\xi)\}$,
$AFSAS_2$) ρ (ч $- \xi) \leq max\{\mu$ (ч)$, \rho(\xi)\}$. And we will symbliz it by AFSAS- Ǣ

Proposition (2.23)[9]. Suppose ρ be an AFSAS- Ǣ of Ǣ.

1- If ρ is an AFSAS- Ǣ of Ꞓ , then it satisfies for any t∈[0, 1], $L(\rho,t) \neq \emptyset$ implies $L(\rho,t)$ is a FSAS- Ǣ of Ꞓ .

2- If $L(\rho,t)$ is a FSAS- Ǣ of Ꞓ, for all t∈[0, 1], $L(\rho,t) \neq \emptyset$, then ρ is an AFSAS- Ǣ of Ꞓ .

Definition (2.24)[9]. Let (Ꞓ; $+,-,0$) be Ǣ, ρ is named **an anti-fuzzy SA-ideal of** Ꞓ if it fulfills the following conditions, for all ч, $\xi, \varsigma \in$ Ꞓ,
($AFSAI_1$) $\rho(0) \leq \rho$ (ч) ,
($AFSAI_2$) ρ(ч $+ \xi) \leq max\{\rho$(ч $+ \varsigma), \rho(\xi - \varsigma)\}$.
And we will symbliz it by AFSAI- Ǣ

Proposition (2.25)[9]. Let ρ be an anti-fuzzy subset of Ǣ.

1- If ρ is an AFSAI- Ǣ of Ꞓ , then it fulfills for any t∈[0, 1], $L(\rho,t) \neq \emptyset$ implies $L(\rho,t)$ is a FSAI- Ǣ of Ꞓ.

2- If $L(\rho,t)$ is a FSAI- Ǣ of Ꞓ, for all t∈[0, 1], $L(\rho,t) \neq \emptyset$, then ρ is an AFSAS- Ǣ of Ꞓ .

Definition (2.26)[9]. Assume $\mathfrak{A}$: (Ꞓ; $+,-,0$) $\rightarrow$ ($\mathfrak{Q}$; $+',-',0'$) be a mapping nonempty SA-algebras Ꞓ and $\mathfrak{Q}$ respectively. If ρ is anti-fuzzy subset of Ꞓ, then the anti-fuzzy subset β of $\mathfrak{Q}$ defined by:

$$\mathfrak{A}(\rho)(\xi) = \begin{cases} inf\{\rho(ч) : ч \in \mathfrak{A}^{-1}(\xi)\}\ if\ \mathfrak{A}^{-1}(\xi) \\ \qquad = \{ч \in Ꞓ, \mathfrak{A}(ч) = \xi\} \neq \emptyset \\ 1 \qquad\qquad otherwise \end{cases}$$

is known as the image of ρ under $\mathfrak{A}$.

Similarly if β is anti-fuzzy subset of $\mathfrak{Q}$, then the fuzzy subset $\rho = (\beta \circ \mathfrak{A})$ of Ꞓ
(i.e the anti-fuzzy subset defined by ρ (ч) $= \beta$ ($\mathfrak{A}$ (ч)), for all ч $\in$ Ꞓ) is named the pre-image of β under $\mathfrak{A}$.

Theorem (2.27)[9]. 1- An onto homomorphic pre-image of AFSAS- Ǣ is also AFSAS- Ǣ.

2- An onto homomorphic pre-image of an AFSAI- Ǣ is also AFSAI- Ǣ.

Definition (2.28)[9]. A fuzzy subset ρ of a set Ꞓ **has inf property** if for any subset T of Ꞓ, there exist $t_0 \in T$ such that $\rho(t_0) = inf\{\rho(t) | t \in T\}$.

Theorem (2.29)[9]. Let $\mathfrak{A}$: (Ꞓ; $+,-,0$) $\rightarrow$ ($\mathfrak{Q}$; $+',-',0'$) be a homomorphism between SA-algebras Ꞓ and $\mathfrak{Q}$ separately.

1- For every AFSAS- Ǣ, ρ of Ꞓ and with inf property, $\mathfrak{A}$ (ρ) is AFSAS- Ǣ of $\mathfrak{Q}$.

2- For every AFSAI- Ǣ, ρ of Ꞓ and with inf property, $\mathfrak{A}$ (ρ) is AFSAI- Ǣ of $\mathfrak{Q}$.

Remark (2.30)[3]. An interval number is $\tilde{\mathfrak{y}} = [\mathfrak{y}^-, \mathfrak{y}^+]$, where $0 \leq \mathfrak{y}^- \leq \mathfrak{y}^+ \leq 1$.

Let I be a closed unit interval, (i.e., I = [0, 1]). Let D[0, 1] denote the family of all closed subintervals of I = [0, 1] , that is,
D[0, 1] $= \{ \tilde{\mathfrak{y}} = [\mathfrak{y}^-, \mathfrak{y}^+] \mid \mathfrak{y}^- \leq \mathfrak{y}^+$, for $\mathfrak{y}^-, \mathfrak{y}^+ \in$ I$\}$.

Now, we describe what is known as cultured minimum (briefly, rmin) of two element in D[0,1].

Definition (2.31)[3]. We also define the symbols ($\succcurlyeq$), ($\preccurlyeq$) , (=) ,"rmin " and "rmax " in situation of two elements in D[0, 1] .
Consider two interval numbers (elements numbers) $\tilde{\mathfrak{y}} = [\mathfrak{y}^-, \mathfrak{y}^+]$, $\tilde{\mathfrak{w}} = [\mathfrak{w}^-, \mathfrak{w}^+]$in D[0, 1] : Then
(1) $\tilde{\mathfrak{y}} \succcurlyeq \tilde{\mathfrak{w}}$ if and only if, $\mathfrak{y}^- \geq \mathfrak{w}^-$ and $\mathfrak{y}^+ \geq \mathfrak{w}^+$,
(2) $\tilde{\mathfrak{y}} \preccurlyeq \tilde{\mathfrak{w}}$ if and only if, $\mathfrak{y}^- \leq \mathfrak{w}^-$ and $\mathfrak{y}^+ \leq \mathfrak{w}^+$,

(3) $\tilde{\mathfrak{y}} = \tilde{\mathfrak{w}}$ if and only if, $\mathfrak{y}^- = \mathfrak{w}^-$ and $\mathfrak{y}^+ = \mathfrak{w}^+$,
(4) rmin $\{\tilde{\mathfrak{y}}, \tilde{\mathfrak{w}}\} = [\min\{\mathfrak{y}^-, \mathfrak{w}^-\}, \min\{\mathfrak{y}^+, \mathfrak{w}^+\}]$,
(5) rmax $\{\tilde{\mathfrak{y}}, \tilde{\mathfrak{w}}\} = [\max\{\mathfrak{y}^-, \mathfrak{w}^-\}, \max\{\mathfrak{y}^+, \mathfrak{w}^+\}]$,

Remark (2.32)[3]. It is obvious that (D[0, 1] , $\preccurlyeq$, ∨ , ∧) is a complete lattice with $\tilde{0}$ = [0, 0] as its least element and $\tilde{1}$ = [1, 1] as its greatest element.
Let $\tilde{\mathfrak{y}}_i \in$D[0, 1] where i ∈Λ. We define
$r\,inf_{i\in\Lambda}\tilde{\mathfrak{y}} = [r\,inf_{i\in\Lambda}\mathfrak{y}^-, r\,inf_{i\in\Lambda}\mathfrak{y}^+]$, $r\,sup_{i\in\Lambda}\tilde{\mathfrak{y}} = [r\,sup_{i\in\Lambda}\mathfrak{y}^-, r\,sup_{i\in\Lambda}\mathfrak{y}^+]$.

Definition (2.33)[7,8]. An **interval-valued fuzzy subset** $\tilde{\rho}_A$ on Ǣ is defined as
$\tilde{\rho}_A = \{<$ ч, $[\rho_A^-$ (ч) , ρ_A^+ (ч) $]>|$ ч ∈Ѥ$\}$.
Where ρ_A^- (ч) $\leq \rho_A^+$ (ч), for all ч ∈ Ѥ. Then the fuzzy subsets ρ_A^-: Ѥ ⟶ [-1, 0] and
ρ_A^+: Ѥ ⟶ [0, 1] are named a **lower fuzzy subset and an upper fuzzy subset** of $\tilde{\rho}_A$ separately .
Let $\tilde{\rho}_A$ (ч) = $[\rho_A^-$ (ч) , ρ_A^+ (ч)] , $\tilde{\rho}_A$:Ѥ ⟶ D[0, 1], then A = $\{<$ ч, $\tilde{\rho}_A$ (ч) $>|$ ч ∈ Ѥ$\}$.

Remark (2.34)[1]. Let Ѥ be the universe of discourse. A **bipolar fuzzy subset** ρ **of** ***Ѥ*** is an object ensuring the form
$\Phi = \{ (ч, \rho_\Phi^N(ч), \rho_\Phi^P(ч)) | ч \in Ѥ\}$,
where μ_Φ^N: Ѥ → [−1, 0] and μ_Φ^P : Ѥ → [0, 1] are mappings.
The positive membership degree ρ_Φ^P (ч) denoted the satisfaction degree of an element Ѥ to the property corresponding to a bipolar-valued fuzzy
$\Phi = \{ (ч, \rho_\Phi^N(ч), \rho_\Phi^P(ч)) | ч \in Ѥ\}$, and the negative membership degree
ρ_Φ^N(ч) means the satisfaction degree of Ѥ to some implicit counter-property of
$\Phi = \{ (ч, \rho_\Phi^N(ч), \rho_\Phi^P(\varkappa)) | ч \in Ѥ\}$. For the sake of plainness, we shall use the symbol
$\Phi = (Ѥ: \rho_\Phi^N, \rho_\Phi^P)$, for the bipolar fuzzy set
$\Phi = \{ (ч, \rho_\Phi^N(ч), \rho_\Phi^P(ч)) | ч \in Ѥ\}$, and use the conception of bipolar fuzzy sets instead of the conception of bipolar-valued fuzzy sets.

Definition (2.35)[5]. A bipolar fuzzy subset $\Phi = (Ѥ: \rho_\Phi^N, \rho_\Phi^P)$ of Ǣ is named **a bipolar fuzzy *SA*-subalgebra of** Ѥ if it fulfills the next properties: for any ч, ξ ∈ Ѥ,

1. $\rho_\Phi^N(ч + \xi) \leq \max\{\rho_\Phi^N(ч), \rho_\Phi^N(\xi)\}$,
2. $\rho_\Phi^N(ч - \xi) \leq \max\{\rho_\Phi^N(ч), \rho_\Phi^N(\xi)\}$,
3. $\rho_\Phi^P(ч + \xi) \geq \min\{\rho_\Phi^P(ч), \rho_\Phi^P(\xi)\}$ and
4. $\rho_\Phi^P(ч - \xi) \geq \min\{\rho_\Phi^P(ч), \rho_\Phi^P(\xi)\}$. And we will symbliz it by BFSAS- Ǣ

Definition (2.36)[5]. A bipolar fuzzy subset $\Phi = (Ѥ: \rho_\Phi^N, \rho_\Phi^P)$ of Ǣ is named **a bipolar fuzzy *SA*-ideal of *Ѥ*** if it fulfills the following: for any ч, ξ, ς ∈ Ѥ,

1. $\rho_\Phi^N(0) \leq \rho_\Phi^N(ч)$,
2. $\rho_\Phi^P(0) \geq \rho_\Phi^P(ч)$,
3. $\rho_\Phi^N(ч + \xi) \leq \max\{\rho_\Phi^N(ч + \varsigma), \rho_\Phi^N(\xi - \varsigma)\}$ and $\rho_\Phi^P(ч + \xi) \geq \min\{\rho_\Phi^P(ч + \varsigma), \rho_\Phi^P(\xi - \varsigma)\}$. And we will symbliz it by BFSAI- **Ǣ.**

Definition (2.37)[7,8]. Assume : $(Ѥ; +, -, 0) \to (\mathfrak{Q}; +', -', 0')$ be a mapping from set Ѥ into a set $\mathfrak{Q}$. let B be a bipolar valued fuzzy subset of $\mathfrak{Q}$. Then the inverse image of B, denoted by $\mathfrak{A}^{-1}(B)$, is a bipolar valued fuzzy subset of Ѥ, with the membership function given by
$\rho_{\mathfrak{A}^{-1}(B)}(ч) = \tilde{\rho}_B(\mathfrak{A}(ч))$, for all ч ∈ Ѥ .

Proposition (2.38)[6]. Assume : $(Ѥ; +, -, 0) \to (\mathfrak{Q}; +', -', 0')$ be a mapping from set Ѥ into set $\mathfrak{Q}$, let $\tilde{\rho}_Ɣ = [(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P]$ *and* $\tilde{n}_Ɣ = [(\tilde{n}_Ɣ)^N, (\tilde{n}_Ɣ)^P]$ be bipolar valued fuzzy subsets of sets Ѥ and $\mathfrak{Q}$ separately. Then

(1) $\mathfrak{A}^{-1}(\tilde{n}_Ɣ) = [\mathfrak{A}^{-1}((\tilde{n}_Ɣ)^N), \mathfrak{A}^{-1}((\tilde{n}_Ɣ)^P)]$,
(2) $\mathfrak{A}(\tilde{\rho}_Ɣ) = [\mathfrak{A}((\tilde{\rho}_Ɣ)^N), \mathfrak{A}((\tilde{\rho}_Ɣ)^P)]$.

3. BIPOLAR VALUED FUZZY SA-SUBALGEBRAS OF SA-ALGEBRA

In the part, the conception of the bipolar valued fuzzy SA-subalgebras of SA-algebra is presented. Some theorems and properties are itemized and ascertained.

Definition (3.1):A interval valued fuzzy subset Ɣ = $\{<$ ч, $\tilde{\rho}_Ɣ$ (ч) $>|$ ч ∈ Ѥ$\}$ = $\{<$ ч, $[\rho_Ɣ^-$ (ч), $\rho_Ɣ^+$ (ч) $]>|$ ч ∈ Ѥ$\}$ of Ǣ is called **a bipolar valued fuzzy SA-subalgebra** denoted by **(BVFSAS- Ǣ) of** Ѥ
$Ɣ^{(N,P)} = \{<$ ч, $(\tilde{\rho}_Ɣ)^N$ (ч), $(\tilde{\rho}_Ɣ)^P$ (ч) $>| \varkappa \in$ Ѥ$\}$
= $\{<$ ч, $\widetilde{(\rho_Ɣ^N)}$ (ч), $\widetilde{(\rho_Ɣ^P)}$ (ч) $>|$ ч ∈ Ѥ $\}$,
$Ɣ^{(N,P)} = <(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P>$, if for all ч, $\xi \in$ Ѥ.
1- $(\tilde{\rho}_Ɣ)^N(ч + \xi) \leq max\{(\tilde{\rho}_Ɣ)^N(ч), (\tilde{\rho}_Ɣ)^N(\xi)\}$,
2- $(\tilde{\rho}_Ɣ)^P(ч + \xi) \geq min\{(\tilde{\rho}_Ɣ)^P(ч), (\tilde{\rho}_Ɣ)^P(\xi)\}$,

3- $(\tilde{\rho}_{Ɣ})^N$ $(ч-\xi)$ $\le max\{(\tilde{\rho}_{Ɣ})^N(ч),(\tilde{\rho}_{Ɣ})^N(\xi)\}$ and
4- $(\tilde{\rho}_{Ɣ})^P$ $(ч-\xi)$ $\ge min\{(\tilde{\rho}_{Ɣ})^P(ч),(\tilde{\rho}_{Ɣ})^P(\xi)\}$.
i.e.,
1- $\widetilde{(\rho_{Ɣ}^N)}$ $(ч+\xi)$ $\preccurlyeq r\, max\{\widetilde{(\rho_{Ɣ}^N)}(ч),\widetilde{(\rho_{Ɣ}^N)}(\xi)\}$,
2- $\widetilde{(\rho_{Ɣ}^P)}$ $(ч+\xi)$ $\succcurlyeq rmin\{\widetilde{(\rho_{Ɣ}^P)}(ч),\widetilde{(\rho_{Ɣ}^P)}(\xi)\}$,
3- $\widetilde{(\rho_{Ɣ}^N)}$ $(ч-\xi)$ $\preccurlyeq rmax\{\widetilde{(\rho_{Ɣ}^N)}(ч),\widetilde{(\rho_{Ɣ}^N)}(\xi)\}$ and
4- $\widetilde{(\rho_{Ɣ}^P)}$ $(ч-\xi)$ $\succcurlyeq r\, min\{\widetilde{(\rho_{Ɣ}^P)}(ч),\widetilde{(\rho_{Ɣ}^P)}(\xi)\}$.
i.e.,
1- $(\rho_{Ɣ}^-)^N$ $(ч+\xi)$ $\le max\{(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^-)^N(\xi)\}$ and
$(\rho_{Ɣ}^-)^P$ $(ч+\xi)$ $\ge min\{(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^-)^P(\xi)\}$.
2- $(\rho_{Ɣ}^+)^N$ $(ч+\xi)$ $\le max\{(\rho_{Ɣ}^+)^N(ч),(\rho_{Ɣ}^+)^N(\xi)\}$ and
$(\rho_{Ɣ}^+)^P$ $(ч+\xi)$ $\ge min\{(\rho_{Ɣ}^+)^P(ч),(\rho_{Ɣ}^+)^P(\xi)\}$.
3- $(\rho_{Ɣ}^-)^N$ $(ч-\xi)$ $\le max\{(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^-)^N(\xi)\}$ and
$(\rho_{Ɣ}^-)^P$ $(ч-\xi)$ $\ge min\{(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^-)^P(\xi)\}$.
4- $(\rho_{Ɣ}^+)^N$ $(ч-\xi)$ $\le max\{(\rho_{Ɣ}^+)^N(ч),(\rho_{Ɣ}^+)^N(\xi)\}$ and
$(\rho_{Ɣ}^+)^P$ $(ч-\xi)$ $\ge min\{(\rho_{Ɣ}^+)^P(ч),(\rho_{Ɣ}^+)^P(\xi)\}$.

Remark (3.2): A bipolar valued fuzzy subset $Ɣ^{(N,P)} = <(\tilde{\rho}_{Ɣ})^N,(\tilde{\rho}_{Ɣ})^P>$ of Ǣ, for all ч ∈ Ѥ, thus,
Since $(\rho_{Ɣ}^-)^N(ч)=(\rho_{Ɣ}^N)^-(ч)$, $(\rho_{Ɣ}^-)^P(ч)=(\rho_{Ɣ}^P)^-(ч)$, $(\rho_{Ɣ}^+)^N(ч)=(\rho_{Ɣ}^N)^+(ч)$ and $(\rho_{Ɣ}^+)^P(ч)=(\rho_{Ɣ}^P)^+(ч)$,
then $(\tilde{\rho}_{Ɣ})^N(ч)=[(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^+)^N(ч)]=[(\rho_{Ɣ}^N)^-(ч),(\rho_{Ɣ}^N)^+(ч)]=\widetilde{(\rho_{Ɣ}^N)}(ч)$ and
$(\tilde{\rho}_{Ɣ})^P(ч)=[(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^+)^P(ч)]=[(\rho_{Ɣ}^P)^-(ч),(\rho_{Ɣ}^P)^+(ч)]=\widetilde{(\rho_{Ɣ}^P)}(ч)$.

Example (3.3): Let **Ѥ** = {0, a, b, c} in which the operations $(+,-)$ be define by the following tables:

+	0	a	b	c
0	0	a	b	c
a	a	b	c	0
b	b	c	0	a
c	c	0	a	b

−	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Then (Ѥ; $+,-,0$) is an SA-algebra. $Ɣ^{(N,P)}=<(\tilde{\rho}_{Ɣ})^N,(\tilde{\rho}_{Ɣ})^P>$ of Ѥ where I ={0,b} is a SAS- Ǣ of Ѥ,such that:
The fuzzy subsets ρ^+: Ѥ→ [0,1] and ρ^-: Ѥ→ [-1,0] by:

$$Ɣ^{(N,P)}(ч)=\begin{cases}[[-0.6,-0.3],[0.3,0.9]] & if\, ч=\{0,b\}\\ [[-0.7,-0.4],[0.2,0.6]] & otherwise\end{cases}$$

, $Ɣ^{(N,P)}$ (ч) is BVFSAS- Ǣ of Ѥ.

Proposition (3.4): If $Ɣ^{(N,P)}=<(\tilde{\rho}_{Ɣ})^N,(\tilde{\rho}_{Ɣ})^P>$ is a **BVFSAS- Ǣ**, then $(\tilde{\rho}_{Ɣ})^N(0)\le(\tilde{\rho}_{Ɣ})^N(ч)$ and $(\tilde{\rho}_{Ɣ})^P(0)\ge(\tilde{\rho}_{Ɣ})^P(ч)$, for all ч ∈ Ѥ .

Proof: For all $\varkappa,\xi$ ∈ Ѥ and $\varkappa=\xi$, we have

$$\widetilde{(\rho_{Ɣ}^N)}(0)=\widetilde{(\rho_{Ɣ}^N)}(ч+\xi)\preccurlyeq r\, max\{\widetilde{(\rho_{Ɣ}^N)}(ч),\widetilde{(\rho_{Ɣ}^N)}(\xi)\}$$

and

$$\widetilde{(\rho_{Ɣ}^P)}(0)=\widetilde{(\rho_{Ɣ}^P)}(ч+\xi)\succcurlyeq r\, min\{\widetilde{(\rho_{Ɣ}^P)}(ч),\widetilde{(\rho_{Ɣ}^P)}(\xi)\},$$

then $(\tilde{\rho}_{Ɣ})^N(0)=[(\rho_{Ɣ}^-)^N(0),(\rho_{Ɣ}^+)^N(0)]$
$=[(\rho_{Ɣ}^N)^-(0),(\rho_{Ɣ}^N)^+(0)]$
$\le \max\{[(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^+)^N(ч)],[(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^+)^N(ч)]\}$
$=[(\rho_{Ɣ}^-)^N(ч),(\rho_{Ɣ}^+)^N(ч)]=(\tilde{\rho}_{Ɣ})^N(ч)$ and
$(\tilde{\rho}_{A})^P(0)=[(\rho_{Ɣ}^-)^P(0),(\rho_{Ɣ}^+)^P(0)]$
$=[(\rho_{Ɣ}^P)^-(0),(\rho_{Ɣ}^P)^+(0)]$

$\ge \min\{[(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^+)^P(ч)],[(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^+)^P(ч)]\}$
$=[(\rho_{Ɣ}^-)^P(ч),(\rho_{Ɣ}^+)^P(ч)]=(\tilde{\rho}_{Ɣ})^P(ч)$
$(\tilde{\rho}_{Ɣ})^N(0)\le(\tilde{\rho}_{Ɣ})^N(ч)$ and $(\tilde{\rho}_{Ɣ})^P(0)\ge(\tilde{\rho}_{Ɣ})^P(ч)$, for all ч ∈ Ѥ .⌂

Proposition (3.5): Let $Ɣ^{(N,P)}=<(\tilde{\rho}_{Ɣ})^N,(\tilde{\rho}_{Ɣ})^P>$ be a BVFSAS- Ǣ,
if there exist a sequence $(\{ч_n\})$ of Ѥ such that $\lim_{n\to\infty}(\tilde{\rho}_{Ɣ})^N(ч_n)=[0,0]$,
and $\lim_{n\to\infty}(\tilde{\rho}_{Ɣ})^P(ч_n)=[1,1]$, then $(\tilde{\rho}_{Ɣ})^N(0)=[0,0]$ and $(\tilde{\rho}_{Ɣ})^P(0)=[1,1]$.

Proof:
By Proposition (3.4), we have $(\tilde{\rho}_{Ɣ})^N(0)\le(\tilde{\rho}_{Ɣ})^N(ч)$, and
$(\tilde{\rho}_{Ɣ})^P(0)\ge(\tilde{\rho}_{Ɣ})^P(ч)$, for all ч ∈ Ѥ, then $(\tilde{\rho}_{Ɣ})^N(0)\le(\tilde{\rho}_{Ɣ})^N(ч_n)$ and
$(\tilde{\rho}_{Ɣ})^P(0)\ge(\tilde{\rho}_{Ɣ})^P(ч_n)$, for every positive integer n.
Consider the inequality $[0,0]\le(\tilde{\rho}_{Ɣ})^N(0)\le\lim_{n\to\infty}(\tilde{\rho}_{Ɣ})^N(ч_n)=[0,0]$
and $[1,1]\ge(\tilde{\rho}_{Ɣ})^P(0)\ge\lim_{n\to\infty}(\tilde{\rho}_{Ɣ})^P(ч_n)=[1,1]$.
Hence $(\tilde{\rho}_{Ɣ})^N(0)=[0,0]$ and $(\tilde{\rho}_{Ɣ})^P(0)=[1,1]$. ⌂

Theorem (3.6): A bipolar valued fuzzy subset $Ɣ^{(N,P)}=<(\tilde{\rho}_{Ɣ})^N,(\tilde{\rho}_{Ɣ})^P>$ of Ǣ is a **BVFSAS- Ǣ** of Ѥ if and only if, $(\rho_{Ɣ}^-)^N$ and $(\rho_{Ɣ}^+)^N$ are **AFSAS- Ǣ** of Ѥ
and $(\rho_{Ɣ}^-)^P$ and $(\rho_{Ɣ}^+)^P$ are **FSAS- Ǣ** of Ѥ.

Proof:

Suppose that $Ɣ^{(N,P)}$ is a **BVFSAS- Ӕ** of Ҝ, then for all ч, $\xi \in$ Ҝ, we have

$[(\rho_{Ɣ}^{-})^{N}(ч+\xi),(\rho_{Ɣ}^{+})^{N}(ч+\xi)] = (\tilde{\rho}_{v})^{N}(ч+\xi)$
$\preccurlyeq r\,max\{(\tilde{\rho}_{Ɣ})^{N}(ч),(\tilde{\rho}_{Ɣ})^{N}(\xi)\} =$
$r\,max\{[(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(ч)],[(\rho_{Ɣ}^{-})^{N}(\xi),(\rho_{Ɣ}^{+})^{N}(\xi)]\}$
$=$
$[max\{(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(ч)\},max\{(\rho_{Ɣ}^{-})^{N}(\xi),(\rho_{Ɣ}^{+})^{N}(\xi)\}] =$
$[max\{(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{-})^{N}(\xi)\},max\{(\rho_{Ɣ}^{+})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(\xi)\}]$
.Therefore ,
$(\rho_{Ɣ}^{-})^{N}(ч+\xi) \leq max\{(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{-})^{N}(\xi)\}$
and $(\rho_{Ɣ}^{+})^{N}(ч+\xi) \leq max\{(\rho_{Ɣ}^{+})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(\xi)\}$.
Also,
$[(\rho_{Ɣ}^{-})^{P}(ч+\xi),(\rho_{Ɣ}^{+})^{P}(ч+\xi)] = (\tilde{\rho}_{Ɣ})^{P}(ч+\xi)$
$\succcurlyeq r\,min\{(\tilde{\rho}_{Ɣ})^{P}(ч),(\tilde{\rho}_{Ɣ})^{P}(ч)\}$
$= r\,min\{[(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(ч)],[(\rho_{Ɣ}^{-})^{P}(\xi),(\rho_{Ɣ}^{+})^{P}(\xi)]\}$
$= [min\{(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(ч)\},$
$min\{(\rho_{Ɣ}^{-})^{P}(\xi),(\rho_{Ɣ}^{+})^{P}(\xi)\}] =$
$[min\{(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{-})^{P}(\xi)\},min\{(\rho_{Ɣ}^{+})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(\xi)\}]$

Therefore,
$(\rho_{Ɣ}^{-})^{P}(ч+\xi) \geq min\{(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{-})^{P}(\xi)\}$
and $(\rho_{Ɣ}^{+})^{P}(ч+\xi) \geq min\{(\rho_{Ɣ}^{+})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(\xi)\}$.
Hence, we get that $(\rho_{Ɣ}^{-})^{N}$ and $(\rho_{Ɣ}^{+})^{N}$ are **AFSAS- Ӕ** of Ҝ and $(\rho_{Ɣ}^{-})^{P}$ and $(\rho_{Ɣ}^{+})^{P}$ are **AFSAS- Ӕ** of Ҝ.
Conversely, if $(\rho_{Ɣ}^{-})^{N}$ and $(\rho_{Ɣ}^{+})^{N}$ are **AFSAS- Ӕ** of Ҝ and $(\rho_{Ɣ}^{-})^{P}$ and $(\rho_{Ɣ}^{+})^{P}$ are **FSAS- Ӕ** of Ҝ,
for all ч, $\xi \in$ Ҝ. Observe :

$(\tilde{\rho}_{Ɣ})^{N}(ч+\xi) = [(\rho_{Ɣ}^{-})^{N}(ч+\xi),(\rho_{Ɣ}^{+})^{N}(ч+\xi)] \leq$
$[max\{(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{-})^{N}(\xi)\},min\{(\rho_{Ɣ}^{+})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(\xi)\}]$
$= r\,max\{[(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(ч)],$
$[(\rho_{Ɣ}^{-})^{N}(\xi),(\rho_{Ɣ}^{+})^{N}(\xi)]\}$
$= r\,max\{(\tilde{\rho}_{Ɣ})^{N}(ч),(\tilde{\rho}_{Ɣ})^{N}(\xi)\}$. and
$(\tilde{\rho}_{Ɣ})^{N}(ч-\xi) = [(\rho_{Ɣ}^{-})^{N}(ч-\xi),(\rho_{Ɣ}^{+})^{N}(ч-\xi)] \leq$
$[max\{(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{-})^{N}(y)\},min\{(\rho_{Ɣ}^{+})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(\xi)\}]$
$=$
$r\,max\{[(\rho_{Ɣ}^{-})^{N}(ч),(\rho_{Ɣ}^{+})^{N}(ч)],[(\rho_{Ɣ}^{-})^{N}(\xi),(\rho_{Ɣ}^{+})^{N}(\xi)]\}$
$= r\,max\{(\tilde{\rho}_{Ɣ})^{N}(ч),(\tilde{\rho}_{Ɣ})^{N}(\xi)\}$. Also,
$(\tilde{\rho}_{Ɣ})^{P}(ч+\xi) = [(\rho_{Ɣ}^{-})^{P}(ч+\xi),(\rho_{Ɣ}^{+})^{P}(ч+\xi)] \geq$

$[min\{(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{-})^{P}(\xi)\},min\{(\rho_{Ɣ}^{+})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(\xi)\}]$
$=$ r
$\min\{[(\rho_{Ɣ}^{-})^{p}(ч),(\rho_{Ɣ}^{+})^{p}(ч)],\ [(\rho_{Ɣ}^{-})^{p}(\xi),(\rho_{Ɣ}^{+})^{p}(\xi)]\}$
$= r\,min\{(\tilde{\rho}_{Ɣ})^{P}(ч),(\tilde{\rho}_{Ɣ})^{P}(\xi)\}$. And
$(\tilde{\rho}_{Ɣ})^{P}(ч-\xi) = [(\rho_{Ɣ}^{-})^{P}(ч-\xi),(\rho_{Ɣ}^{+})^{P}(ч-\xi)]$
$\geq [min\{(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{-})^{P}(\xi)\},min\{(\rho_{Ɣ}^{+})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(\xi)\}]$
$= r\,min\{[(\rho_{Ɣ}^{-})^{P}(ч),(\rho_{Ɣ}^{+})^{P}(ч)],[(\rho_{Ɣ}^{-})^{P}(\xi),(\rho_{Ɣ}^{+})^{P}(\xi)]\}$
$= r\,min\{(\tilde{\rho}_{Ɣ})^{P}(ч),(\tilde{\rho}_{Ɣ})^{P}(\xi)\}$.
Thus, we can conclude that $Ɣ^{(N,P)}$ is a **BVFSAS- Ӕ** of Ҝ.⌂

Definition (3.7): In Ӕ. A bipolar valued fuzzy subset $Ɣ^{(N,P)} = <(\tilde{\rho}_{Ɣ})^{N},(\tilde{\rho}_{Ɣ})^{P}>$ of Ҝ ,
for all $\tilde{t} = [t_1, t_2] \in$ D[0, 1], the set $\tilde{U}(Ɣ^{(N,P)};\tilde{t})$ is **a level set** of Ҝ such that
$\tilde{U}(Ɣ^{(N,P)};\tilde{t}) = \{ч \in Ҝ \,|\tilde{\rho}_{Ɣ}(x) \succcurlyeq \tilde{t}\} =$
$\{ч \in Ҝ \,|[(\tilde{\rho}_{Ɣ})^{N}ч,(\tilde{\rho}_{Ɣ})^{P}(ч)] \succcurlyeq [t_1, t_2]\}$
$= \{ч \in Ҝ \,|(\tilde{\rho}_{Ɣ})^{N}(ч) \leq t_1,\ (\tilde{\rho}_{Ɣ})^{P}(ч) \geq t_2\}$.

Proposition (3.8): Assume (Ҝ ; +, −,0) be Ӕ. A bipolar valued fuzzy subset $Ɣ^{(N,P)} = <(\tilde{\rho}_{Ɣ})^{N},(\tilde{\rho}_{Ɣ})^{P}>$ of Ҝ. If $Ɣ^{(N,P)}$ is a **BVFSAS- Ӕ** of Ҝ,
then for any $\tilde{t} = [t_1, t_2] \in$ D[0, 1], the set $\tilde{U}(Ɣ^{(N,P)};\tilde{t})$ is a Ӕ of Ҝ .

Proof.

Assume that $Ɣ^{(N,P)}$ is a **BVFSAS- Ӕ** of Ҝ and let $\tilde{t} = [t_1, t_2] \in$ D[0, 1]
such that $\tilde{U}(Ɣ^{(N,P)};\tilde{t}) \neq \emptyset$, and suppose ч , $\xi \in$ Ҝ such that
ч , $\xi \in \tilde{U}(Ɣ^{(N,P)};\tilde{t})$, then $(\tilde{\rho}_{Ɣ})^{N}(ч) \leq t_1$, $(\tilde{\rho}_{Ɣ})^{N}(\xi) \leq t_1$ $(\tilde{\rho}_{Ɣ})^{P}(ч) \geq t_2$
and $(\tilde{\rho}_{Ɣ})^{P}(\xi) \geq t_2$. Since $Ɣ^{(N,P)}$ is a **BVFSAS- Ӕ** of Ҝ, we get

1- $(\tilde{\rho}_{Ɣ})^{N}(ч+\xi) \leq max\{(\tilde{\rho}_{Ɣ})^{N}(ч),(\tilde{\rho}_{Ɣ})^{N}(\xi)\} \leq t_1$,
2- $(\tilde{\rho}_{Ɣ})^{P}(ч+\xi) \geq min\{(\tilde{\rho}_{Ɣ})^{P}(ч),(\tilde{\rho}_{Ɣ})^{P}(\xi)\} \geq t_2$,
3- $(\tilde{\rho}_{Ɣ})^{N}(ч-\xi) \leq max\{(\tilde{\rho}_{Ɣ})^{N}(ч),(\tilde{\rho}_{Ɣ})^{N}(\xi)\} \leq t_1$
4- $(\tilde{\rho}_{Ɣ})^{P}(ч-\xi) \geq min\{(\tilde{\rho}_{Ɣ})^{P}(ч),(\tilde{\rho}_{Ɣ})^{P}(\xi)\} \geq t_2$.

Therefor, $ч+\xi$, $ч-\xi \in \tilde{U}(Ɣ^{(N,P)};\tilde{t})$
Hence the set $\tilde{U}(Ɣ^{(N,P)};\tilde{t})$ is a Ӕ of Ҝ. ⌂

Proposition (3.9): In Ӕ. A bipolar valued fuzzy subset $Ɣ^{(N,P)} = <(\tilde{\rho}_{Ɣ})^{N},(\tilde{\rho}_{Ɣ})^{P}>$ of Ҝ .

If for all $\tilde{t} = [t_1, t_2] \in$ D[0, 1], the set $\widetilde{U}$ (Ɣ$^{(N,P)}$; $\tilde{t}$) is **SAS- Ǣ** of Ҽ, then Ɣ$^{(N,P)}$ is **BVFSAS- Ǣ** of Ҽ.

Proof.

Suppose that $\widetilde{U}$ (Ɣ$^{(N,P)}$; $\tilde{t}$) is a **SAS- Ǣ** of Ҽ and ч , ξ ∈ Ҽ be such that

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) > $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\}$,

Consider α = 1 / 2 { $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) + $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N$ (ξ)} }

and β = 1/2 { $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) + $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N$ (ξ)} }

We have $\alpha, \beta \in$ [0, 1], $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) >α> max { $(\tilde{\rho}_Ɣ)^N$ (ч),$(\tilde{\rho}_Ɣ)^N$ (ξ) },

and $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) >α> max { $(\tilde{\rho}_Ɣ)^N$ (ч),$(\tilde{\rho}_Ɣ)^N$ (ξ)}.

It follows thatч, $\xi \in \widetilde{U}$ (Ɣ$^{(N,P)}$; $\tilde{t}$) and (ч + ξ) ∉ $\widetilde{U}$ (Ɣ$^{(N,P)}$; $\tilde{t}$). This is a contradiction.

Hence, $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq t_1$. Summarily,

2- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq t_2$,

3- $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq t_1$ and

4- $(\tilde{\rho}_Ɣ)^P$ (ч − ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq t_2$.

Therefore Ɣ$^{(N,P)}$ is a **BVFSAS- Ǣ** of Ҽ. ⌂

Theorem (3.10): Any **SAS- Ǣ** of Ǣ can be realized as the upper [t_1,t_2]-Level of some **BVFSAS- Ǣ** of Ҽ.

Proof.

Suppose I be a **SAS- Ǣ** of Ҽ and Ɣ$^{(N,P)}$ = < $(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P$ > be bipolar valued fuzzy subset on Ҽ defined by

$\tilde{\rho}_Ɣ$(ч)=$\begin{cases}[\alpha_1, \alpha_2], & \text{if } ч \in I\\ [0,0], & \text{otherwise}\end{cases}$.

For all $[\alpha_1, \alpha_2] \in D[0,1]$,

we deliberate the following cases

Case 1) If ч , ξ ∈ I, then $(\tilde{\rho}_Ɣ)^N$ (ч) ≤ α_1 , $(\tilde{\rho}_Ɣ)^N$ (ξ) ≤ α_1, $(\tilde{\rho}_Ɣ)^P$ (ч) ≥ α_2 *and* $(\tilde{\rho}_Ɣ)^P$ (ξ) ≥ α_2 , thus

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$

2- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq \alpha_2$,

3- $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$ and

4- $(\tilde{\rho}_Ɣ)^P$ (ч − ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq \alpha_2$.

Case 2) If ч ∈ I and ξ ∉ I,then $(\tilde{\rho}_Ɣ)^N$ (ч) ≤ α_1 , $(\tilde{\rho}_Ɣ)^N$ (ξ) ≤ 0, $(\tilde{\rho}_Ɣ)^P$ (ч) ≥ α_2 *and* $(\tilde{\rho}_Ɣ)^P$ (ξ) ≥ 0 , thus

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$,

2- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq 0$,

3- $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$ and

4- $(\tilde{\rho}_Ɣ)^P$ (ч − ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq 0$.

Case 3) If ч ∉ I and y∈ I,then $(\tilde{\rho}_Ɣ)^N$ (ч) ≤ 0 , $(\tilde{\rho}_Ɣ)^N$ (ξ) ≤ α_1, $(\tilde{\rho}_Ɣ)^P$ (ч) ≥ 0 *and* $(\tilde{\rho}_Ɣ)^P$ (ξ) ≥ α_2 , thus

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\}$ ≤ α_1 ,

2- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq 0$,

3- $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$ and

4- $(\tilde{\rho}_Ɣ)^P$ (ч − ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq 0$.

Case 4) If ч ∉ I, ξ ∉ I and y,then $(\tilde{\rho}_Ɣ)^N$ (ч) ≤ 0 , $(\tilde{\rho}_Ɣ)^N$ (ξ) ≤ 0and $(\tilde{\rho}_Ɣ)^P$ (ч) ≥ 0 *and* $(\tilde{\rho}_Ɣ)^P$ (ξ) ≥ 0 , thus

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq 0$,

2- $(\tilde{\rho}_Ɣ)^P$ (ч + y) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(y)\} \geq 0$,

3- $(\tilde{\rho}_Ɣ)^N$ (ч − ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq 0$ and

4- $(\tilde{\rho}_Ɣ)^P$ (ч − ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq 0$.

Therefore, Ɣ$^{(N,P)}$ is a **BVFSAS- Ǣ** of Ҽ.⌂

Corollary (3.11): In Ǣ, Ɑ be a subset of Ҽ and let Ɣ$^{(N,P)}$ = < $(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P$ > be an bipolar valued fuzzy subset on Ҽ defined by :

$\tilde{\rho}_Ɣ$(ч) = $\begin{cases}[\alpha_1, \alpha_2] & if \quad ч \in Ɑ\\ [0,0] & ot\square erwise\end{cases}$.

Where α_1 , α_2 ∈ (0, 1] with $\alpha_1 < \alpha_2$. If Ɣ$^{(N,P)}$ is a **BVFSAS- Ǣ** of Ҽ,

then Ɑ is a SA-subalgebra of Ҽ.

Proof:

Since that Ɣ$^{(N,P)}$ is a **BVFSAS- Ǣ** of Ҽ. Let ч, ξ ∈ Ɑ, then by Definition(3.1)

1- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) ≤ $max\{(\tilde{\rho}_Ɣ)^N$(ч), $(\tilde{\rho}_Ɣ)^N(\xi)\} \leq \alpha_1$,

2- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) ≥ $min\{(\tilde{\rho}_Ɣ)^P$(ч), $(\tilde{\rho}_Ɣ)^P(\xi)\} \geq \alpha_2$,

3- $(\tilde{\rho}_{Ɣ})^N$ (ч − ξ) $\leq max\{(\tilde{\rho}_{Ɣ})^N(ч), (\tilde{\rho}_{Ɣ})^N(\xi)\} \leq \alpha_1$
and
4- $(\tilde{\rho}_{Ɣ})^P$ (ч − ξ) $\geq min\{(\tilde{\rho}_{Ɣ})^P(ч), (\tilde{\rho}_{Ɣ})^P(\xi)\} \geq \alpha_2$.
This implies that ч + ξ, ч − ξ ∈ Ꝋ. Hence Ꝋ is a **SAS-Ǣ** of Ꞓ. ⌂

Proposition (3.12): Assume𝔄: (Ꞓ; +, −,0) → (Ꝋ; +′, −′, 0′) be homomorphism of SA-algebras. If B is a **BVFSAS- Ǣ** of Ꝋ,
then the inverse image $𝔄^{-1}(B)$ of B is a **BVFSAS- Ǣ** of Ꞓ.

Proof:

Since $B^{(N,P)} = < (\tilde{\rho}_B)^N, (\tilde{\rho}_B)^P >$ is a **BVFSAS- Ǣ** of Ꝋ,
it follows from Theorem (3.6), that $(\rho_B^-)^N$ and $(\rho_B^+)^N$ are **AFSAS- Ǣ** of Ꝋ and $(\rho_B^-)^P$ and $(\rho_B^+)^P$ are **FSAS- Ǣ** of Ꝋ.
Using Theorem (2.19) and Theorem (2.27), we discern $𝔄^{-1}((\rho_B^-)^N)$ and $𝔄^{-1}((\rho_B^+)^N)$ are **AFSAS- Ǣ** of Ꞓ and $𝔄^{-1}((\rho_B^-)^P)$ and $𝔄^{-1}((\rho_B^+)^P)$ are **FSAS- Ǣ** of Ꞓ.
Hence $𝔄^{-1}(B) = [𝔄^{-1}((\tilde{\rho}_B)^N), 𝔄^{-1}((\tilde{\rho}_B)^P)]$ is a **BVFSAS- Ǣ** of Ꞓ. ⌂

Definition (3.13): Assume 𝔄: (Ꞓ; +, −,0) → (Ꝋ; +′, −′, 0′)be a mapping from a set Ꞓ into a set Ꝋ. $Ɣ^{(N,P)} = < (\tilde{\rho}_{Ɣ})^N, (\tilde{\rho}_{Ɣ})^P >$ is a bipolar valued subset of Ꞓ **has sup and inf properties** if for any subset T of Ꞓ,
there exist t, s ∈ T such that $\tilde{\rho}_{Ɣ}(t) = \underset{t0\in T}{rsup}\ \tilde{\rho}_{Ɣ}(t_0)$ and $\tilde{\rho}_{Ɣ}(t) = \underset{t0\in T}{rinf}\ \tilde{\rho}_{Ɣ}(t_0)$

Proposition (3.14): Let 𝔄: (Ꞓ; +, −,0) → (Ꝋ; +′, −′, 0′) be an epimorphism of SA-algebras.
If $Ɣ^{(N,P)} = < (\tilde{\rho}_{Ɣ})^N, (\tilde{\rho}_{Ɣ})^P >$ is a **BVFSAS- Ǣ** of Ꞓ with inf-sup property, then𝔄(Ɣ) is a **BVFSAS- Ǣ** of Ꝋ.

Proof:

Assume that $Ɣ^{(N,P)} = < (\tilde{\rho}_{Ɣ})^N, (\tilde{\rho}_{Ɣ})^P >$ is a **BVFSAS- Ǣ** of Ꞓ.
It follows from Theorem (3.6), that $(\rho_{Ɣ}^-)^N$ and $(\rho_{Ɣ}^+)^N$ are **AFSAS- Ǣ** of Ꞓ and $(\rho_{Ɣ}^-)^P$ and $(\rho_{Ɣ}^+)^P$ are **FSAS- Ǣ** of Ꞓ. of Ꞓ.
Using (2.21), Theorem (2.29), the images $𝔄((\rho_B^-)^N)$ and $𝔄((\rho_B^+)^N)$ are **AFSAS- Ǣ** of Ꝋ and $𝔄((\rho_B^-)^P)$ and $𝔄((\rho_B^+)^P)$ are **BVFSAS- Ǣ** of Ꝋ. Hence $𝔄(Ɣ^{(N,P)}) = < 𝔄((\tilde{\rho}_{Ɣ})^N), 𝔄((\tilde{\rho}_{Ɣ})^P) >$ is a **BVFSAS- Ǣ** of Ꝋ.⌂

4. BIPOLAR VALUED FUZZY SA-IDEALS OF SA-ALGEBRA

In the part, the conception of the bipolar valued fuzzy SA-ideals of SA-algebra is introduced. Some theorems and properties are detailed and evidenced.

Definition (4.1): A interval valued fuzzy subset
Ɣ = {< ч, $\tilde{\rho}_{Ɣ}$ (ч) >| ч ∈ Ꞓ}
= {< ч, [$\rho_{Ɣ}^-$ (ч), $\rho_{Ɣ}^+$ (ч)]>| ч ∈ Ꞓ} of SA-algebra (Ꞓ; +, −,0)
is named **a bipolar valued fuzzy SA-ideal (BVFSAI- Ǣ) of** Ꞓ signified by
$Ɣ^{(N,P)}$ = {< ч, $(\tilde{\rho}_{Ɣ})^N$ (ч), $(\tilde{\rho}_{Ɣ})^P$ (ч) >| ч ∈ Ꞓ}
= {< ч, $\widetilde{(\rho_{Ɣ}^N)}$ (ч), $\widetilde{(\rho_{Ɣ}^P)}$ (ч) >| ч ∈ Ꞓ } , $Ɣ^{(N,P)}$
$= < (\tilde{\rho}_{Ɣ})^N, (\tilde{\rho}_{Ɣ})^P >$, if for all ч, ξ, ς ∈ Ꞓ.
1- $(\tilde{\rho}_{Ɣ})^N$ (0) $\leq (\tilde{\rho}_{Ɣ})^N$(ч), $(\tilde{\rho}_{Ɣ})^P$ (0) $\geq (\tilde{\rho}_{Ɣ})^P$(ч).
2- $(\tilde{\rho}_{Ɣ})^N$ (ч + ξ) $\leq max\{(\tilde{\rho}_{Ɣ})^N(ч + ς), (\tilde{\rho}_{Ɣ})^N(\xi - ς)$
and
3- $(\tilde{\rho}_{Ɣ})^P$ (ч + ξ) $\geq min\{(\tilde{\rho}_{Ɣ})^P(ч + ς), (\tilde{\rho}_{Ɣ})^P(\xi - ς)\}$.
i.e.,
1- $\widetilde{(\rho_{Ɣ}^N)}$ (0) $\preccurlyeq \widetilde{(\rho_{Ɣ}^N)}$(ч) and $\widetilde{(\rho_{Ɣ}^P)}$ (0) $\succcurlyeq \widetilde{(\rho_{Ɣ}^P)}$(ч) ,
2- $\widetilde{(\rho_{Ɣ}^N)}$ (ч + ξ) $\preccurlyeq rmax\{\widetilde{(\rho_{Ɣ}^N)}(ч + ς), \widetilde{(\rho_{Ɣ}^N)}(\xi - ς)\}$,
3- $\widetilde{(\rho_{Ɣ}^P)}$ (ч + ξ) $\succcurlyeq rmin\{\widetilde{(\rho_{Ɣ}^P)}(ч + ς), \widetilde{(\rho_{Ɣ}^P)}(\xi - ς)\}$.
i.e.,
1-$(\rho_{Ɣ}^-)^N$ (0) $\leq (\rho_{Ɣ}^-)^N$(ч) and $(\rho_{Ɣ}^-)^P$ (0) $\geq (\rho_{Ɣ}^-)^P$(ч),
2- $(\rho_{Ɣ}^-)^N$ (ч + ξ) $\leq max\{(\rho_{Ɣ}^-)^N(ч + ς), (\rho_{Ɣ}^-)^N(\xi - ς)\}$ and
3- $(\rho_{Ɣ}^-)^P$ (ч + ξ) $\geq min\{(\rho_{Ɣ}^-)^P(ч + ς), (\rho_{Ɣ}^-)^P(\xi - ς)\}$.

Example (4.2): Let Ⱪ = {0, 1, 2, 3} in which the operations $(+,-)$ be define by the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

−	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then (Ⱪ; $+,-,0$) is Ӕ. Define $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ of Ⱪ where I ={0,1} is a SA-ideal of Ⱪ,such that:
The fuzzy subsets ρ^+: Ⱪ→ [0,1] and ρ^-: Ⱪ → $[-1,0]$ by:

$$Ɣ^{(N,P)}\ (ч) = \begin{cases} [[-0.4,-0.3],[0.3,0.7]] & if\, ч = \{0,1\} \\ [[-0.3,-0.2],[0.2,0.5]] & otherwise \end{cases}$$

Then $Ɣ^{(N,P)}$ (ч) is is a **BVFSAI-** Ӕ of Ⱪ.

Theorem (4.3): A bipolar valued fuzzy subset $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ of Ӕ is a **BVFSAI-** Ӕ of Ⱪ if and only if, $(\rho_Ɣ^-)^N$ and $(\rho_Ɣ^+)^N$ are **AFSAI-** Ӕ of Ⱪ and $(\rho_Ɣ^-)^P$ and $(\rho_Ɣ^+)^P$ are **FSAI-** Ӕ of Ⱪ.

Proof:

Suppose that $Ɣ^{(N,P)}$ is a **BVFSAI-** Ӕ of Ⱪ, then for all ч ∈ Ⱪ,
$[(\tilde{\rho}_Ɣ)^N(0), (\tilde{\rho}_Ɣ)^P(0)] \geq [(\tilde{\rho}_Ɣ)^N(ч), (\tilde{\rho}_Ɣ)^P(ч)]$, then $(\rho_Ɣ^-)^N(0) \leq (\rho_Ɣ^-)^N(ч)$ and $(\rho_Ɣ^-)^P(0) \geq (\rho_Ɣ^-)^P(ч)$. For all ч ∈ Ⱪ, we have

$[(\rho_Ɣ^-)^N(ч+\xi), (\rho_Ɣ^+)^N(ч+\xi)] = (\tilde{\rho}_Ɣ)^N(ч+\xi)$
$\leq r\,max\left\{(\tilde{\rho}_Ɣ)^N(ч+\varsigma), (\tilde{\rho}_Ɣ)^N(\xi-\varsigma)\right\}$
$= r\,max\{[(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^+)^N(ч+\varsigma)], [(\rho_Ɣ^-)^N(\xi-\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)]\}$
$= [max\{(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^+)^N(ч+\varsigma)\}, max\{(\rho_Ɣ^-)^N(\xi-\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)\}]$
$= [max\{(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^-)^N(\xi-\varsigma)\}, max\{(\rho_Ɣ^+)^N(ч+\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)\}]$

Therefore , $(\rho_Ɣ^-)^N(ч+\xi) \leq max\{(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^-)^N(\xi-\varsigma)\}$ and $(\rho_Ɣ^+)^N(ч+\xi) \leq max\{(\rho_Ɣ^+)^N(ч+\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)\}$.

Also,
$[(\rho_Ɣ^-)^P(ч+\xi), (\rho_Ɣ^+)^P(ч+\xi)] = (\tilde{\rho}_Ɣ)^P(ч+\xi) \succcurlyeq r\,min\{(\tilde{\rho}_Ɣ)^P(ч+\varsigma), (\tilde{\rho}_Ɣ)^P(\xi-\varsigma)\}$
$= r\,min\{[(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^+)^P(ч+\varsigma)], [(\rho_Ɣ^-)^P(\xi-\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)]\}$

$= [min\{(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^+)^P(ч+\varsigma)\}, min\{(\rho_Ɣ^-)^P(\xi-\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)\}]$

$= [min\{(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^-)^P(\xi-\varsigma)\}, min\{(\rho_Ɣ^+)^P(ч+\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)\}]$

Therefore, $(\rho_Ɣ^-)^P(ч+\xi) \geq min\{(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^-)^P(\xi-\varsigma)\}$ and $(\rho_Ɣ^+)^P(ч+\xi) \geq min\{(\rho_Ɣ^+)^P(ч+\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)\}$.

Hence, we become that $(\rho_Ɣ^-)^N$ and $(\rho_Ɣ^+)^N$ are **AFSAI-** Ӕ of Ⱪ and $(\rho_Ɣ^-)^P$ and $(\rho_Ɣ^+)^P$ are **FSAI-** Ӕ of Ⱪ.

Conversely, if $(\rho_Ɣ^-)^N$ and $(\rho_Ɣ^+)^N$ are **AFSAI-** Ӕ of Ⱪ and $(\rho_Ɣ^-)^P$ and $(\rho_Ɣ^+)^P$ are **FSAI-** Ӕ of Ⱪ, for all ч, ξ, ς ∈ Ⱪ. Observe :

$(\tilde{\rho}_Ɣ)^N(ч+\xi) = [(\rho_Ɣ^-)^N(ч+\xi), (\rho_Ɣ^+)^N(ч+\xi)]$
$\leq [max\{(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^-)^N(\xi-\varsigma)\}, min\{(\rho_Ɣ^+)^N(ч+\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)\}]$
$= r\,max\{[(\rho_Ɣ^-)^N(ч+\varsigma), (\rho_Ɣ^+)^N(ч+\varsigma)], [(\rho_Ɣ^-)^N(\xi-\varsigma), (\rho_Ɣ^+)^N(\xi-\varsigma)]\}$
$= r\,max\{(\tilde{\rho}_Ɣ)^N(ч+\varsigma), (\tilde{\rho}_Ɣ)^N(\xi-\varsigma)\}$. Also
$(\tilde{\rho}_Ɣ)^P(ч+\xi) = [(\rho_Ɣ^-)^P(ч+\xi), (\rho_Ɣ^+)^P(ч+\xi)]$
$\geq [min\{(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^-)^P(\xi-\varsigma)\}, min\{(\rho_Ɣ^+)^P(ч+\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)\}]$
$= r\,min\{[(\rho_Ɣ^-)^P(ч+\varsigma), (\rho_Ɣ^+)^P(ч+\varsigma)], [(\rho_Ɣ^-)^P(\xi-\varsigma), (\rho_Ɣ^+)^P(\xi-\varsigma)]\}$
$= r\,min\{(\tilde{\rho}_Ɣ)^P(ч+\varsigma), (\tilde{\rho}_Ɣ)^P(\xi-\varsigma)\}$.Thus, we can settle that $Ɣ^{(N,P)}$ is a **BVFSAI-** Ӕ of Ⱪ.⌂

Proposition (4.4): In Ӕ. A bipolar valued fuzzy subset $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ of Ⱪ. If $Ɣ^{(N,P)}$ is a **BVFSAI-** Ӕ of Ⱪ, then for any $\tilde{t} = [t_1, t_2]$∈ D[0, 1], the set $\tilde{U}$ ($Ɣ^{(N,P)}; \tilde{t}$) is a **SAI-** Ӕ of Ⱪ .

Proof.

Assume that $Ɣ^{(N,P)}$ is a **BVFSAI-** Ӕ of Ⱪ and let $\tilde{t} = [t_1, t_2]$∈ D[0, 1] be such that $\tilde{U}$ ($Ɣ^{(N,P)}; \tilde{t}$)≠∅, and assume ч, ξ, ς ∈ Ⱪ such that

$ч + ς , ξ - ς \in \widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$, then $(\tilde{\rho}_Ɣ)^N(ч + ς) \le t_1$, $(\tilde{\rho}_Ɣ)^N(\xi - ς) \le t_1$,
$(\tilde{\rho}_Ɣ)^P(ч + ς) \ge t_2$ and $(\tilde{\rho}_Ɣ)^P(\xi - z) \ge t_2$.
Since $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ꞓ, we get
1- $(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(ч) \le t_1$ and $(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(ч) \ge t_2$, implies that :
$0 \in \widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$,
2- $(\tilde{\rho}_Ɣ)^N(ч + \xi) \le max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\} \le t_1$,
3- $(\tilde{\rho}_Ɣ)^P(ч + \xi) \ge min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\} \ge t_2$,
Therefore, $ч + \xi \in \widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$, Hence the set $\widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$ is a **SAI-** Ǣ of Ꞓ. ⌂

Proposition (4.5): In Ǣ. A bipolar valued fuzzy subset $Ɣ^{(N,P)} = <(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P>$ of Ꞓ . If for all $\tilde{t} = [t_1, t_2] \in D[0, 1]$,
the set $\widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$ is an SAI-Ǣ of Ꞓ, then $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ꞓ.

roof.

Assume that $\widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$ is a **SAI-** Ǣ of Ꞓ ,for any $ч \in$ Ꞓ, $(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(ч) \le t_1$ and $(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(ч), \ge t_2$. And assume $ч, \xi, ς \in$ Ꞓ be such that $(\tilde{\rho}_Ɣ)^N(ч + \xi) > max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\}$,
Consider:
$\alpha = 1/2\ \{(\tilde{\rho}_Ɣ)^N(ч + \xi) + max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\}\}$ and
$\beta = 1/2\ \{(\tilde{\rho}_Ɣ)^P(ч + \xi) + \min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\}\}$

We have $\alpha, \beta \in [0,1]$, $(\tilde{\rho}_Ɣ)^N(ч + \xi) > \alpha > max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\}$ and $(\tilde{\rho}_Ɣ)^P(ч + \xi) < \beta < \min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\}$.
It follows that $ч + ς,\ \xi - ς \in \widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$ and $(ч + \xi) \notin \widetilde{U}\,(Ɣ^{(N,P)}; \tilde{t})$. This is a contradiction.
Therefore $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ꞓ. ⌂

Theorem (4.6): Any **SAI-** Ǣ of Ǣ can be realized as the upper $[t_1,t_2]$-Level of some **BVFSAI-** Ǣ of Ꞓ.

Proof.

Assume I be a **SAI-** Ǣ of Ꞓ and $Ɣ^{(N,P)} = <(\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P>$ be bipolar valued fuzzy subset on Ꞓ defined by

$$\tilde{\rho}_Ɣ(ч) = \begin{cases} [\alpha_1, \alpha_2], & \text{if } ч \in I \\ [0,0], & \text{otherwise} \end{cases}.$$

For all $[\alpha_1, \alpha_2] \in D[0,1]$, we contemplate the following cases:

Case 1) If $ч + ς, \xi - ς \in$ I, then
$(\tilde{\rho}_Ɣ)^N(ч) \le \alpha_1$, $(\tilde{\rho}_Ɣ)^N(\xi) \le \alpha_1$,
$(\tilde{\rho}_Ɣ)^P(ч) \ge \alpha_2\ and\ (\tilde{\rho}_Ɣ)^P(\xi) \ge \alpha_2$,
$(\tilde{\rho}_Ɣ)^N(ч + ς) \le \alpha_1$, $(\tilde{\rho}_Ɣ)^N(\xi - ς) \le \alpha_1$, and
$(\tilde{\rho}_Ɣ)^P(ч + ς) \ge \alpha_2\ and\ (\tilde{\rho}_Ɣ)^P(\xi - ς) \ge \alpha_2$, thus
1- $(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(ч) \le \alpha_1$ and $(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(ч) \ge \alpha_2$,
2- $(\tilde{\rho}_Ɣ)^N(ч + \xi) \le max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\} \le \alpha_1$,
3- $(\tilde{\rho}_Ɣ)^P(ч + \xi) \ge min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\} \ge \alpha_2$.

Case 2) If $ч \in$ I and $\xi \notin$ I,then
$(\tilde{\rho}_Ɣ)^N(ч) \le \alpha_1$, $(\tilde{\rho}_Ɣ)^N(\xi) \le 0$
$(\tilde{\rho}_Ɣ)^P(ч) \ge \alpha_2$ and $(\tilde{\rho}_Ɣ)^P(\xi) \ge 0$, thus
1- $(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(ч) \le \alpha_1$,
$(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(ч) \ge \alpha_2$,
$(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(\xi) \le 0$ and
$(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(\xi) \ge 0$,
$2 - (\tilde{\rho}_Ɣ)^N(ч + \xi) \le max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\} \le \alpha_1$,
3- $(\tilde{\rho}_Ɣ)^P(ч + \xi) \ge min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\} \ge 0$.

Case 3) If $ч \notin$ I and $\xi \in$ I,then
$(\tilde{\rho}_Ɣ)^N(ч) \le 0$, $(\tilde{\rho}_Ɣ)^N(\xi) \le \alpha_1$
and $(\tilde{\rho}_Ɣ)^P(ч) \ge 0\ and\ (\tilde{\rho}_Ɣ)^P(\xi) \ge \alpha_2$, thus
1- $(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(\xi) \le \alpha_1$,
$(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(\xi) \ge \alpha_2$,
$(\tilde{\rho}_Ɣ)^N(0) \le (\tilde{\rho}_Ɣ)^N(ч) \le 0$ and
$(\tilde{\rho}_Ɣ)^P(0) \ge (\tilde{\rho}_Ɣ)^P(ч) \ge 0$
$1 - (\tilde{\rho}_Ɣ)^N(ч + \xi) \le max\{(\tilde{\rho}_Ɣ)^N(ч + ς), (\tilde{\rho}_Ɣ)^N(\xi - ς)\} \le \alpha_1$
2- $(\tilde{\rho}_Ɣ)^P(ч + \xi) \ge min\{(\tilde{\rho}_Ɣ)^P(ч + ς), (\tilde{\rho}_Ɣ)^P(\xi - ς)\} \ge 0$.

Case 4) If $ч \notin$ Iand$\xi \notin$ I,then

$(\tilde{\rho}_Ɣ)^N$ (ч) ≤ 0 , $(\tilde{\rho}_Ɣ)^N (\xi) \leq 0$, $(\tilde{\rho}_Ɣ)^P$ (ч) $\geq$ 0 *and* $(\tilde{\rho}_Ɣ)^P (\xi) \geq 0$, thus

1- $(\tilde{\rho}_Ɣ)^N (0) \leq (\tilde{\rho}_Ɣ)^N$(ч) ≤ 0,
$(\tilde{\rho}_Ɣ)^P (0) \geq (\tilde{\rho}_Ɣ)^P$(ч) ≥ 0,
$(\tilde{\rho}_Ɣ)^N (0) \leq (\tilde{\rho}_Ɣ)^N(\xi) = 0$ and
$(\tilde{\rho}_Ɣ)^P (0) \geq (\tilde{\rho}_Ɣ)^P(\xi) = 0$,

2 − $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) $\leq max\{(\tilde{\rho}_Ɣ)^N$(ч + ς), $(\tilde{\rho}_Ɣ)^N(\xi - \varsigma)\} \leq 0$,

3- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) $\geq min\{(\tilde{\rho}_Ɣ)^P$(ч + ς), $(\tilde{\rho}_Ɣ)^P(\xi - \varsigma)\} \geq 0$.

Therefore, $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ҝ.⌂

Corollary (4.7): Let (Ҝ ; +, −, 0) be a **SAS-** Ǣ, Ꝙ be a subset of Ҝ
and let $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ be an bipolar valued fuzzy subset on Ҝ

defined by : $\tilde{\rho}_Ɣ$(ч) $= \begin{cases} [\alpha_1, \alpha_2] & if \quad ч \in Ꝙ \\ [0,0] & otherwise \end{cases}$.

Where $\alpha_1, \alpha_2 \in (0, 1]$ with $\alpha_1 < \alpha_2$. If $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ҝ, then Ꝙ is a SAI- Ǣ of Ҝ.

Proof:

Since that $Ɣ^{(N,P)}$ is a **BVFSAI-** Ǣ of Ҝ. Assume ч, $\xi, \varsigma \in$ Ꝙ,
then $\tilde{\rho}_Ɣ$(ч − 0 = ч) $= [\alpha_1, \alpha_2] = \tilde{\rho}_Ɣ(\xi - 0 = \xi)$, so we have by Theorem (4.6),

1- $(\tilde{\rho}_Ɣ)^N (0) \leq (\tilde{\rho}_Ɣ)^N(x) \leq \alpha_1,$ $(\tilde{\rho}_Ɣ)^P (0) \geq (\tilde{\rho}_Ɣ)^P$(ч) $\geq \alpha_2$,
$(\tilde{\rho}_Ɣ)^N (0) \leq (\tilde{\rho}_Ɣ)^N(\xi) \leq \alpha_1$ and $(\tilde{\rho}_Ɣ)^P (0) \geq (\tilde{\rho}_Ɣ)^P(\xi) \geq \alpha_2$,

2- $(\tilde{\rho}_Ɣ)^N$ (ч + ξ) $\leq max\{(\tilde{\rho}_Ɣ)^N$(ч + ς), $(\tilde{\rho}_Ɣ)^N(\xi - \varsigma)\} \leq \alpha_1$,

3- $(\tilde{\rho}_Ɣ)^P$ (ч + ξ) $\geq min\{(\tilde{\rho}_Ɣ)^P$(ч + ς), $(\tilde{\rho}_Ɣ)^P(\xi - \varsigma)\} \geq \alpha_2$,

this implies that 0, ч + $\xi \in$ Ꝙ. Hence Ꝙ is a **SAI-** Ǣ of Ҝ.⌂

Proposition (4.8): Every **BVFSAI-** Ǣ of Ǣ is **BVFSAS-** Ǣ of Ǣ .

Proof:

Since $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ is **BVFSAI-** Ǣ of Ǣ, then by Theorem (4.6) ,
$\tilde{U}$ ($Ɣ^{(N,P)}; \tilde{t}$) is a **SAI-** Ǣ of Ҝ. By Proposition (2.7), $\tilde{U}$ ($Ɣ^{(N,P)}; \tilde{t}$)is a **SAS-** Ǣ of Ҝ. Hence $Ɣ^{(N,P)}$is **BVFSAS-** Ǣ of Ҝ by Proposition (3.9) . ⌂

Remark (4.9): The convers of Proposition (4.8) is not true as shows in the example (3.3),
it is easy to show that (Ҝ ; +, −, 0)is Ǣ .And the fuzzy subsets ρ^+: Ҝ→ [0,1] and ρ^-: Ҝ → [−1,0] by:
Define a bipolar valued subset $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ of Ҝ is a BVFSAS- Ǣ of Ҝ as:

$Ɣ^{(N,P)}$ (ч) = $\begin{cases} [[-0.6, -0.3], [0.3, 0.9]] & if\, ч = \{0, a\} \\ [[-0.7, -0.4], [0.2, 0.6]] & otherwise \end{cases}$

It is easy to check that $Ɣ^{(N,P)}$ is a **BVFSAS-** Ǣ, but not **BVFSAI-** Ǣ.

Proposition (4.10): Let 𝔄: (Ҝ; +, −,0) → (Ꝙ; +′, −′, 0′) be homomorphism of SA-algebras.
If B is a **BVFSAI-** Ǣ of Ꝙ, then the inverse image $𝔄^{-1}(B)$ of B is a **BVFSAI-** Ǣ of Ҝ.

Proof:

Since $B^{(N,P)} = < (\tilde{\rho}_B)^N, (\tilde{\rho}_B)^P >$ is a **BVFSAI-** Ǣ of Ꝙ, it follows from Theorem (4.3), that $(\rho_B^-)^N$ and $(\rho_B^+)^N$ are **AFSAI-** Ǣ of Ꝙ
and $(\rho_B^-)^P$ and $(\rho_B^+)^P$ are **FSAI-** Ǣ of Ꝙ.
Using Theorem (2.19) and Theorem (2.27),
we know $𝔄^{-1}((\rho_B^-)^N)$ and $𝔄^{-1}((\rho_B^+)^N)$ are **AFSAI-** Ǣ of Ҝ
and $𝔄^{-1}((\rho_B^-)^P)$ and $𝔄^{-1}((\rho_B^+)^P)$ are **FSAI-** Ǣ of Ҝ.
Hence $𝔄^{-1}(B) = [𝔄^{-1}((\tilde{\rho}_B)^N), 𝔄^{-1}((\tilde{\rho}_B)^P)]$ is a **BVFSAI-** Ǣ of Ҝ. ⌂

Proposition (4.11): Assume 𝔄: (Ҝ; +, −,0) → (Ꝙ; +′, −′, 0′) be an epimorphism
of SA-algebras. If $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ is **BVFSAI-** Ǣ of Ҝ with inf-sup property, then f(Ɣ) is a **BVFSAI-** Ǣ of Ꝙ.

Proof:

Assume that $Ɣ^{(N,P)} = < (\tilde{\rho}_Ɣ)^N, (\tilde{\rho}_Ɣ)^P >$ is a **BVFSAI-** Ǣ of Ҝ, it follows from Theorem (4.3), that $(\rho_Ɣ^-)^N$ and $(\rho_Ɣ^+)^N$ are **AFSAI-** Ǣ of Ҝ and $(\rho_Ɣ^-)^P$ and $(\rho_Ɣ^+)^P$ are **FSAI-** Ǣ. of Ҝ. Using Theorem (2.21) andTheorem (2.29), the images $𝔄((\rho_B^-)^N)$ and $𝔄((\rho_B^+)^N)$ are **AFSAI-** Ǣ of Ꝙ and $𝔄((\rho_B^-)^P)$ and $𝔄((\rho_B^+)^P)$ are **FSAI-** Ǣ of Ꝙ. Hence $𝔄(Ɣ^{(N,P)}) = < 𝔄((\tilde{\rho}_Ɣ)^N), 𝔄((\tilde{\rho}_Ɣ)^P) >$ is a **BVFSAI-** Ǣ of Ꝙ.⌂

CONCLLUSION

The idea of this study avails as abasis for of new readings in the SA-algebra. We started by explaining of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals on SA-algebras with their properties and substantial examples and theorems and The image and inverse image of them are defined.

REFERENCES

[1] Akrm, M.,2011. Bipolar fuzzy graph, S. inf. Sci. ,181,5548-5564.

[2] Areej T. Hameed,2015." Fuzzy ideal of some algebras," Faculty of Science, Ain Shams University, Egypt; PH.D.SC.Thesis.

[3] charye P. Bhatta and Mukheriee, N.P. ,1985. Fuzzy Relations and Fuzzy Group, Inform Sci, vol.36, PP:267-282.

[4] Hameed, A.T. and Abed,A.S. ,2023.Some Fuzzy Properties on SA-algebra, to appear.

[5] Hameed, A.T. and Abed, A.S**,** 2023. Bipolar Fuzzy *SA*-ideals of *SA*-algebra,to appear.

[6] Hameed, A.T., Ali, S.H. and Flayyih, R.A.,2021The Bipolar-valued of Fuzzy Ideals on AT-algebra. Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-9.

[7] Hameed, A.T. and Kadhim, E.K., 2020. Interval-valued IFAT-ideals of AT-algebra. Journal of Physics: Conference Series (IOP Publishing), pp:1-5.

[8] Hameed, A.T. and Raheem, N.J. ,2021. Interval-valued Fuzzy SA-ideals with Degree (λ,κ) of SA-algebra. Journal of Physics: Conference Series (IOP Publishing), pp:1-13.

[9] Hameed, A.T, Raheem, N.J. and Abed, A.H.2021. Anti-fuzzy SA-ideals with Degree (λ,κ) of SA-algebra. Journal of Physics: Conference Series (IOP Publishing), pp:1-16.

[10] Zadeh, L.A, 1965. Fuzzy sets, Inform. and control ,vol.8 , 338-353 .

فترات ثنائي القطب للجبر الضبابي الجزئي-SA والمثالي الضبابي -SA في جبر -SA

الاء صالح عبد[1] ، اريج توفيق حميد[2]

[2,1]قسم الرياضيات، كلية التربية للبنات، جامعة الكوفة ، النجف، العراق
alaas.abed@uokufa.edu.iq , areej.tawfeeq@uokufa.edu.iq

الملخص:

في هذا البحث قدمنا تعريف لفترات ثنائي القطب للجبر الجزئي- SA وفترات ثنائي القطب للمثالي-SA في جبر-SA مع ذكر الخواص لكل منهم كما قمنا باعطاء وبرهان مجموعة من المبرهنات مع ذكر مجموعة من الامثلة الخاصة بهم. ايضا قمنا بتعريف الصورة والصورة العكسية ضمن تعريف التشاكل من جبر -SA الى جبر-SA واثبتنا ان الصورة لفترة ثنائي القطب للجبر الجزئي-SA هي ايضا فترة ثنائي القطب للجبر الجزئي -SA وكذلك الصورة العكسية لفترة ثنائي القطب للجبر الجزئي-SA هي ايضا فترة ثنائي القطب للجبر الجزئي -SA .كما قدمنا تعريف جديد و هو القيم السالبة للجبر الجزئي الضد ضبابي مع ذكر الامثلة والمبرهنات الخاصة به والمرتبطة بفترات ثنائي القطب للجبر الجزئي -SA .ثم بعد ذلك انتقلنا الى اثبات ان الصورة والصورة العكسية لفترات ثنائي القطب للمثالي -SA انها فترات ثنائي القطب للمثالي -SA وقمنا بتعريف جديد هو القيم السالبة للمثالي الضد العكسي مع ذكر الامثلة والمبرهنات المتعلقة بربط هذا التعريف بفترات ثنائي القطب للمثالي -SA من خلال مجموعة من المبرهنات.

الكلمات المفتاحية: جبر-SA وفترات ثنائي القطب للجبر الجزئي-SA وفترات ثنائي القطب للمثالي -SA.