

A Review on the Integral Transforms

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ARTICLE INFO

Received: 24 / 06 /2023

Accepted: 07 / 08 / 2023

Available online: 18 / 12 / 2023

DOI: [10.37652/juaps.2023.141302.1090](https://doi.org/10.37652/juaps.2023.141302.1090)

Keywords:

Integral Transforms, Double of Double of Integral Transforms.

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ABSTRACT

Many integral transformations have been proposed, tasted and proven to solve many applications in various scientific fields. The diversity of integral transformations comes from their unique ability to solve problems by transforming them from one domain where the solution is given by a complex mathematical procedure to another domain where simple algebraic methods can solve them. On this basis, many of these transformations and knowledge appeared on the field of real numbers and the field of complex numbers. The reason for the diversity of these transforms is the need for them in solving some mathematical equations and physical equations or that pertain to any branch of applied sciences, so they have proven their strength in solving differential and complementary equations and differential systems in addition to scientific applications. The Laplace Transform was one of the first transformations that contributed to the solution of mathematical equations, and then dozens of transformations appeared, which depended mainly on the Laplace Transform. This work demonstrates the temporal diversity of the integral transformations by presenting a group of integral transformations with their basic characteristics. We dealt with single and double integrals.

Introduction:

The mathematical technique responsible for transforming differential equation into algebraic equation is referred to as the integral transform and this procedure is responsible for converting a complicated problem into a simpler one to be solved mathematically. The integral transform as a mathematical operator can produce a result function by integrating the product of a function with another function called the kernel of the integral transform. The general format of the integral transform can be written as follows:

$$F(s) = \int_a^b k(x, s)f(x)dx.$$

Where $F(s)$ is the function resulting from the integral transform, $k(x, s)$ is their kernel function and $a, b \in (-\infty, \infty), \exists a < b$.

Integral transform can convert function from one domain where some mathematical procedures are fairly difficult into another domain where they become more malleable and mathematically smoother to process.

The resulting function is usually transformed back into its original domain via an inverse form of the used integral transformed [1, 2].

In (1763), Euler introduced the integral transformation to the world for the first time [3]. Since that time, mathematicians have been eager to invest their time and effort to investigate, propose and test the capabilities of using existing and new integral transforms in different aspects of life applications [4-7].

Over the years, many integral transforms have been proposed and most of these transforms have been named after the mathematicians who proposed them. Some integral transforms and their properties have been chronologically displayed in this work.

Experimental:

1.Integral transforms

Here, the integral transforms have been displayed with some their properties arranged according to chronology of their establishment from the oldest to the most recent.

1. Laplace Transformation [8]

One of the origin of the integral transforms is the Laplace transform can be traced back to celebrated work

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of P.S. Laplace (1749-1827) on probability theory in (1780s). In fact, Laplace's classic book includes some basic results of the Laplace transform which is one of the oldest and most commonly used integral transforms available in the mathematical literature.

Laplace transform is defined for the function $f(t)$ as:

$$L\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt = F(s), \quad \text{Re}(s) > 0.$$

The Laplace transform of some elementary functions are:

1. $L\{t^n\} = \frac{n!}{s^{n+1}}, \quad n \in N.$
2. $L\{e^{\alpha t}\} = \frac{1}{s-\alpha}, \quad \alpha \in R.$
3. $L\{\sin \sin (\alpha t)\} = \frac{\alpha}{s^2+\alpha^2}.$
4. $L\{\cos \cos (\alpha t)\} = \frac{s}{s^2+\alpha^2}.$
5. $L\{\sinh \sinh (\alpha t)\} = \frac{\alpha}{s^2-\alpha^2}.$
6. $L\{\cosh \cosh (\alpha t)\} = \frac{s}{s^2-\alpha^2}.$

The Laplace Transform of Derivatives:

Let $L\{f(t)\} = F(s)$, then

- i. $L\{f'(t)\} = s F(s) - f(0).$
- ii. $L\{f''(t)\} = s^2 F(s) - sf(0) - f'(0).$
- iii. $L\{f^{(n)}(t)\} = s^n F(s) - \sum_{j=0}^{n-1} s^{n-j-1} f^{(j)}(0), \quad n = 1, 2, 3, \dots$

2. Sumudu Transformation [9]

In (1993), G.K. Watugala introduced this transform. Sumudu Transform is defined for the function $f(t)$ as:

$$S\{f(t)\} = \frac{1}{v} \int_0^{\infty} f(t)e^{-\frac{t}{v}} dt = G(v), \quad v \in (-k_1, k_2), k_1, k_2 > 0, v \neq 0.$$

The Sumudu transform of some elementary functions are:

1. $S\{t^n\} = \frac{v^n}{n!}, \quad n \in N.$
2. $S\{e^{\alpha t}\} = \frac{1}{1-\alpha v}, \quad \alpha \in R.$
3. $S\{\sin \sin (\alpha t)\} = \frac{\alpha v}{1+\alpha^2 v^2}.$
4. $S\{\cos \cos (\alpha t)\} = \frac{1}{1+\alpha^2 v^2}.$
5. $S\{\sinh \sinh (\alpha t)\} = \frac{\alpha v}{1-\alpha^2 v^2}.$
6. $S\{\cosh \cosh (\alpha t)\} = \frac{1}{1-\alpha^2 v^2}.$

The Sumudu Transform of Derivatives:

Let $S\{f(t)\} = G(v)$, then

- i. $S\{f'(t)\} = \frac{G(v)-f(0)}{v}.$
- ii. $S\{f''(t)\} = \frac{G(v)-f(0)}{v^2} - \frac{f'(0)}{v}.$
- iii. $S\{f^{(n)}(t)\} = \frac{G(v)}{v^n} - \sum_{j=0}^{n-1} \frac{f^{(j)}(0)}{v^{n-j}}, \quad n = 1, 2, 3, \dots$

3. Natural Transform [10]

In (2008), Khan, Z. H. et al. presented an integral transform named the Nature Transform. This transform is defined for the function $f(t)$ as:

$$N^+\{f(t)\} = \int_{t=0}^{\infty} f(vt)e^{-st} dt = R(v, s), \quad s, v > 0.$$

The Nature transform of some elementary functions are:

1. $N^+\{t^n\} = \frac{n!v^n}{s^{n+1}}, \quad n \in N.$
2. $N^+\{e^{\alpha t}\} = \frac{1}{s-\alpha v}, \quad \alpha \in R.$
3. $N^+\{\sin \sin (\alpha t)\} = \frac{\alpha v}{s^2+\alpha^2 v^2}.$
4. $N^+\{\cos \cos (\alpha t)\} = \frac{s}{s^2+\alpha^2 v^2}.$
5. $N^+\{\sinh \sinh (\alpha t)\} = \frac{\alpha v}{s^2-\alpha^2 v^2}.$
6. $N^+\{\cosh \cosh (\alpha t)\} = \frac{s}{s^2+\alpha^2 v^2}.$

The Natural Transform of Derivatives:

Let $S\{f(t)\} = G(v)$, then

- i. $N^+\{f'(t)\} = \frac{s}{v} R(v, s) - \frac{f(0)}{v}.$
- ii. $N^+\{f''(t)\} = \frac{s^2 R(v, s) - sf(0) - f'(0)}{v^2}.$
- iii. $N^+\{f^{(n)}(t)\} = \frac{s^n R(v, s)}{v^n} - \sum_{j=0}^{n-1} \frac{s^{n-j-1} f^{(j)}(0)}{v^{n-j}}, \quad n = 1, 2, 3, \dots$

4. Al-Tememe Transformation [11]

In (2008), another transform presented by Ali Hassan Mohammed et al. is called Al-Tememe Transform. Al-Tememe Transform is convergent and it is defined for the function $f(t)$ as:

$$T\{f(t)\} = \int_1^{\infty} f(t)t^{-p} dt = F(p), \quad p > 0.$$

Al-Tememe transform of some elementary functions are:

1. $T\{x^n\} = \frac{1}{p-(n+1)}, \quad p > n + 1.$
2. $T\{\ln \ln t\} = \frac{1}{(p-1)^2}, \quad p > 1.$
3. $T\{t^n \ln \ln t\} = \frac{1}{(p-(n+1))^2}, \quad n \in R, p > n + 1.$
4. $T\{\sin \sin (\alpha \ln \ln t)\} = \frac{\alpha}{(p-1)^2+\alpha^2}.$
5. $T\{\cos \cos (\alpha \ln \ln t)\} = \frac{p-1}{(p-1)^2+\alpha^2}.$

$$6. T\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{\alpha}{(p-1)^2-\alpha^2}.$$

$$7. T\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{p-1}{(p-1)^2-\alpha^2}.$$

Al-Tememe Transform of Derivatives:

Let $T\{f(t)\} = F(p)$, then

$$T\{t^n f^{(n)}(t)\} = -f^{(n-1)}(1) - (p-n)f^{(n-2)}(1) - \dots - (p-n)(p-(n-1))(p-(n-2)) \dots (p-2) f(1) + (p-n)! F(p),$$

$$n = 1, 2, 3, \dots$$

5. Tarig Transform [12]

In (2011) Tarig M. E. and Salih M. E. introduced a new integral transform named Tarig transform. This Transform is defined for the function $f(t)$ as:

$$T\{f(t)\} = \frac{1}{u} \int_0^\infty f(t) e^{-\frac{t}{u^2}} dt = F(u), \quad u \neq 0.$$

Tarig transform of some elementary functions are:

$$1. T\{t^n\} = n! u^{2n+1}, \quad n \in N.$$

$$2. T\{e^{\alpha t}\} = \frac{u}{1-\alpha u^2}, \quad \alpha \in R.$$

Tarig Transform of Derivatives:

Let $T\{f(t)\} = F(u)$, then

$$i. T\{f'(t)\} = \frac{1}{u^2} F(u) - \frac{1}{u} f(0).$$

$$ii. T\{f''(t)\} = \frac{1}{u^4} F(u) - \frac{1}{u^3} f(0) - \frac{1}{u} f'(0).$$

$$iii. T\{f^{(n)}(t)\} = \frac{1}{u^{2n}} F(u) - \sum_{j=1}^n u^{2(j-n)-1} f^{(j-1)}(0),$$

$$n = 1, 2, 3, \dots$$

6. Elzaki Transformation [13]

In (2011), Tarig. M. Elzaki introduced Elzaki integral Transform. This transform is defined for the function $f(t)$ as:

$$E\{f(t)\} = v \int_{t=0}^\infty f(t) e^{-\frac{t}{v}} dt = T(v), \quad v \in (k_1, k_2), k_1, k_2 > 0.$$

Elzaki transform of some elementary functions are:

$$1. E\{t^n\} = n! v^{n+2}, \quad n \in N.$$

$$2. E\{e^{\alpha t}\} = \frac{v^2}{1-\alpha v}, \quad \alpha \in R.$$

$$3. E\{\sin \sin (\alpha t)\} = \frac{\alpha v^3}{1+\alpha^2 v^2}.$$

$$4. E\{\cos \cos (\alpha t)\} = \frac{v^2}{1+\alpha^2 v^2}.$$

$$5. E\{\sinh \sinh (\alpha t)\} = \frac{\alpha v^3}{1-\alpha^2 v^2}.$$

$$6. E\{\cosh \cosh (\alpha t)\} = \frac{v^2}{1-\alpha^2 v^2}.$$

Elzaki Transform of Derivatives:

Let $E\{f(t)\} = T(v)$, then

$$i. E\{f'(t)\} = \frac{T(v)}{v} - v f(0).$$

$$ii. E\{f''(t)\} = \frac{T(v)}{v^2} - f(0) - v f'(0).$$

$$iii. E\{f^{(n)}(t)\} = \frac{T(v)}{v^n} - \sum_{j=0}^{n-1} v^{2-n+j} f^{(j)}(0), \quad n = 1, 2, 3, \dots$$

$$E\{f^{(n)}(t)\} = \frac{T(v)}{v^n} - \sum_{j=0}^{n-1} v^{2-n+j} f^{(j)}(0), \quad n = 1, 2, 3, \dots$$

7. Aboodh Transformation [14]

In (2013), Khalid Suliman Aboodh presented an integral transform called the Aboodh Transform. This transform is defined for the function $f(t)$ as:

$$A\{f(t)\} = \frac{1}{v} \int_0^\infty f(t) e^{-vt} dt = K(v), \quad t \geq 0, k_1 \leq v \leq k_2.$$

Aboodh transform of some elementary functions are:

$$1. A\{t^n\} = \frac{n!}{v^{n+2}}, \quad n \geq 0.$$

$$2. A\{e^{\alpha t}\} = \frac{1}{v^2-\alpha v}, \quad \alpha \in R.$$

$$3. A\{\sin \sin (\alpha t)\} = \frac{\alpha}{v(\alpha^2+v^2)}.$$

$$4. A\{\cos \cos (\alpha t)\} = \frac{1}{(\alpha^2+v^2)}.$$

$$5. A\{\sinh \sinh (\alpha t)\} = \frac{\alpha}{v(v^2-\alpha^2)}.$$

$$6. A\{\cosh \cosh (\alpha t)\} = \frac{1}{(v^2-\alpha^2)}.$$

Aboodh Transform of Derivatives:

Let $A\{f(t)\} = K(v)$, then

$$i. A\{f'(t)\} = vK(v) - \frac{f(0)}{v}.$$

$$ii. A\{f''(t)\} = v^2K(v) - \frac{f'(0)}{v} - f(0).$$

$$iii. A\{f^{(n)}(t)\} = v^nK(v) - \sum_{j=0}^{n-1} \frac{f^{(j)}(0)}{v^{2-n+j}}, \quad n = 1, 2, 3, \dots$$

8. Kashuri and Fundo Integral Transform [15]

A new integral transform was introduced in (2013) by Artion Kashuri and Akli Fundo. This transform is defined for the function $f(t)$ as:

$$K\{f(t)\} = \frac{1}{v} \int_{t=0}^\infty f(t) e^{-\frac{t}{v^2}} dt = A(v), \quad t \geq 0, k_1 \leq v \leq k_2.$$

Kashuri and Fundo transform of some elementary functions are:

$$1. K\{t^n\} = n! v^{2n+1}, \quad n \in N.$$

2. $K\{e^{\alpha t}\} = \frac{v}{1-\alpha v^2}, \alpha \in R.$
3. $K\{\sin \sin (\alpha t)\} = \frac{\alpha v^3}{1+\alpha^2 v^4}.$
4. $K\{\cos \cos (\alpha t)\} = \frac{v}{1+\alpha^2 v^4}.$
5. $K\{\sinh \sinh (\alpha t)\} = \frac{\alpha v^3}{1-\alpha^2 v^4}.$
6. $K\{\cosh \cosh (\alpha t)\} = \frac{v}{1-\alpha^2 v^4}.$

Kashuri and Fundo Transform of Derivatives:

Let $K\{f(t)\} = A(v)$, then

- i. $K\{f'(t)\} = \frac{A(v)}{v^2} - \frac{f(0)}{v}.$
- ii. $K\{f''(t)\} = \frac{A(v)}{v^4} - \frac{f(0)}{v^3} - \frac{f'(0)}{v}.$
- iii. $K\{f^{(n)}(t)\} = \frac{A(v)}{v^{2n}} - \sum_{j=0}^{n-1} \frac{f^{(j)}(0)}{v^{2(n-j)-1}}, n = 1, 2, 3, \dots$

After while integral transformations have emerged in math ' which were used in solving mathematical equations and other scientific specialties.

Here are some transformations would be reviewed according to the years of their emersion.

In (2016)

1. ZZ Integral Transformation [16]

Zain Ulabadin Zafar introduced an integral transform named the ZZ Transform. This transform is defined for the function $f(t)$ as:

$$H\{f(t)\} = s \int_0^\infty f(vt)e^{-st} dt = Z(v, s), \quad t \geq 0, k_1 \leq v \leq k_2.$$

ZZ transform of some elementary functions are:

1. $H\{t^n\} = \frac{n!v^n}{s^n}, n \in N.$
2. $H\{e^{\alpha t}\} = \frac{s}{s-\alpha v}, \alpha \in R.$

ZZ Transform of Derivatives:

Let $H\{f(t)\} = Z(v, s)$, then

- i. $H\{f'(t)\} = \frac{s}{v}Z(v, s) - f(0).$
- ii. $H\{f''(t)\} = \frac{s^2}{v^2}Z(v, s) - \frac{sf'(0)}{v} - \frac{sf(0)}{v^2}.$
- iii. $H\{f^{(n)}(t)\} = \frac{s^n}{v^n}Z(v, s) - \sum_{j=0}^{n-1} \frac{s^{n-1}}{v^{n-j}}f^{(j)}(0), n = 1, 2, 3, \dots$
 $H\{f^{(n)}(t)\} = \frac{s^n}{v^n}Z(v, s) - \sum_{j=0}^{n-1} \frac{s^{n-1}}{v^{n-j}}f^{(j)}(0), n = 1, 2, 3, \dots$

10. Ramadan Group (RG) Transform [17]

Mohamed A. Ramadan et al. presented an integral transform named the Ramadan Group (RG) Transform. This transform is defined for the function $f(t)$ as:

$$RG\{f(t)\} = \int_0^\infty f(vt)e^{-st} dt = K(v, s), \quad s, v > 0.$$

The RG transform of some elementary functions are:

1. $RG\{t^n\} = \frac{v^n n!}{s^{n+1}}, n \in N.$
2. $RG\{e^{\alpha t}\} = \frac{1}{s+\alpha v}, \alpha \in R.$
3. $RG\{\sin \sin (\alpha t)\} = \frac{\alpha v}{s^2+\alpha^2 v^2}.$
4. $RG\{\cos \cos (\alpha t)\} = \frac{s}{s^2+\alpha^2 v^2}.$
5. $RG\{\sinh \sinh (\alpha t)\} = \frac{\alpha v}{s^2-\alpha^2 v^2}.$
6. $RG\{\cosh \cosh (\alpha t)\} = \frac{s}{s^2-\alpha^2 v^2}.$

Ramadan Group (RG) Transform of Derivatives:

Let $RG\{f(t)\} = K(v, s)$, then

- i. $RG\{f'(t)\} = \frac{s}{v}K(v, s) - \frac{f(0)}{v}.$
- ii. $RG\{f''(t)\} = \frac{s^2}{v^2}K(v, s) - \frac{f'(0)}{v} - \frac{sf(0)}{v^2}.$
- iii. $RG\{f^{(n)}(t)\} = \frac{s^n}{v^n}K(v, s) - \sum_{j=0}^{n-1} \frac{s^{n-j-1}f^{(j)}(0)}{v^{n-j}},$
 $n = 1, 2, 3, \dots$
 $RG\{f^{(n)}(t)\} = \frac{s^n}{v^n}K(v, s) - \sum_{j=0}^{n-1} \frac{s^{n-j-1}f^{(j)}(0)}{v^{n-j}}, n = 1, 2, 3, \dots$

11. Mahgoub Transformation [18]

A transform suggested by Mohand M. et al. named Mahgoub transform. This transform is defined for the function $f(t)$ as:

$$M\{f(t)\} = v \int_0^\infty f(t)e^{-vt} dt = H(v), \quad v \in [k_1, k_2], t \geq 0.$$

Mahgoub transform of some elementary functions are:

1. $M\{t^n\} = \frac{n!}{v^n}, n \in N.$
2. $M\{e^{\alpha t}\} = \frac{v}{v-\alpha}, \alpha \in R.$
3. $M\{\sin \sin (\alpha t)\} = \frac{\alpha v}{\alpha^2+v^2}.$
4. $M\{\cos \cos (\alpha t)\} = \frac{v^2}{\alpha^2+v^2}.$
5. $M\{\sinh \sinh (\alpha t)\} = \frac{\alpha v}{v^2-\alpha^2}.$
6. $M\{\cosh \cosh (\alpha t)\} = \frac{v^2}{v^2-\alpha^2}.$

Mahgoub Transform of Derivatives:

Let $M\{f(t)\} = H(v)$, then

- i. $M\{f'(t)\} = vH(v) - vf(0).$
- ii. $M\{f''(t)\} = v^2H(v) - v^2f(0) - vf'(0).$
- iii. $M\{f^{(n)}(t)\} = v^nH(v) - \sum_{j=0}^{n-1} v^{n-j}f^{(j)}(0), n = 1, 2, 3, \dots$

12. Kamal Transformation [19]

This transform proposed by Abdelilah Kamal et al. and called the Kamal transform. The integral transform is defined for a function $f(t)$ as:

$$K\{f(t)\} = \int_0^\infty f(t)e^{-\frac{t}{v}} dt = G(v), \quad v \in (k_1, k_2), k_1, k_2 > 0.$$

The Kamal transform of some elementary functions are:

1. $K\{t^n\} = n! v^{n+1}, n \in N.$
2. $K\{e^{at}\} = \frac{v}{1-av}, \alpha \in R.$
3. $K\{\sin \sin (at)\} = \frac{\alpha v^2}{1+\alpha^2 v^2}.$
4. $K\{\cos \cos (at)\} = \frac{v}{1+\alpha^2 v^2}.$
5. $K\{\sinh \sinh (at)\} = \frac{\alpha v^2}{1-\alpha^2 v^2}.$
6. $K\{\cosh \cosh (at)\} = \frac{v}{1-\alpha^2 v^2}.$

Kamal Transform of Derivatives:

Let $K\{f(t)\} = G(v)$, then

- i. $K\{f'(t)\} = \frac{1}{v}F(v) - f(0).$
- ii. $K\{f''(t)\} = \frac{1}{v^2}F(v) - \frac{1}{v}f(0) - f'(0).$
- iii. $K\{f^{(n)}(t)\} = \frac{1}{v^n}F(v) - \sum_{j=0}^{n-1} \frac{f^{(j)}(0)}{v^{n-j-1}}, n = 1, 2, 3, \dots$

13. Polynomial Transform [20]

The transform that proposed by Benedict Barnes and called polynomial integral transform. Polynomial Transform is defined for the function $f(t)$ as:

$$B\{f(t)\} = \int_1^\infty f(\ln \ln t)t^{-s-1} dt = F(s), \quad t \in [1, \infty).$$

The polynomial Transform of Derivatives:

Let $B\{f(t)\} = F(s)$, then

- i. $B\{f'(t)\} = sF(s) - f(0).$
- ii. $B\{f''(t)\} = s^2F(s) - sf(0) - f'(0).$
- iii. $B\{f^{(n)}(t)\} = s^n F(s) - \sum_{j=0}^{n-1} s^{n-j-1} f^{(j)}(0), n = 1, 2, 3, \dots$
 $B\{f^{(n)}(t)\} = s^n F(s) - \sum_{j=0}^{n-1} s^{n-j-1} f^{(j)}(0), n = 1, 2, 3, \dots$

In (2017)

14. Mohand Transformation [21]

Mohand M. et al. proposed an integral transform named Mohand transform. The integral transform is defined for a function $f(t)$ as:

$$M\{f(t)\} = v^2 \int_0^\infty f(t)e^{-vt} dt = R(v), \quad v \in [k_1, k_2].$$

Mohand transform of some elementary functions are:

1. $M\{t^n\} = \frac{n!}{v^{n-1}}, n \in N.$
2. $M\{e^{at}\} = \frac{v^2}{v-\alpha}, \alpha \in R.$
3. $M\{\sin \sin (at)\} = \frac{\alpha v^2}{\alpha^2+v^2}.$
4. $M\{\cos \cos (at)\} = \frac{v^3}{\alpha^2+v^2}.$
5. $M\{\sinh \sinh (at)\} = \frac{\alpha v^2}{v^2-\alpha^2}.$
6. $M\{\cosh \cosh (at)\} = \frac{v^3}{v^2-\alpha^2}.$

Mohand Transform of Derivatives:

Let $M\{f(t)\} = R(v)$, then

- i. $M\{f'(t)\} = vR(v) - v^2f(0).$
- ii. $M\{f''(t)\} = v^2R(v) - v^3f(0) - v^2f'(0).$
- iii. $M\{f^{(n)}(t)\} = v^n R(v) - \sum_{j=0}^{n-1} v^{n-j+1} f^{(j)}(0), n = 1, 2, 3, \dots$
 $M\{f^{(n)}(t)\} = v^n R(v) - \sum_{j=0}^{n-1} v^{n-j+1} f^{(j)}(0), n = 1, 2, 3, \dots$

15. Rangaig Transformation [22]

This integral transform introduced by Norodin A. Rangaig et al. This transform is defined for a function $f(t)$ as:

$$\eta\{f(t)\} = \frac{1}{\mu} \int_{-\infty}^0 f(t)e^{\mu t} dt = \Lambda(\mu), \quad \mu \in \left[\frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right].$$

Rangaig transform of some elementary functions are:

1. $\eta\{t^n\} = \frac{(-1)^n n!}{\mu^{n+2}}, n \in N.$
2. $\eta\{e^{at}\} = \frac{1}{\mu(\mu+\alpha)}, \alpha \in R.$
3. $\eta\{\sin \sin (at)\} = -\frac{1}{\mu} \left(\frac{1}{(\mu^2+\alpha)}\right).$
4. $\eta\{\cos \cos (t)\} = \frac{1}{\mu^2+1}.$

Rangaig Transform of Derivatives:

Let $\eta\{f(t)\} = \Lambda(\mu)$, then

- i. $\eta\{f'(t)\} = -\mu\Lambda(\mu) + \frac{1}{\mu}f(0).$
- ii. $\eta\{f''(t)\} = \mu^2\Lambda(\mu) + f(0) - \frac{1}{\mu}f'(0).$

$$\begin{aligned} \text{iii. } \eta\{f^{(n)}(t)\} &= \\ &(-1)^n \mu^n \Lambda(\mu) + \\ &(-1)^n \sum_{j=0}^{n-1} (-1)^j \mu^{n-j-2} f^{(j)}(0), \\ &n = 1, 2, 3, \dots \eta\{f^{(n)}(t)\} \\ &= (-1)^n \mu^n \Lambda(\mu) \\ &+ (-1)^n \sum_{j=0}^{n-1} (-1)^j \mu^{n-j-2} f^{(j)}(0), \\ &n = 1, 2, 3, \dots \end{aligned}$$

16. G-Transform [23]

Hj. Kim presented a definition of Laplace-typed integral transform and called G-transform. This transform is defined for the function $f(t)$ as:

$$G\{f(t)\} = v^\beta \int_0^\infty f(t)e^{-\frac{t}{v}} dt = F(v), \quad v > 0.$$

G-Transform of some elementary functions are:

1. $G\{t^n\} = v^{n+\beta+1} n!$, $n \in N$.
2. $G\{e^{at}\} = \frac{v^{\beta+1}}{1-\alpha v}$, $\alpha \in R$.
3. $G\{\sin \sin (at)\} = \frac{\alpha v^{\beta+2}}{1+v^2 \alpha^2}$.
4. $G\{\cos \cos (at)\} = \frac{v^{\beta+1}}{1+v^2 \alpha^2}$.
5. $G\{\sinh \sinh (at)\} = \frac{\alpha v^{\beta+2}}{1-v^2 \alpha^2}$.
6. $G\{\cosh \cosh (at)\} = \frac{v^{\beta+1}}{1-v^2 \alpha^2}$.

G-Transform of Derivatives:

Let $G\{f(t)\} = F(v)$, then

- i. $G\{f'(t)\} = \frac{1}{v} F(v) - v^\beta f(0)$.
- ii. $G\{f''(t)\} = \frac{1}{v^2} F(v) - \frac{1}{v} f(0)v^\beta - f'(0)v^\beta$.
- iii. $G\{f^{(n)}(t)\} = \frac{1}{v^n} F(v) - v^\beta \sum_{j=0}^{n-1} \frac{1}{v^{n-j-1}} f^{(j)}(0)$,
 $n = 1, 2, 3, \dots G\{f^{(n)}(t)\} = \frac{1}{v^n} F(v) -$
 $v^\beta \sum_{j=0}^{n-1} \frac{1}{v^{n-j-1}} f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

17. G_α -Transform [24]

Patarawadee Prasertsang et al. presented the intrinsic structure of G_α -transform. This transform is defined for the function $f(t)$ as:

$$G_\alpha\{f(t)\} = u^\alpha \int_0^\infty f(t)e^{-\frac{1}{u}t} dt = F(u), \quad \alpha \in Z,$$

where u is complex variable.

G_α -transform of some elementary functions are:

1. $G_\alpha\{t^n\} = n! u^{n+\beta+1}$, $n \in N$.
2. $G_\alpha\{e^{\alpha t}\} = \frac{u^{\alpha+1}}{1-\beta u}$, $\beta \in R$.
3. $G_\alpha\{\sin \sin (\beta t)\} = \frac{\beta u^{\alpha+2}}{1+\beta^2 u^2}$.
4. $G_\alpha\{\cos \cos (\beta t)\} = \frac{\beta u^{\alpha+2}}{1+\beta^2 u^2}$.
5. $G_\alpha\{\sinh \sinh (\beta t)\} = \frac{\beta u^{\alpha+2}}{1-\beta^2 u^2}$.
6. $G_\alpha\{\cosh \cosh (\beta t)\} = \frac{u^{\alpha+2}}{1-\beta^2 u^2}$.

G_α -Transform of Derivatives:

Let $G_\alpha\{f(t)\} = F(u)$, then

- i. $G_\alpha\{f'(t)\} = \frac{1}{u} F(u) - u^\alpha f(0)$.
- ii. $G_\alpha\{f''(t)\} = \frac{1}{u^2} F(u) - \frac{1}{u} f(0)u^\alpha - u^\alpha f'(0)$.
- iv. $G_\alpha\{f^{(n)}(t)\} =$
 $\frac{1}{u^n} F(u) - \sum_{j=0}^{n-1} u^{\alpha-n+(j+1)} f^{(j)}(0)$, $n =$
 $1, 2, 3, \dots G_\alpha\{f^{(n)}(t)\} =$
 $\frac{1}{u^n} F(u) - \sum_{j=0}^{n-1} u^{\alpha-n+(j+1)} f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

18. The Complex Al-Tememe Transform [25]

Ali Hassan Mohammed and Sarah Falih Makttoof presented a new integral transform called the complex Al-Tememe integral transform. The complex Al-Tememe integral transform is convergent and it is defined for the function $f(t)$ as:

$$T^c\{f(t)\} = \int_1^\infty t^{-ip} f(t) dt = F(ip), \quad t > 1.$$

The complex Al-Tememe integral transform of some elementary functions are:

1. $T^c\{x^n\} = \frac{-(n+1)}{p^2+(n+1)^2} - \frac{ip}{p^2+(n+1)^2}$, $n \in R$.
2. $T^c\{\ln \ln t\} = \frac{1-p^2}{(p^2+1)^2} + \frac{2ip}{(p^2+1)^2}$.
3. $T^c\{\sin \sin (\alpha \ln \ln t)\} = \frac{-\alpha[(p^2-1)-\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} +$
 $\frac{2iap}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} \cdot T^c\{\sin \sin (\alpha \ln \ln t)\} =$
 $\frac{-\alpha[(p^2-1)-\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} + \frac{2iap}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4}$.
4. $T^c\{\cos \cos (\alpha \ln \ln t)\} = \frac{-\alpha[(p^2+1)+\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} -$
 $\frac{-ip[(p^2+1)-\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} \cdot T^c\{\cos \cos (\alpha \ln \ln t)\} =$
 $\frac{-\alpha[(p^2+1)+\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4} - \frac{-ip[(p^2+1)-\alpha^2]}{(p^2+1)^2-2\alpha^2(p^2-1)+\alpha^4}$.

$$5. T^c\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{-\alpha[(p^2-1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} + \frac{2iap}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} \cdot T^c\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{-\alpha[(p^2-1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} + \frac{2iap}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4}$$

$$6. T^c\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{-[(p^2+1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} - \frac{-ip[(p^2+1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} \cdot T^c\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{-[(p^2+1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4} - \frac{-ip[(p^2+1)+\alpha^2]}{(p^2+1)^2+2\alpha^2(p^2-1)+\alpha^4}$$

The complex Al-Tememe Transforms of Derivatives:

Let $T^c\{f(t)\} = F(ip)$, then

- i. $T^c\{xf'(t)\} = (ip - 1)F(ip) - f(1)$.
- ii. $T^c\{x^2f''(t)\} = (ip - 2)(ip - 1)F(ip) - (ip - 2)f(1) - f'(1)$. $T^c\{x^2f''(t)\} = (ip - 2)(ip - 1)F(ip) - (ip - 2)f(1) - f'(1)$.
- iii. $T^c\{x^n f^{(n)}(t)\} = -f^{(n-1)}(1) - (ip - n)f^{(n-2)}(1) - \dots - (ip - n)(ip - (n - 1))(ip - (n - 2)) \dots (ip - 2) f(1) + (-ip - n)! F(ip)$, $n = 1, 2, 3, \dots$. $T^c\{x^n f^{(n)}(t)\} = -f^{(n-1)}(1) - (ip - n)f^{(n-2)}(1) - \dots - (ip - n)(ip - (n - 1))(ip - (n - 2)) \dots (ip - 2) f(1) + (-ip - n)! F(ip)$, $n = 1, 2, 3, \dots$. **In (2018)**

19. Al-Zughair Transform [26]

Al-Zughair integral transform was introduced by Ali H.M. et al. This transform is convergent and defined for a function $f(t)$ as:

$$Z\{f(t)\} = \int_1^e f(t) \frac{[\ln \ln t]^v}{t} dt = F(v), \quad v > -1.$$

Al-Zughair transform of some elementary functions are:

- 1. $Z\{(\ln \ln (t))^n\} = \frac{1}{v+(n+1)}$, $n \in R, v > -(n + 1)$.
- 2. $Z\{\ln \ln ((\ln \ln (t)))^n\} = \frac{(-1)^n n!}{(v+1)^{n+1}}$, $n \in N$.
- 3. $Z\{\sin \sin (\alpha \ln \ln (\ln \ln (t)))\} = \frac{-\alpha}{(v+1)^2+\alpha^2}$, α is constant.
- 4. $Z\{\cos \cos (\alpha \ln \ln (\ln \ln (t)))\} = \frac{v+1}{(v+1)^2+\alpha^2}$, α is constant.
- 5. $\{ \sinh \sinh (\alpha \ln \ln (\ln \ln (t))) \} = \frac{-\alpha}{(v+1)^2-\alpha^2}, |v + 1| > \alpha, \alpha$ is constant.

$$6. Z\{\cosh \cosh (\alpha \ln \ln (\ln \ln (t)))\} = \frac{v+1}{(v+1)^2-\alpha^2}, |v + 1| > \alpha \alpha$$
 is constant.

Al-Zughair Transform of Derivatives:

Let $f(\ln \ln t)$ is defined for $t \in [1, e]$, $Z\{f(t)\} = F(v)$, then

- i. $Z\{(\ln \ln t) f'((\ln \ln t))\} = -(v + 1)! Z\{f(\ln \ln t)\} + f(1)$.
- ii. $Z\{(\ln \ln t)^2 f''(\ln \ln t)\} = (v + 2)! Z\{f(\ln \ln t)\} + f'(1) - (v + 2)f(1)$. $Z\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} = f^{(n-1)}(1) + (-1)^{2n+1}(v + n)f^{(n-2)}(1) + (-1)^{2n+2}(v + n)(v + (n - 1))$
- iii. $Z\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} = f^{(n-1)}(1) + (-1)^{2n+1}(v + n)f^{(n-2)}(1) + (-1)^{2n+2}(v + n)(v + (n - 1))f^{(n-3)}(1) + \dots + (-1)^{3n-1}(v + n)(v + (n - 1)) \dots (v + 2)f(1) + (-1)^{3n}(v + n)! F(v)$, $n = 1, 2, 3, \dots$. $Z\{(\ln \ln t)^2 f''(\ln \ln t)\} = (v + 2)! Z\{f(\ln \ln t)\} + f'(1) - (v + 2)f(1)$.

20. Sadik Transform [27]

S.L. Shaikh proposed an integral transform named Sadik Transform. This Transform is defined for the function $f(t)$ as:

$$S_\sigma\{f(t)\} = \frac{1}{v^\beta} \int_0^\infty f(vt)e^{-tv^\sigma} dt =$$

$F(v^\sigma, \beta)$, $\alpha, \beta \in R, \sigma \neq 0$ and α is complex variable.

Sadik transform of some elementary functions are:

- 1. $S_\sigma\{t^n\} = \frac{n!}{v^{n\sigma+(\sigma+\beta)}}$, $n \in N$.
- 2. $S_\sigma\{e^{\alpha t}\} = \frac{v^{-\beta}}{v^\sigma-\alpha}$, $\alpha \in R$.
- 3. $S_\sigma\{\sin \sin (\alpha t)\} = \frac{\alpha v^{-\beta}}{v^{2\sigma}+\alpha^2}$.
- 4. $S_\sigma\{\cos \cos (\alpha t)\} = \frac{v^{\sigma-\beta}}{v^{2\sigma}+\alpha^2}$.
- 5. $S_\sigma\{\sinh \sinh (\alpha t)\} = \frac{\alpha v^{-\beta}}{v^{2\sigma}-\alpha^2}$.
- 6. $S_\sigma\{\cosh \cosh (\alpha t)\} = \frac{v^{\sigma-\beta}}{v^{2\sigma}-\alpha^2}$.

Sadik Transform of Derivatives:

Let $S_\sigma\{f(t)\} = F(v^\sigma, \beta)$, then

- i. $S_\sigma\{f'(t)\} = v^\sigma F(v^\sigma, \beta) - v^{-\beta} f(0)$.
- ii. $S_\sigma\{f''(t)\} = v^{2\sigma} F(v^\sigma, \beta) - v^{-\beta} f'(0) - v^{\sigma-\beta} f(0)$.
- iii. $S_\sigma\{f^{(n)}(t)\} = v^{n\sigma} F(v^\sigma, \beta) - \sum_{j=0}^{n-1} v^{j\sigma-\beta} f^{(n-1-j)}(0)$,

$$n = 1, 2, 3, \dots$$

In (2019)

21. Shehu Transform [28]

Shehu Maitama and Weidong Zhao presented a definition of Laplace-typed integral transform which is a generalization of the Laplace and the Sumudu transforms that is called Shehu transformation. This transform is defined for the function $f(t)$ as:

$$S\{f(t)\} = \int_0^\infty f(t)e^{-\frac{st}{v}} dt = F(s, v), \quad s, v > 0.$$

Shehu Transform of some elementary functions are:

1. $S\{t^n\} = \frac{v^{n+1}n!}{s^{n+1}}, n \in N.$
2. $S\{e^{\alpha t}\} = \frac{v}{s-\alpha v}, \alpha \in R.$
3. $S\{\sin \sin(\alpha t)\} = \frac{\alpha v^2}{s^2+v^2\alpha^2}.$
4. $S\{\cos \cos(\alpha t)\} = \frac{vs}{s^2+v^2\alpha^2}.$
5. $S\{\sinh \sinh(\alpha t)\} = \frac{\alpha v^2}{s^2-v^2\alpha^2}.$
6. $S\{\cosh \cosh(\alpha t)\} = \frac{vs}{s^2+v^2\alpha^2}.$

Shehu Transform of Derivatives:

Let $S\{f(t)\} = F(s, v)$, then

- i. $S\{f'(t)\} = \frac{s}{v}F(s, v) - f(0).$
- ii. $S\{f''(t)\} = \frac{s^2}{v^2}F(s, v) - \frac{s}{v}f(0) - f'(0).$
- iv. $S\{f^{(n)}(t)\} = \frac{s^n}{v^n}F(s, v) - \sum_{j=0}^{n-1} \frac{s^{n-j-1}}{v^{n-j-1}}f^{(j)}(0),$
 $n = 1, 2, 3, \dots S\{f^{(n)}(t)\} = \frac{s^n}{v^n}F(s, v) -$
 $\sum_{j=0}^{n-1} \frac{s^{n-j-1}}{v^{n-j-1}}f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

22. Sawi Transform [29]

Mahgoub et al. presented an integral transform named Sawi transform. This transform is defined for the function $f(t)$ as:

$$S\{f(t)\} = \frac{1}{\sigma^2} \int_0^\infty f(t)e^{-\frac{t}{\sigma}} dt = T(\sigma), \quad \sigma > 0.$$

Sawi transform of some elementary functions are:

7. $S\{t^n\} = n! \sigma^{n-1}, n \in N.$
8. $S\{e^{\alpha t}\} = \frac{1}{\sigma(1-\alpha\sigma)}, \alpha \in R.$
9. $S\{\sin \sin(\alpha t)\} = \frac{\alpha}{1+\alpha^2\sigma^2}.$
10. $S\{\cos \cos(\alpha t)\} = \frac{1}{\sigma(1+\alpha^2\sigma^2)}.$
11. $S\{\sinh \sinh(\alpha t)\} = \frac{\alpha}{1-\alpha^2\sigma^2}.$

$$12. S\{\cosh \cosh(\alpha t)\} = \frac{1}{\sigma(1-\alpha^2\sigma^2)}.$$

Sawi Transform of Derivatives:

Let $S\{f(t)\} = T(\sigma)$, then

- i. $S\{f'(t)\} = \left(\frac{1}{\sigma}\right)T(\sigma) - \frac{1}{\sigma^2}f(0).$
- ii. $S\{f''(t)\} = \frac{1}{\sigma^2}T(\sigma) - \frac{1}{\sigma^2}f'(0) - \left(\frac{1}{\sigma^3}\right)f(0).$
- iii. $S\{f^{(n)}(t)\} = \frac{1}{\sigma^n}T(\sigma) - \sum_{j=0}^{n-1} \frac{1}{\sigma^{n-(j-1)}}f^{(j)}(0),$
 $n = 1, 2, 3, \dots S\{f^{(n)}(t)\} = \frac{1}{\sigma^n}T(\sigma) -$
 $\sum_{j=0}^{n-1} \frac{1}{\sigma^{n-(j-1)}}f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

23. Upadhyaya Transform [30]

Lalit M. Upadhyaya introduced a new integral transform named Upadhyaya transform. This Transform is defined for the function $f(t)$ as:

$$\begin{aligned} U\{f(\ln \ln t)\} &= \lambda_1 \int_0^\infty f(\lambda_3 \ln \ln t)t^{-\lambda_2-1} dt \\ &= u(\lambda_1, \lambda_2, \lambda_3), \quad \lambda_1, \lambda_2, \lambda_3 U\{f(\ln \ln t)\} \\ &= \lambda_1 \int_0^\infty f(\lambda_3 \ln \ln t)t^{-\lambda_2-1} dt \\ &= u(\lambda_1, \lambda_2, \lambda_3), \end{aligned}$$

$\lambda_1, \lambda_2, \lambda_3$ are complex parameters

Upadhyaya transform of some elementary functions are:

1. $U\{t^n\} = \frac{\lambda_1(\lambda_3)^{n!}}{(\lambda_2)^{n+1}}, n \in N.$
2. $U\{e^{\alpha t}\} = \frac{\lambda_1}{\lambda_2-\alpha\lambda_3}, \alpha \in R.$
3. $U\{\sin \sin(\alpha t)\} = \frac{\alpha\lambda_1\lambda_3}{\lambda_2^2+\alpha^2\lambda_3^2}.$
4. $U\{\cos \cos(\alpha t)\} = \frac{\lambda_1\lambda_2}{\lambda_2^2+\alpha^2\lambda_3^2}.$
5. $U\{\sinh \sinh(\alpha t)\} = \frac{\alpha\lambda_1\lambda_3}{\lambda_2^2-\alpha^2\lambda_3^2}.$
6. $U\{\cosh \cosh(\alpha t)\} = \frac{\lambda_1\lambda_2}{\lambda_2^2-\alpha^2\lambda_3^2}.$

Upadhyaya Transform of Derivatives:

Let $U\{f(t)\} = u(\lambda_1, \lambda_2, \lambda_3)$, then

- i. $U\{f'(t)\} = \frac{\lambda_2}{\lambda_3}u(\lambda_1, \lambda_2, \lambda_3) - \frac{\lambda_1}{\lambda_3}f(0).$
- ii. $U\{f''(t)\} = \left(\frac{\lambda_2}{\lambda_3}\right)^2 u(\lambda_1, \lambda_2, \lambda_3) - \frac{\lambda_1}{\lambda_3}f'(0) -$
 $\frac{\lambda_1\lambda_2}{\lambda_3}f(0).$
- iii. $U\{f^{(n)}(t)\} =$
 $\left(\frac{\lambda_2}{\lambda_3}\right)^n u(\lambda_1, \lambda_2, \lambda_3) - \lambda_1 \sum_{j=0}^{n-1} \frac{\lambda_2^{n-j-1}}{\lambda_3^{n-j}} f^{(j)}(0), n =$

$$1, 2, 3, \dots U\{f^{(n)}(t)\} = \left(\frac{\lambda_2}{\lambda_3}\right)^n u(\lambda_1, \lambda_2, \lambda_3) - \lambda_1 \sum_{j=0}^{n-1} \frac{\lambda_2^{n-j-1}}{\lambda_3^{n-j}} f^{(j)}(0),$$

$$n = 1, 2, 3, \dots$$

24. An Extension of Al-Zughair Transform [31]

Ali H.M. et al. introduced an integral transform , that is an extension of the Al-Zughair integral transform. This Transform is defined for the function $f(t)$ as:

$$EZ\{f(\ln \ln(t))\} = \int_{e^{-1}}^e f(\ln \ln(t)) \frac{(\ln \ln(t))^w}{t} dt = F(w), \quad w \text{ is constant.}$$

The extension of the Al-Zughair transform of some elementary functions are:

- $EZ\{(\ln \ln(t))^n\} = \frac{2}{w+(n+1)}, n \geq 0, (w+n)$ is an even number.
- $EZ\{\ln \ln(\ln \ln(t))\} = \frac{-2}{(w+1)^2}, w$ is an even number.
- $EZ\{[\ln \ln(\ln \ln(t))]^n\} = \frac{(-1)^n 2 * n!}{(w+1)^{n+1}}, n \in N, w$ is an even number.
- $EZ\{\sin \sin(\alpha \ln \ln(\ln \ln(t)))\} = \frac{-2\alpha}{(w+1)^2 + \alpha^2}, (w \mp \alpha)$ is an even number.
- $EZ\{\cos \cos(\alpha \ln \ln(\ln \ln(t)))\} = \frac{2(w+1)}{(w+1)^2 + \alpha^2}, (w \mp \alpha)$ is an even number.
- $EZ\{\sinh \sinh(\alpha \ln \ln(\ln \ln(t)))\} = \frac{-2\alpha}{(w+1)^2 - \alpha^2}, (w \mp \alpha)$ is an even number.
- $EZ\{\cosh \cosh(\alpha t)\} = \frac{2(w+1)}{(w+1)^2 + \alpha^2}, (w \mp \alpha), (w \mp \alpha)$ is an even number.

An Extension of Al-Zughair Transform of Derivatives:

Let $EZ\{f(t)\} = F(w)$, then

- $EZ\{\ln \ln(t) f'(\ln \ln(t))\} = -(w+1)EZ\{f(\ln \ln(t))\}.$
- $EZ\{(\ln \ln(t))^2 f''(\ln \ln(t))\} = 2f'(1) + (w+2)(w+1)EZ\{f(\ln \ln(t))\}.$
 $EZ\{(\ln \ln(t))^2 f''(\ln \ln(t))\} = 2f'(1) + (w+2)(w+1)EZ\{f(\ln \ln(t))\}.$
- $EZ\{(\ln \ln(t))^3 f'''(\ln \ln(t))\} = -2(w+3)f'(1) +$

$$(w+3)(w+2)(w+1)EZ\{f(\ln \ln(t))\}.$$

25. The Natural Logarithmic integral Transform [32]

Emad Kuffi et al. proposed the Natural Logarithmic integral transform. This Transform is defined for the function $f(t)$ as:

$$IL\{f(t)\} = \int_{\frac{1}{\sigma}}^1 \ln \ln(\sigma t) f(t) dt = F(\sigma), \quad \sigma = 2, 3, 4, \dots \text{ and } t > 0.$$

The Natural Logarithmic integral transform of some elementary functions are:

- $IL\{t^n\} = \frac{\ln \ln(\sigma)}{n+1} - \frac{[1 - (\frac{1}{\sigma})^{n+1}]}{(n+1)^2}, n \neq -1.$
- $IL\{\sin \sin(\alpha t)\} = \sin \sin \alpha (\ln \ln(\sigma) - 1) - \frac{1}{\sigma} \sin \sin \left(\frac{\alpha}{\sigma}\right) + \frac{\sin \sin \left(\frac{\alpha}{\sigma}\right) + \alpha \sigma \sin \sin \left(\frac{\alpha}{\sigma}\right)}{\sigma^2} + \frac{\cos \cos(\alpha) - \alpha^2 \cos \cos(\alpha)}{\alpha} + \frac{(\alpha) + \sin \sin(\alpha)}{\sigma} + \frac{\cos \cos \left(\frac{\alpha}{\sigma}\right) + \alpha^2 \cos \cos \left(\frac{\alpha}{\sigma}\right)}{\alpha}.$
 $IL\{\sin \sin(\alpha t)\} = \sin \sin \alpha (\ln \ln(\sigma) - 1) - \frac{1}{\sigma} \sin \sin \left(\frac{\alpha}{\sigma}\right) + \frac{\sin \sin \left(\frac{\alpha}{\sigma}\right) + \alpha \sigma \sin \sin \left(\frac{\alpha}{\sigma}\right)}{\sigma^2} + \frac{\cos \cos(\alpha) - \alpha^2 \cos \cos(\alpha)}{\alpha} + \frac{(\alpha) + \sin \sin(\alpha)}{\sigma} + \frac{\cos \cos \left(\frac{\alpha}{\sigma}\right) + \alpha^2 \cos \cos \left(\frac{\alpha}{\sigma}\right)}{\alpha}$

$$2. IL\{e^{-\alpha t}\} = \ln \ln(\sigma) \left[\frac{1}{\sigma} - \frac{e^{-\alpha}}{\alpha}\right].$$

The Natural Logarithmic integral Transform of Derivatives:

Let $IL\{f(t)\} = F(\sigma)$, then

- $IL\{f'(t)\} = \ln \ln(\sigma) f(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f(1))^3 - k^3}, f'(1) > k.$
- $IL\{f'(t)\} = \ln \ln(\sigma) f(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f(1))^3 - k^3}, f'(1) > k.$

ii. $IL\{f''(t)\} =$
 $ln ln (\sigma)f'(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f'(1))^3 - k^3}, f''(1) >$
 $k. IL\{f''(t)\} =$
 $ln ln (\sigma)f'(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f'(1))^3 - k^3}, f''(1) >$
 $k. IL\{f''(t)\} =$
 $ln ln (\sigma)f'(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f'(1))^3 - k^3}, f''(1) >$
 $k.$
 iii. $S_a\{f^{(n)}(t)\} =$
 $ln ln (\sigma)f^{(n-1)}(1) -$
 $\sqrt{\frac{\sigma-1}{3}} \sqrt{(f^{(n-1)}(1))^3 - k^3}, f^{(n)}(1) > k,$
 $S_a\{f^{(n)}(t)\} =$
 $ln ln (\sigma)f^{(n-1)}(1) - \sqrt{\frac{\sigma-1}{3}} \sqrt{(f^{(n-1)}(1))^3 - k^3},$
 $f^{(n)}(1) > k, \quad n = 1, 2, 3, \dots$

In (2020)

26. Rohit Integral Transform [33]

S.L. Rohit Gupta proposed an integral transform called the Rohit Transform. This Transform is defined for the function $f(t)$ as:

$$R\{f(t)\} = r^3 \int_0^\infty f(t)e^{-rt} dt = F(r), \quad r \text{ is a real or complex parameter.}$$

Rohit transform of some elementary functions are:

1. $R\{t^n\} = \frac{n!}{r^{n-2}}, n \in N.$
2. $R\{e^{at}\} = \frac{r^3}{r-\alpha}, r > \alpha.$
3. $R\{\sin \sin (at)\} = \frac{\alpha r^3}{r^2+\alpha^2}.$
4. $R\{\cos \cos (at)\} = \frac{r^4}{r^2+\alpha^2}, r > 0.$
5. $R\{\sinh \sinh (at)\} = \frac{\alpha r^3}{r^2-\alpha^2}, r > |\alpha|.$
6. $R\{\cosh \cosh (at)\} = \frac{r^4}{r^2-\alpha^2}, r > |\alpha|.$

Rohit Transform of Derivatives:

Let $R\{f(t)\} = F(r)$, then

- i. $R\{f'(t)\} = rF(r) - r^3f(0).$
- ii. $R\{f''(t)\} = r^2F(r) - r^4f(0) - r^3f'(0).$
- iii. $R\{f^{(3)}(t)\} = r^3F(r) - r^5f(0) - r^4f'(0) - r^3f''(0).$

27. The Dinesh Verma Integral Transform [34]

Dinesh Verma integral transform (DVT) was proposed. The transform is convergent and defined for the function $f(t)$ as:

$$D\{f(t)\} = p^5 \int_{t=0}^\infty f(t)e^{-pt} dt = \underline{f}(p), \quad p \text{ is a real or complex parameter.}$$

The Dinesh Verma transform of some elementary functions are:

1. $D\{t^n\} = \frac{n!}{p^{n-4}}, n \in N.$
2. $D\{e^{at}\} = \frac{p^5}{p-\alpha}, \alpha \text{ is constant.}$
3. $D\{\sin \sin (at)\} = \frac{\alpha p^5}{p^2+\alpha^2}, p > 0.$
4. $D\{\cos \cos (at)\} = \frac{p^6}{p^2+\alpha^2}, p > 0.$
5. $D\{\sinh \sinh (at)\} = \frac{\alpha p^5}{p^2-\alpha^2}, p > |\alpha|.$
6. $D\{\cosh \cosh (at)\} = \frac{p^6}{p^2-\alpha^2}, p > |\alpha|.$

The Dinesh Verma Integral Transform of Derivatives:

Let $D\{f(t)\} = \underline{f}(p)$, then

- i. $D\{f'(t)\} = p\underline{f}(p) - p^5f(0).$
- ii. $D\{f''(t)\} = p^2\underline{f}(p) - p^6f(0) - p^5f'(0).$
- iii. $D\{f^{(3)}(t)\} = p^3\underline{f}(p) - p^7f(0) - p^6f'(0) - p^5f''(0).$

28. Gupta Transform [35]

A transform introduced by Rahul Gupta et al. that called the Gupta transform. The transform is convergent and defined for the function $f(t)$ as:

$$\dot{R}\{f(t)\} = \frac{1}{p^3} \int_0^\infty f(t)e^{-pt} dt = \dot{F}(p), \quad p \text{ is a real or complex parameter.}$$

The Gupta transform of some elementary functions are:

1. $\dot{R}\{t^n\} = \frac{n!}{p^{n+4}}, n \in N.$
2. $\dot{R}\{e^{at}\} = \frac{1}{p^3(p-\alpha)}, p > \alpha.$
3. $\dot{R}\{\sin \sin (at)\} = \frac{\alpha}{p^3(p^2+\alpha^2)}, p > 0.$
4. $\dot{R}\{\cos \cos (at)\} = \frac{1}{p^2(p^2+\alpha^2)}, p > 0.$
5. $\dot{R}\{\sinh \sinh (at)\} = \frac{\alpha}{p^3(p^2-\alpha^2)}, p > |\alpha|.$
6. $\dot{R}\{\cosh \cosh (at)\} = \frac{1}{p^2(p^2-\alpha^2)}, p > |\alpha|.$

Gupta Transform of Derivatives:

Let $\dot{R}\{f(t)\} = \dot{F}(p)$, then

- i. $\dot{R}\{f'(t)\} = p\dot{F}(p) - \frac{1}{p^3}f(0).$
- ii. $\dot{R}\{f''(t)\} = p^2\dot{F}(p) - \frac{1}{p^2}f(0) - \frac{1}{p^3}f'(0).$

29. Kharrat-Toma Transform [36]

B. Kharrat and G. Toma presented a definition of a new integral transform which is called Kharrat-Toma transformation. This transform is defined for the function $f(t)$ as:

$$B\{f(t)\} = s^3 \int_0^\infty f(t)e^{-\frac{t}{s^2}} dt = G(s), \quad s > 0.$$

Kharrat-Toma Transform of some elementary functions are:

1. $B\{t^n\} = s^{2n+5}n!$, $n \in N$.
2. $B\{\sin \sin (at)\} = \frac{\alpha s^7}{1+\alpha^2 s^4}$.
3. $B\{\cos \cos (at)\} = \frac{s^5}{1+\alpha^2 s^4}$.
4. $B\{\sinh \sinh (at)\} = \frac{\alpha s^7}{1-\alpha^2 s^4}$.
5. $B\{\cosh \cosh (at)\} = \frac{s^5}{1-\alpha^2 s^4}$.

Kharrat-Toma Transform of Derivatives:

Let $B\{f(t)\} = G(s)$, then

- i. $B\{f'(t)\} = \frac{1}{s^2}G(s) - s^3f(0)$.
- ii. $B\{f''(t)\} = \frac{1}{s^4}G(s) - sf(0) - s^3f'(0)$.
- iii. $B\{f^{(n)}(t)\} = \frac{1}{s^{2n}}G(s) - \sum_{j=0}^{n-1} s^{-2n+2j+5}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$. $B\{f^{(n)}(t)\} = \frac{1}{s^{2n}}G(s) - \sum_{j=0}^{n-1} s^{-2n+2j+5}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

30. J-Transform [37]

V. Srinivas and C.H. Jayanthi presented an integral transform named J-transform. This transform is defined for the function $f(t)$ as:

$$J\{f(t)\} = v^2 \int_0^\infty f(t)e^{-\frac{t}{v}} dt = F(v), \quad k_1 < v < k_2.$$

J-transform of some elementary functions are:

1. $J\{t^n\} = n! v^{n+3}$, $n \in N$.
2. $J\{e^{\alpha t}\} = \frac{v^3}{1-\alpha v}$, $\alpha \in R$.
3. $J\{\sin \sin (at)\} = \frac{\alpha v^4}{1+\alpha^2 v^2}$.
4. $J\{\cos \cos (at)\} = \frac{v^3}{1+\alpha^2 v^2}$.
5. $J\{\sinh \sinh (at)\} = \frac{\alpha v^4}{1-\alpha^2 v^2}$.
6. $J\{\cosh \cosh (at)\} = \frac{v^3}{1-\alpha^2 v^2}$.

31. Jafari Transform [38]

H. Jafari introduced a new integral transform named Jafari transform. This Transform is defined for the function $f(t)$ as:

$$T\{f(t)\} = p(s) \int_0^\infty f(t)e^{-q(s)t} dt = T(s), \quad p(s) \neq 0, q(s) > 0.$$

Jafari transform of some elementary functions are:

1. $T\{t^n\} = \frac{n!p(s)}{[q(s)]^{n+1}}$, $n \in N$.
2. $T\{e^{\alpha t}\} = \frac{p(s)}{q(s)-1}$, $q(s) > 1$.
3. $T\{\sin \sin (at)\} = \frac{\alpha p(s)}{(q(s))^2 + \alpha^2}$.
4. $T\{\cos \cos (at)\} = \frac{p(s)q(s)}{(q(s))^2 + \alpha^2}$.
5. $T\{\sinh \sinh (at)\} = \frac{\alpha p(s)}{(q(s))^2 - \alpha^2}$.
6. $T\{\cosh \cosh (at)\} = \frac{p(s)q(s)}{(q(s))^2 - \alpha^2}$.

Jafari Transform of Derivatives:

Let $T\{f(t)\} = T(s)$, then

- i. $T\{f'(t)\} = q(s)T(s) - p(s)f(0)$.
- ii. $T\{f''(t)\} = [q(s)]^2T(s) - q(s)p(s)f(0) - p(s)f'(0)$.
- i. $T\{f^{(n)}(t)\} = [q(s)]^nT(s) - p(s) \sum_{j=0}^{n-1} [q(s)]^{n-1-j}f^{(j)}(0)$, $n = 1, 2, 3, \dots$. $T\{f^{(n)}(t)\} = [q(s)]^nT(s) - p(s) \sum_{j=0}^{n-1} [q(s)]^{n-1-j}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

32. ARA Transform [39]

Rania Saadeh et al. introduced a new integral transform named the ARA transform. This Transform is defined for the function $f(t)$ as:

$$G_m\{f(t)\} = s \int_0^\infty t^{m-1}e^{-st}f(t)dt = G(n, s), \quad s > 0.$$

ARA transform of some elementary functions are:

1. $G_m\{t^n\} = \frac{\Gamma(n+m)}{s^{n+m-1}}$, $n \in N$.
2. $G_m\{e^{\alpha t}\} = \frac{s\Gamma(m)}{(s-\alpha)^m}$, $\alpha \in R$.
3. $G_m\{\sin \sin (at)\} = \left(1 + \frac{\alpha^2}{s^2}\right)^{-\frac{m}{2}} s^{1-m}\Gamma(m) \sin \sin \left(m\left(\frac{\alpha}{s}\right)\right)$.

4. $G_m\{\cos \cos (\alpha t)\} = \left(1 + \frac{\alpha^2}{s^2}\right)^{-\frac{m}{2}} s^{1-m} \Gamma(m) \cos \cos \left(m\left(\frac{\alpha}{s}\right)\right).$
5. $G_m\{\sinh \sinh (\alpha t)\} = \frac{s}{2} \Gamma(m) \left(\frac{1}{(s-\alpha)^m} - \frac{1}{(s+\alpha)^m}\right).$
6. $G_m\{\cosh \cosh (\alpha t)\} = \frac{s}{2} \Gamma(m) \left(\frac{1}{(s-\alpha)^m} + \frac{1}{(s+\alpha)^m}\right).$

ARA Transform of Derivatives:

Let $G_m\{f(x, t)\} = G(n, s)$, then

$$G_m\{f^{(n)}(t)\} = (-1)^{n-1} s \frac{d^{n-1}}{ds^{n-1}} \left(s^{n-1} G_1[f(t)](s) - \sum_{j=1}^n s^{n-j} f^{(j-1)}(0) \right),$$

$$n = 1, 2, 3, \dots$$

33. SEE Transform [40]

Eman A.Mansour et al. proposed (Sadik-Emad-Eman) Transform named SEE integral transform. This Transform is defined for the function $f(t)$ as:

$$S\{f(t)\} = \frac{1}{v^m} \int_0^\infty f(t) e^{-vt} dt = T(v), \quad m \in Z, v \in [l_1, l_2].$$

SEE transform of some elementary functions are:

1. $S\{t^n\} = \frac{n!}{v^{m+n+1}}, \quad n \in N.$
2. $S\{e^{\alpha t}\} = \frac{1}{v^m(v-\alpha)}, \quad \alpha \in R.$
3. $S\{\sin \sin (\alpha t)\} = \frac{\alpha}{v^m(v^2+\alpha^2)}.$
4. $S\{\cos \cos (\alpha t)\} = \frac{v}{v^m(v^2+\alpha^2)}.$
5. $S\{\sinh \sinh (\alpha t)\} = \frac{\alpha}{v^m(v^2-\alpha^2)}.$
6. $S\{\cosh \cosh (\alpha t)\} = \frac{v}{v^m(v^2-\alpha^2)}.$

SEE Transform of Derivatives:

Let $S\{f(t)\} = T(v)$, then

- i. $S\{f'(t)\} = vT(v) - \frac{1}{v^m} f(0).$
- ii. $S\{f''(t)\} = v^2T(v) - \frac{1}{v^m} f'(0) - \frac{1}{v^{m-1}} f(0).$
- iii. $S\{f^{(n)}(t)\} = v^n T(v) - \sum_{j=0}^{n-1} \frac{1}{v^{m-j}} f^{(n-j-1)}(0),$
 $n = 1, 2, 3, \dots S\{f^{(n)}(t)\} = v^n T(v) - \sum_{j=0}^{n-1} \frac{1}{v^{m-j}} f^{(n-j-1)}(0),$
 $n = 1, 2, 3, \dots$

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34. Alenezi Transform [41]

An integral transform had been introduced by Ahmed M. Alenezi. The integral transform is defined for the function $f(t)$ as:

$$J\{f(t)\} = m(s) \int_0^\infty f(t) e^{-tn(s)} dt = J(s), \quad n(s) \neq 0, m(s) \neq 0 \text{ are real functions.}$$

Alenezi transform of some elementary functions are:

1. $J\{t^k\} = \frac{k!m(s)}{[n(s)]^{k+1}}, \quad n \in N.$
2. $J\{e^{\alpha t}\} = \frac{m(s)}{n(s)-\alpha}, \quad \alpha \in R.$
3. $J\{\sin \sin (\alpha t)\} = \frac{\alpha m(s)}{[n(s)]^2+\alpha^2}.$
4. $J\{\cos \cos (\alpha t)\} = \frac{n(s)m(s)}{[n(s)]^2+\alpha^2}.$

Alenezi Transform of Derivatives:

Let $J\{f(t)\} = J(s)$, then

- i. $J\{f'(t)\} = n(s)J(s) - m(s)f(0).$
- ii. $J\{f''(t)\} = [n(s)]^2 J(s) - m(s)n(s)f(0) - m(s)f'(0).$
- i. $J\{f^{(k)}(t)\} = [n(s)]^k J\{f(t)\} - m(s) \sum_{j=0}^{k-1} [n(s)]^{k-1-j} f^{(j)}(0),$
 $k = 1, 2, 3, \dots J\{f^{(k)}(t)\} = [n(s)]^k J\{f(t)\} - m(s) \sum_{j=0}^{k-1} [n(s)]^{k-1-j} f^{(j)}(0),$
 $J\{f^{(k)}(t)\} = [n(s)]^k J\{f(t)\} - m(s) \sum_{j=0}^{k-1} [n(s)]^{k-1-j} f^{(j)}(0),$
 $k = 1, 2, 3, \dots$

35. Emad- Sara Transform [42]

A transform was introduced by Emad Abbas and Sara Falih named (Emad- Sara) (ES) integral transform. This Transform is defined for the function $f(t)$ as:

$$ES\{f(t)\} = \frac{1}{\beta^2} \int_0^\infty f(t) e^{-\beta t} dt = T(\beta), \quad \beta \in [m_1, m_2].$$

ES transform of some elementary functions are:

1. $ES\{t^n\} = \frac{n!}{\beta^{n+3}}, \quad n \in N.$
2. $ES\{e^{\alpha t}\} = \frac{1}{\beta^2(\beta-\alpha)}, \quad \alpha \in R.$
3. $ES\{\sin \sin (\alpha t)\} = \frac{\alpha}{\beta^2(\beta^2+\alpha^2)}.$
4. $ES\{\cos \cos (\alpha t)\} = \frac{1}{\beta^2(\beta^2+\alpha^2)}.$
5. $ES\{\sinh \sinh (\alpha t)\} = \frac{\alpha}{\beta^2(\beta^2-\alpha^2)}.$
6. $ES\{\cosh \cosh (\alpha t)\} = \frac{1}{\beta^2(\beta^2-\alpha^2)}.$

ES Transform of Derivatives:

Let $ES\{f(t)\} = T(\beta)$, then

- i. $ES\{f'(t)\} = \beta T(\beta) - \frac{1}{\beta^2} f(0).$
- ii. $ES\{f''(t)\} = \beta^2 T(\beta) - \frac{1}{\beta^2} f'(0) - \frac{1}{\beta} f(0).$
- iii. $ES\{f^{(n)}(t)\} = \beta T\{f^{(n-1)}(t)\} - \frac{1}{\beta^2} f^{(n-1)}(0), n = 1, 2, 3, \dots$
 $ES\{f^{(n)}(t)\} = \beta T\{f^{(n-1)}(t)\} - \frac{1}{\beta^2} f^{(n-1)}(0),$
 $n = 1, 2, 3, \dots$

36. Complex SEE Integral Transform [43]

The complex (Sadiq- Emad- Jinan) SEE Transform is introduced by Eman A.Mansour. This Transform is defined for the function $f(t)$ as:

$$S^c\{f(t)\} = \frac{1}{v^m} \int_{t=0}^{\infty} f(t)e^{-ivt} dt = T(iv), \quad m \in Z, v \in [l_1, l_2], l_1, l_2 > 0.$$

Complex SEE transform of some elementary functions are:

- 1. $S^c\{t^n\} = \frac{(-1)^n(i)^{n-1}n!}{v^{m+n+1}}, n \in N.$
- 2. $S^c\{e^{at}\} = \frac{1}{v^m} \left[\frac{\alpha}{v^2+\alpha^2} + i \frac{v}{v^2+\alpha^2} \right], \alpha \in R.$
- 3. $S^c\{\sin \sin (\alpha t)\} = \frac{-\alpha}{v^m(v^2-\alpha^2)}.$
- 4. $S_a\{\cos \cos (\alpha t)\} = \frac{-iv}{v^m(v^2-\alpha^2)}.$
- 5. $S_a\{\sinh \sinh (\alpha t)\} = \frac{-\alpha}{v^m(v^2+\alpha^2)}.$
- 6. $S_a\{\cosh \cosh (\alpha t)\} = \frac{-iv}{v^m(v^2+\alpha^2)}.$

Complex SEE Transform of Derivatives:

Let $S^c\{f(t)\} = T(iv)$, then

- i. $S^c\{f'(t)\} = (iv)T(iv) - \frac{1}{v^m} f(0).$
- ii. $S^c\{f''(t)\} = (iv)^2T(iv) - \frac{1}{v^m} f'(0) - \frac{1}{v^m} (iv)f(0).$
- iv. $S^c\{f^{(n)}(t)\} = (iv)^n T(iv) - \frac{1}{v^m} \sum_{j=0}^{n-1} (iv)^j f^{(n-j-1)}(0), n = 1, 2, 3, \dots$
 $S^c\{f^{(n)}(t)\} = (iv)^n T(iv) - \frac{1}{v^m} \sum_{j=0}^{n-1} (iv)^j f^{(n-j-1)}(0),$
 $n = 1, 2, 3, \dots$

37. Soham Transform [44]

A transform was introduced by D.P.Patil et al. named Soham transform. This Transform is defined for the function $f(t)$ as:

$$S\{f(t)\} = \frac{1}{v} \int_0^{\infty} f(t)e^{-v\beta t} dt = P(v), \quad \beta \neq 0, v \in [k_1, k_2].$$

Soham transform of some elementary functions are:

- 1. $S\{t^n\} = \frac{n!}{v^{\beta(n+1)+1}}, n \in N.$
- 2. $S\{e^{\alpha t}\} = \frac{1}{v(v\beta+\alpha)}, \alpha \in R.$
- 3. $S\{\sin \sin (\alpha t)\} = \frac{\alpha}{v(v^2\beta+\alpha^2)}.$
- 4. $S\{\cos \cos (\alpha t)\} = \frac{v\beta}{v(v^2\beta+\alpha^2)}.$
- 5. $S\{\sinh \sinh (\alpha t)\} = \frac{\alpha v}{v^2\beta-\alpha^2}.$
- 6. $S\{\cosh \cosh (\alpha t)\} = \frac{v\beta}{v^2\beta-\alpha^2}.$

Soham Transform of Derivatives:

Let $S\{f(t)\} = P(v)$, then

- i. $S\{f'(t)\} = v\beta P(v) - \frac{1}{v} f(0).$
- ii. $S\{f''(t)\} = v^2\beta P(v) - v\beta^{-1} f(0) - \frac{1}{v} f'(0).$
- i. $S\{f^{(n)}(t)\} = v^{n\beta} P(v) - \frac{1}{v} \sum_{j=0}^{n-1} v^{\beta(n-1-j)} f^{(j)}(0), n = 1, 2, 3, \dots$
 $S\{f^{(n)}(t)\} = v^{n\beta} P(v) - \frac{1}{v} \sum_{j=0}^{n-1} v^{\beta(n-1-j)} f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

38. AMK Transformation [45]

A transform suggested by M. Kashif et al. called the AMK transform. This transform is defined for the function $f(t)$ as:

$$AMK\{f(\sin \sin \varrho)\} = \int_0^{\frac{\pi}{2}} (\sin \sin \varrho)^\sigma \cos \cos \varrho f(\sin \sin \varrho) d\varrho = F(\sigma), \quad v \in [k_1, k_2], t \geq 0.$$

AMK transform of some elementary functions are:

- 1. $AMK\{(\sin \sin \varrho)^n\} = \frac{1}{\sigma+(n+1)}, \sigma > -(n+1).$
- 2. $AMK\{[\ln(\sin \sin \varrho)]^n\} = \frac{n!(-1)^n}{(\sigma+1)^{1+n}}, n \in N.$
- 3. $AMK\{\cos \cos (\ln \sin \sin \varrho)\} = \frac{\sigma+1}{(\sigma+1)^2+\alpha^2}, \sigma > -1.$
- 4. $AMK\{\sin \sin (\alpha \ln \sin \sin \varrho)\} = \frac{\alpha}{(\sigma+1)^2+\alpha^2}.$
- 5. $AMK\{\sinh \sinh (\alpha \ln(\sin \sin \varrho))\} = \frac{-\alpha}{(\sigma+1)^2-\alpha^2}.$
- 6. $AMK\{\cosh \cosh (\alpha \ln(\sin \sin \varrho))\} = \frac{\sigma+1}{(\sigma+1)^2-\alpha^2}.$

AMK Transforms of Derivatives:

Let $AMK\{f(\sin \sin \varrho)\} = F(\sigma)$, then

$$M\{(\sin \sin \varrho)^n f^{(n)}(\sin \sin \varrho)\} =$$

$$f^{(n-1)}(1) + (-1)^n(n + \sigma)f^{(n-2)}(1) + (-1)^{n-1}(n + \sigma)((n - 1) + \sigma)f^{(n-3)}(1) + \dots + (n + \sigma)((n - 1) + \sigma) + \dots + (2 + \sigma)f(1) - (n + \sigma)!F(\sigma),$$

$$n = 1, 2, 3, \dots$$

39. g-Transform [46]

Yusra Al-Ameri and Methaq Hamza presented an integral transform called g-transformation. This transform is defined for the function $f(t)$ as:

$$G_{hk}\{f(t)\} = s^h \int_0^\infty f(t)e^{-s^k t} dt = F_{hk}(s), \quad s > 0.$$

g-Transform of some elementary functions are:

1. $G_{hk}\{t^n\} = n! s^{h-(n+1)k}, \quad n \in N.$
2. $G_{hk}\{e^{\alpha t}\} = \frac{s^h}{s^k - \alpha}, \quad \alpha \in R.$
3. $G_{hk}\{\sin \sin(\alpha t)\} = \frac{\alpha s^h}{s^{2k} + \alpha^2}.$
4. $G_{hk}\{\cos \cos(\alpha t)\} = \frac{s^{h+k}}{s^{2k} + \alpha^2}.$
5. $G_{hk}\{\sinh \sinh(\alpha t)\} = \frac{\alpha s^h}{s^{2k} - \alpha^2}.$
6. $G_{hk}\{\cosh \cosh(\alpha t)\} = \frac{s^{h+k}}{s^{2k} - \alpha^2}.$

g-Transformation of Derivatives:

Let $G_{hk}\{f(t)\} = F_{hk}(s)$, then

- i. $G_{hk}\{f'(t)\} = s^k F_{hk}(s) - s^h f(0).$
- ii. $G_{hk}\{f''(t)\} = s^{2k} F_{hk}(s) - s^h [s^k f(0) - f'(0)].$
- i. $G_{hk}\{f^{(n)}(t)\} = s^{nk} F_{hk}(s) - s^h \sum_{j=0}^{n-1} s^{(n-j-1)k} f^{(j)}(0), \quad n = 1, 2, 3, \dots$
 $G_{hk}\{f^{(n)}(t)\} = s^{nk} F_{hk}(s) - s^h \sum_{j=0}^{n-1} s^{(n-j-1)k} f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

40. Kushare Transformation [47]

Sachin Kushare and Dinkar Patil presented an integral transform named Kushare transform. This transform is defined for the function $f(t)$ as:

$$K\{f(t)\} = v \int_0^\infty f(t)e^{-tv^\beta} dt = S(v), \quad v \in [\tau_1, \tau_2], \tau_1, \tau_2, \beta \neq 0.$$

Kushare transform of some elementary functions are:

1. $K\{t^n\} = \frac{n!}{v^{\beta(n+1)-1}}, \quad n \in N.$
2. $K\{e^{\alpha t}\} = \frac{v}{v^\beta - \alpha}, \quad \alpha \in R.$

$$3. K\{\sin \sin(\alpha t)\} = \frac{\alpha v}{v^{2\beta} + \alpha^2}.$$

$$4. K\{\cos \cos(\alpha t)\} = \frac{v^{\beta+1}}{v^{2\beta} + \alpha^2}.$$

Kushare Transform of Derivatives:

Let $K\{f(t)\} = S(v)$, then

- i. $K\{f'(t)\} = v^\beta S(v) - v f(0).$
- ii. $K\{f''(t)\} = v^{2\beta} S(v) - v^{\beta+1} f(0) - v f'(0).$
- i. $K\{f^{(n)}(t)\} = v^{n\beta} S(v) - v \sum_{j=0}^{n-1} v^{\beta(n-j-1)} f^{(j)}(0), \quad n = 1, 2, 3, \dots$
 $K\{f^{(n)}(t)\} = v^{n\beta} S(v) - v \sum_{j=0}^{n-1} v^{\beta(n-j-1)} f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

41. The General Polynomial Transform [48]

Emad A. Kuffi and Sara F. Maktoof proposed a new transform and called the new general polynomial integral transform. The new general polynomial Transform is defined for the function $f(t)$ as:

$$P_g\{f(t)\} = \int_{t=1}^\infty f(t)t^{-(q(p)+1)} dt = F(q(p)), \quad t \in [1, \infty).$$

The new general polynomial transform of some elementary functions are:

1. $P_g\{t^n\} = \frac{1}{q(p)-n}, \quad 0 \leq n < q(p).$
2. $P_g\{\ln \ln t\} = \frac{1}{(q(p))^2}, \quad q(p) > 0.$
3. $P_g\{t^n \ln \ln t\} = \frac{1}{(q(p)-n)^2}, \quad q(p) > n.$
4. $P_g\{\sin \sin(\alpha \ln \ln t)\} = \frac{\alpha}{(q(p))^2 + \alpha^2}.$
5. $P_g\{\cos \cos(\alpha \ln \ln t)\} = \frac{q(p)}{(q(p))^2 + \alpha^2}.$
6. $P_g\{\sinh \sinh(\alpha \ln \ln t)\} = \frac{\alpha}{(q(p))^2 - \alpha^2}.$
7. $P_g\{\cosh \cosh(\alpha \ln \ln t)\} = \frac{q(p)}{(q(p))^2 - \alpha^2}.$

The New General polynomial Transform of Derivatives:

Let $P_g\{f(t)\} = F(q(p))$, then

- i. $P_g\{t f'(t)\} = q(p)! F(s) - f(1).$
- ii. $P_g\{t^2 f''(t)\} = (q(p) - 1)! F(s) - (q(p) - 1)f(1) - f'(1).$
 $P_g\{t^2 f''(t)\} = (q(p) - 1)! F(s) - (q(p) - 1)f(1) - f'(1).$
- iv. $P_g\{t^n f^{(n)}(t)\} = (q(p) - (n - 1))! F(s) - f^{(n-1)}(1) - (q(p) - (n - 1))f^{(n-2)}(1) - (q(p) - (n - 1))(q(p) - (n - 2))f^{(n-3)}(1) -$

$$\begin{aligned} & (q(p) - (n - 1))(q(p) - (n - 2)) \dots (q(p) - \\ & 1)f(1), \quad n = 1, 2, 3, \dots P_g \{ t^n f^{(n)}(t) \} = \\ & (q(p) - (n - 1))! F(s) - f^{(n-1)}(1) - \\ & (q(p) - (n - 1))f^{(n-2)}(1) \\ & \quad - (q(p) - (n - 1))(q(p) \\ & \quad - (n - 2))f^{(n-3)}(1) \\ & - (q(p) - (n - 1))(q(p) - (n - 2)) \dots (q(p) \\ & - 1)f(1), \\ & \quad n = 1, 2, 3, \dots \end{aligned}$$

42. ZMA Transform [49]

Zainab M. Alwan presented an integral transform named ZMA transform. This transform is defined for the function $f(t)$ as:

$$\Lambda\{f(t)\} = \frac{1}{s} \int_0^{\infty} f(vt) e^{-\frac{t}{s}} dt = Z_{MA}(v, s).$$

ZMA transform of some elementary functions are:

1. $\Lambda\{t^n\} = n! s^n v^n, \quad n \in N.$
2. $\Lambda\{e^{\alpha t}\} = \frac{1}{1 - \alpha sv}, \quad \alpha \in R.$
3. $\Lambda\{\sin \sin(\alpha t)\} = \frac{\alpha sv}{1 + \alpha^2 s^2 v^2}.$
4. $\Lambda\{\cos \cos(\alpha t)\} = \frac{1}{1 + \alpha^2 s^2 v^2}.$

ZMA Transform of Derivatives:

Let $\Lambda\{f(t)\} = Z_{MA}(v, s)$, then

- i. $\Lambda\{f'(t)\} = \frac{1}{sv} Z_{MA}(v, s) - \frac{1}{sv} f(0).$
- ii. $\Lambda\{f''(t)\} = \left(\frac{1}{sv}\right)^2 Z_{MA}(v, s) - \frac{1}{sv} f'(0) - \left(\frac{1}{sv}\right)^2 f(0).$
- i. $\Lambda\{f^{(n)}(t)\} = \left(\frac{1}{sv}\right)^n Z_{MA}(v, s) - \sum_{j=0}^{n-1} \left(\frac{1}{sv}\right)^{n-j} f^{(j)}(0), \quad n = 1, 2, 3, \dots$
 $\Lambda\{f^{(n)}(t)\} = \left(\frac{1}{sv}\right)^n Z_{MA}(v, s) - \sum_{j=0}^{n-1} \left(\frac{1}{sv}\right)^{n-j} f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

43. Emad- Falih Transform [50]

Emad A.Kuffi and Sara F.Maktoof introduced a new integral transform named (Emad- Falih) integral transform. This Transform is defined for the function $f(t)$ as:

$$EF\{f(t)\} = \frac{1}{\varphi} \int_0^{\infty} f(t) e^{-\varphi^2 \alpha t} dt = T(\varphi), \quad \varphi \in [m_1, m_2].$$

Emad- Falih transform of some elementary functions are:

1. $EF\{t^n\} = \frac{n!}{\varphi^{2n+3}}, \quad n \in N.$
2. $EF\{e^{\alpha t}\} = \frac{1}{\varphi(\varphi^2 - \alpha)}, \quad \alpha \in R.$
3. $EF\{\sin \sin(\alpha t)\} = \frac{\alpha}{\varphi(\varphi^4 + \alpha^2)}.$
4. $EF\{\cos \cos(\alpha t)\} = \frac{\varphi}{\varphi^4 + \alpha^2}.$
5. $EF\{\sinh \sinh(\alpha t)\} = \frac{\alpha}{\varphi(\varphi^4 - \alpha^2)}.$
6. $EF\{\cosh \cosh(\alpha t)\} = \frac{\varphi}{\varphi^4 - \alpha^2}.$

Emad- Falih Transform of Derivatives:

Let $EF\{f(t)\} = T(\varphi)$, then

- i. $EF\{f'(t)\} = \varphi^2 T(\varphi) - \frac{1}{\varphi} f(0).$
- ii. $ES\{f''(t)\} = \varphi^2 EF\{f'(t)\} - \frac{1}{\varphi} f'(0). EF\{f''(t)\} = \varphi^2 EF\{f'(t)\} - \frac{1}{\varphi} f'(0).$
- i. $ES\{f^{(n)}(t)\} = \varphi^2 EF\{f^{(n-1)}(t)\} - \frac{1}{\varphi} f^{(n-1)}(0),$
 $n = 1, 2, 3, \dots EF\{f^{(n)}(t)\} = \varphi^2 EF\{f^{(n-1)}(t)\} - \frac{1}{\varphi} f^{(n-1)}(0),$
 $n = 1, 2, 3, \dots$

44. Shaban Transform [51]

Rehab A. Khudair et al. introduced a new integral transform named Shaban transform. This Transform is defined for the function $f(t)$ as:

$$Sh\{f(t)\} = \int_0^{\frac{\pi}{2}} \sin \sin t (\cos \cos t)^p f(t) dt, \quad p \in R, \quad t \in \left[0, \frac{\pi}{2}\right].$$

Shaban transform of some elementary functions are:

1. $Sh\{(\cos \cos t)^n\} = \frac{1}{p+(n+1)}, \quad p > -(n+1).$
2. $Sh\{\cos \cos(\alpha \ln \ln \cos \cos t)\} = \frac{p+1}{(p+1)^2 + \alpha^2}, \quad p > -1.$
3. $Sh\{\sin \sin(\alpha \ln \ln \cos \cos t)\} = \frac{-\alpha}{(p+1)^2 + \alpha^2}, \quad p > -1.$
4. $Sh\{\cosh \cosh(\alpha \ln \ln \cos \cos t)\} = \frac{p+1}{(p+1)^2 - \alpha^2}, \quad p + 1 > \alpha.$

$$5. Sh\{sinh\ sinh(\alpha \ln \ln \cos \cos t)\} = \frac{-\alpha}{(p+1)^2 - \alpha^2},$$

$$p + 1 > \alpha.$$

Shaban Transform of Derivatives:

Let $Sh\{f(t)\}$ be the Shaban transform of $f(t)$, then

- i. $Sh\{\cos \cos t f'(\cos \cos t)\} = f(1) - (p + 1) Sh\{f(\cos \cos t)\}.$
- ii. $Sh\{(\cos \cos t)^2 f''(\cos \cos t)\} = f'(1) - (p + 2)f(1) + (p + 2)(p + 1) Sh\{f(\cos \cos t)\}.$
 $Sh\{(\cos \cos t)^2 f''(\cos \cos t)\} = f'(1) - (p + 2)f(1) + (p + 2)(p + 1) Sh\{f(\cos \cos t)\}.$
- i. $S\{(\cos \cos t)^n f^{(n)}(\cos \cos t)\} = f^{(n-1)}(1) - (p + n)f^{(n-2)}(1) + \dots + (p + n)(p + (n - 1))(p + (n - 2)) \dots (p + 2) f(1) + (-1)^{(n)}(p + n)(p + (n - 1))(p + (n - 2)) + \dots + (p + 2)(p + 1) Sh\{f(t)\},$
 $n = 1, 2, 3, \dots. S\{(\cos \cos t)^n f^{(n)}(\cos \cos t)\} = f^{(n-1)}(1) - (p + n)f^{(n-2)}(1) + \dots + (p + n)(p + (n - 1))(p + (n - 2)) \dots (p + 2) f(1) + (-1)^{(n)}(p + n)(p + (n - 1))(p + (n - 2)) + \dots + (p + 2)(p + 1) Sh\{f(t)\},$
 $n = 1, 2, 3, \dots$

In (2022)

45. AMJ Transformation [52]

Adil Mousa presented an integral transform named AMJ transform. This transform is defined for the function $f(t)$ as:

$$A\{f(t)\} = v \int_0^\infty f\left(\frac{t}{v}\right) e^{-vt} dt = J(v), \quad v > 0, 0 < a < b.$$

AMJ transform of some elementary functions are:

- 1. $A\{t^n\} = \frac{n!}{v^{2n+1}}, \quad n \in N.$
- 2. $A\{e^{\alpha t}\} = \frac{v}{v^2 - \alpha}, \quad \alpha \in R.$
- 3. $A\{\sin \sin(\alpha t)\} = \frac{\alpha v}{v^4 + \alpha^2}.$
- 4. $A\{\cos \cos(\alpha t)\} = \frac{v^3}{v^4 + \alpha^2}.$
- 5. $A\{\sinh \sinh(\alpha t)\} = \frac{\alpha v}{v^4 - \alpha^2}.$
- 6. $A\{\cosh \cosh(\alpha t)\} = \frac{v^3}{v^4 - \alpha^2}.$

AMJ Transform of Derivatives:

Let $A\{f(t)\} = J(v)$, then

- i. $A\{f'(t)\} = v^2 J(v) - v f(0).$
- ii. $A\{f''(t)\} = v^4 J(v) - v^3 f(0) - v f'(0).$
- iii. $A\{f^{(n)}(t)\} = v^{2n} J(v) - \sum_{j=0}^{n-1} v^{2(n-j)-1} f^{(j)}(0),$
 $n = 1, 2, 3, \dots. A\{f^{(n)}(t)\} = v^{2n} J(v) - \sum_{j=0}^{n-1} v^{2(n-j)-1} f^{(j)}(0),$
 $n = 1, 2, 3, \dots$

46. Anuj Transformation [53]

Anuj Kumar et al. presented an integral transform in 2022 named Anuj transform. This transform is defined for the function $f(t)$ as:

$$A\{f(t)\} = v^2 \int_0^\infty f(t) e^{-\frac{t}{v}} dt = F(v), \quad v > 0.$$

Anuj transform of some elementary functions are:

- 1. $A\{t^n\} = v^{n+3} n!, \quad n \in N.$
- 2. $A\{e^{\alpha t}\} = \frac{v^3}{1 - \alpha v}, \quad \alpha \in R.$
- 3. $A\{\sin \sin(\alpha t)\} = \frac{\alpha v^4}{\alpha^2 v^2 + 1}.$
- 4. $A\{\cos \cos(\alpha t)\} = \frac{v^3}{\alpha^2 v^2 + 1}.$
- 5. $A\{\sinh \sinh(\alpha t)\} = \frac{\alpha v^4}{1 - \alpha^2 v^2}.$
- 6. $A\{\cosh \cosh(\alpha t)\} = \frac{v^3}{1 - \alpha^2 v^2}.$

Anuj Transform of Derivatives:

Let $A\{f(t)\} = F(v)$, then

- i. $A\{f'(t)\} = \frac{1}{v} F(v) - v^2 f(0).$
- ii. $A\{f''(t)\} = \frac{1}{v^2} F(v) - v f(0) - v^2 f'(0).$
- iii. $A\{f^{(3)}(t)\} = \frac{1}{v^3} F(v) - f(0) - v f'(0) - v^2 f''(0).$

47. Formable Transformation [54]

Rania Zohair and Bayan Fu'ad presented an integral transform called Formable transform. This transform is defined for the function $f(t)$ as:

$$R\{f(t)\} = s \int_0^\infty f(vt) e^{-st} dt = B(s, v), \quad s, v > 0.$$

Formable transform of some elementary functions are:

- 1. $R\{t^n\} = \frac{v^n n!}{s^n}, \quad n \in N.$
- 2. $R\{e^{\alpha t}\} = \frac{s}{s - \alpha v}, \quad \alpha \in R.$
- 3. $R\{\sin \sin(\alpha t)\} = \frac{\alpha s v}{s^2 + \alpha^2 v^2}.$
- 4. $R\{\cos \cos(\alpha t)\} = \frac{s^2}{s^2 + \alpha^2 v^2}.$
- 5. $R\{\sinh \sinh(\alpha t)\} = \frac{\alpha s v}{s^2 - \alpha^2 v^2}.$
- 6. $R\{\cosh \cosh(\alpha t)\} = \frac{s^2}{s^2 - \alpha^2 v^2}.$

Formable Transform of Derivatives:

Let $R\{f(t)\} = B(s, v)$, then

- i. $R\{f'(t)\} = \frac{s}{v}B(s, v) - \frac{s}{v}f(0)$.
- ii. $R\{f''(t)\} = \frac{s^2}{v^2}B(s, v) - \frac{s^2}{v^2}f(0) - \frac{s}{v}f'(0)$.
- i. $R\{f^{(n)}(t)\} = \frac{s^n}{v^n}B(s, v) - \sum_{j=0}^{n-1} \frac{s^{n-j}}{v^{n-j}}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$. $R\{f^{(n)}(t)\} = \frac{s^n}{v^n}B(s, v) -$
 $\sum_{j=0}^{n-1} \frac{s^{n-j}}{v^{n-j}}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

48. Khalouta Transform [55]

Ali Khalouta presented a definition of a new integral transform which is called Khalouta transformation. This transform is defined for the function $f(t)$ as:

$$KH\{f(t)\} = s \int_0^\infty f(\gamma\eta t)e^{-st} dt = K(s, \gamma, \eta),$$

$$s, \gamma, \eta > 0.$$

Khalouta Transform of some elementary functions are:

- 1. $KH\{t^n\} = \frac{(\gamma\eta)^{n!}}{s^n}, n \in N$.
- 2. $KH\{e^{at}\} = \frac{s}{s-\alpha\gamma\eta}, \alpha \in R$.
- 3. $KH\{\sin \sin (at)\} = \frac{\alpha s \gamma \eta}{s^2 + \alpha^2 (\gamma \eta)^2}$.
- 4. $KH\{\cos \cos (at)\} = \frac{s^2}{s^2 + \alpha^2 (\gamma \eta)^2}$.
- 5. $KH\{\sinh \sinh (at)\} = \frac{\alpha s \gamma \eta}{s^2 - \alpha^2 (\gamma \eta)^2}$.
- 6. $KH\{\cosh \cosh (at)\} = \frac{s^2}{s^2 - \alpha^2 (\gamma \eta)^2}$.

Khalouta Transform of Derivatives:

Let $KH\{f(t)\} = K(s, \gamma, \eta)$, then

- i. $KH\{f'(t)\} = \frac{s}{\gamma\eta}K(s, \gamma, \eta) - \frac{s}{\gamma\eta}f(0)$.
- ii. $KH\{f''(t)\} = \frac{s^2}{(\gamma\eta)^2}K(s, \gamma, \eta) - \frac{s^2}{(\gamma\eta)^2}f(0) -$
 $\frac{s}{\gamma\eta}f'(0)$.
- iii. $KH\{f^{(n)}(t)\} =$
 $\frac{s^n}{(\gamma\eta)^n}K(s, \gamma, \eta) - \sum_{j=0}^{n-1} \frac{s^{n-j}}{(\gamma\eta)^{n-j}}f^{(j)}(0)$,
 $n = 1, 2, 3, \dots$

49. KKAT Transformation [56]

Karry Iqbal et al. presented an integral transform named KKAT transform. This transform is defined for the function $f(t)$ as:

$$K\{f(t)\} = \frac{1}{\delta\beta} \int_0^\infty f(t)e^{-\frac{\beta t}{\delta}} dt = F\left(\frac{\beta}{\delta}\right), \beta, \delta \neq 0.$$

KKAT transform of some elementary functions are:

- 1. $K\{t^n\} = \frac{n!\delta^n}{\beta^{n+2}}, n \in N$.
- 2. $K\{e^{at}\} = \frac{1}{\beta(\beta-\alpha\delta)}, \alpha \in R$.
- 3. $K\{\sin \sin (at)\} = \frac{\alpha\delta}{\beta(\beta^2 + \alpha^2\delta^2)}$.
- 4. $K\{\cos \cos (at)\} = \frac{1}{\beta^2 + \alpha^2\delta^2}$.
- 5. $K\{\sinh \sinh (at)\} = \frac{\alpha\delta}{\beta(\beta^2 - \alpha^2\delta^2)}$.
- 6. $K\{\cosh \cosh (at)\} = \frac{1}{\beta^2 - \alpha^2\delta^2}$.

KKAT Transform of Derivatives:

Let $K\{f(t)\} = F\left(\frac{\beta}{\delta}\right)$, then

- i. $K\{f'(t)\} = \left(\frac{\beta}{\delta}\right)F\left(\frac{\beta}{\delta}\right) - \left(\frac{1}{\beta\delta}\right)f(0)$.
- ii. $K\{f''(t)\} = \left(\frac{\beta}{\delta}\right)^2 F\left(\frac{\beta}{\delta}\right) - \left(\frac{1}{\delta\beta}\right)f'(0) - \left(\frac{1}{\delta^2}\right)f(0)$.
- i. $K\{f^{(n)}(t)\} = \frac{\beta}{\delta}K\{f^{(n-1)}(t)\} - \frac{1}{\delta\beta}f^{(n-1)}(0), n =$
 $1, 2, 3, \dots$. $K\{f^{(n)}(t)\} = \frac{\beta}{\delta}K\{f^{(n-1)}(t)\} -$
 $\frac{1}{\delta\beta}f^{(n-1)}(0)$,
 $n = 1, 2, 3, \dots$

50. KAJ Transform [57]

Emad A. Kuffi, Elaf S. Abbas and Alyaa A. Jawad presented an integral transform named the "Kuffi-Abbas-Jawad" KAJ integral transform. This transform is defined for the function $f(t)$ as:

$$S_m\{f(t)\} = \frac{1}{v^m} \int_0^\infty f\left(\frac{t}{v}\right)e^{-t} dt = K(v), 0 < l_1$$

$$\leq v \leq l_2.$$

KAJ transform of some elementary functions are:

- 1. $S_m\{t^n\} = \frac{n!}{v^{m+n}}, n \in N$.
- 2. $S_m\{e^{at}\} = \frac{v}{v^m(v-\alpha)}, \alpha \in R$.
- 3. $S_m\{\sin \sin (at)\} = \frac{\alpha v}{v^m(v^2 + \alpha^2)}$.
- 4. $S_m\{\cos \cos (at)\} = \frac{v^2}{v^m(v^2 + \alpha^2)}$.
- 5. $S_m\{\sinh \sinh (at)\} = \frac{\alpha v}{v^m(v^2 - \alpha^2)}$.
- 6. $S_m\{\cosh \cosh (at)\} = \frac{v^2}{v^m(v^2 - \alpha^2)}$.

KAJ Transform of Derivatives:

Let $S_m\{f(t)\} = K(v)$, then

- i. $S_m\{f'(t)\} = vK(v) - \frac{v}{v^m}f(0).$
- ii. $S_m\{f''(t)\} = v^2K(v) - \frac{v^2}{v^m}f(0) - \frac{v}{v^m}f'(0), n = 1, 2, 3, \dots$
 $S_m\{f''(t)\} = v^2K(v) - \frac{v^2}{v^m}f(0) - \frac{v}{v^m}f'(0).$

51. Rishi Transform [58]

R. Kumar et al. presented an integral transform named Rishi transform. This transform is defined for the function $f(t)$ as:

$$R\{f(t)\} = \left(\frac{\sigma}{\varepsilon}\right) \int_0^\infty f(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t} dt = T(\varepsilon, \sigma), \quad \varepsilon, \sigma > 0.$$

Rishi transform of some elementary functions are:

- 1. $R\{t^n\} = n! \left(\frac{\sigma}{\varepsilon}\right)^{n+2}, n \in N.$
- 2. $R\{e^{\alpha t}\} = \frac{\sigma^2}{\varepsilon(\varepsilon - \alpha\sigma)}, \alpha \in R.$
- 3. $R\{\sin \sin(\alpha t)\} = \frac{\alpha\sigma^3}{\varepsilon(\varepsilon^2 + \alpha^2\sigma^2)}.$
- 4. $R\{\cos \cos(\alpha t)\} = \frac{\sigma^2}{\varepsilon^2 + \alpha^2\sigma^2}.$
- 5. $R\{\sinh \sinh(\alpha t)\} = \frac{\alpha\sigma^3}{\varepsilon(\varepsilon^2 - \alpha^2\sigma^2)}.$
- 6. $R\{\cosh \cosh(\alpha t)\} = \frac{\sigma^2}{\varepsilon^2 - \alpha^2\sigma^2}.$

Rishi Transform of Derivatives:

Let $R\{f(t)\} = T(\varepsilon, \sigma)$, then

- i. $R\{f'(t)\} = \left(\frac{\varepsilon}{\sigma}\right)T(\varepsilon, \sigma) - \left(\frac{\sigma}{\varepsilon}\right)f(0).$
- ii. $R\{f''(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^2 T(\varepsilon, \sigma) - f(0) - \left(\frac{\sigma}{\varepsilon}\right)f'(0).$
- iii. $R\{f^{(3)}(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^3 T(\varepsilon, \sigma) - \left(\frac{\varepsilon}{\sigma}\right)f(0) - f'(0) - \left(\frac{\sigma}{\varepsilon}\right)f''(0).$
 $R\{f^{(3)}(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^3 T(\varepsilon, \sigma) - \left(\frac{\varepsilon}{\sigma}\right)f(0) - f'(0) - \left(\frac{\sigma}{\varepsilon}\right)f''(0).$

52. SEA Transform [59]

Emad A. Kuffi et al. presented a new integral transform named the (Sherifa-Emad-Ali) SEA integral transform. This transform is defined for the function $f(t)$ as:

$$H^c\{f(t)\} = \frac{s}{v} \int_0^\infty f(t)e^{-i\left(\frac{s}{v}\right)t} dt = Z(v, s),$$

$s, v > 0.$

SEA transform of some elementary functions are:

- 1. $H^c\{t^n\} = (-1)^n(i)^{n-1}n! \left(\frac{v}{s}\right)^n, n \in N.$
- 2. $H^c\{e^{\alpha t}\} = -\left[\frac{\alpha sv}{s^2 + \alpha^2 v^2} + i \frac{s^2}{s^2 + \alpha^2 v^2}\right], \alpha \in R.$

- 3. $H^c\{\sin \sin(\alpha t)\} = \frac{-\alpha sv}{s^2 - \alpha^2 v^2}.$
- 4. $H^c\{\cos \cos(\alpha t)\} = \frac{-is^2}{s^2 - \alpha^2 v^2}.$

SEA Transform of Derivatives:

Let $H^c\{f(t)\} = Z(v, s)$, then

- i. $H^c\{f'(t)\} = i \frac{s}{v}Z(v, s) - \frac{s}{v}f(0).$
- ii. $H^c\{f''(t)\} = -\left(\frac{s}{v}\right)^2 Z(v, s) - \frac{s}{v}f'(0) - i \left(\frac{s}{v}\right)^2 f(0).$
- i. $H^c\{f^{(n)}(t)\} = (i)^n \left(\frac{s}{v}\right)^n Z(v, s) - \sum_{j=1}^n (i)^{j-1} \left(\frac{s}{v}\right)^j f^{(n-j)}(0),$
 $n = 1, 2, 3, \dots$
 $H^c\{f^{(n)}(t)\} = (i)^n \left(\frac{s}{v}\right)^n Z(v, s) - \sum_{j=1}^n (i)^{j-1} \left(\frac{s}{v}\right)^j f^{(n-j)}(0),$
 $n = 1, 2, 3, \dots$

53. Complex EE Transform [60]

Emad A. Kuffi and Elaf S. Abbas presented an integral transform named Complex EE transform. This transform is defined for the function $f(t)$ as:

$$E^c\{f(t)\} = v \int_0^\infty f\left(\frac{t}{v}\right)e^{-iv^m t} dt = E(iv), \quad v \in [l_1, l_2].$$

Complex EE transform of some elementary functions are:

- 1. $E^c\{t^n\} = \frac{n!}{(iv)^{(n+1)m}}, n \in N.$
- 2. $E^c\{e^{\alpha t}\} = -\left[\frac{\alpha}{\alpha^2 + v^{2m}} + i \frac{v^m}{\alpha^2 + v^{2m}}\right], \alpha \in R.$
- 3. $E^c\{\sin \sin(\alpha t)\} = \frac{-\alpha}{v^{2m} - \alpha^2}.$
- 4. $E^c\{\cos \cos(\alpha t)\} = \frac{-iv^m}{v^{2m} - \alpha^2}.$
- 5. $E^c\{\sinh \sinh(\alpha t)\} = \frac{-\alpha}{v^{2m} + \alpha^2}.$
- 6. $E^c\{\cosh \cosh(\alpha t)\} = \frac{-iv^m}{v^{2m} + \alpha^2}.$

Complex EE Transform of Derivatives:

Let $E^c\{f(t)\} = E(iv)$, then

- i. $E^c\{f'(t)\} = iv^m E(iv) - f(0).$
- ii. $E^c\{f''(t)\} = -v^{2m} E(iv) - iv^m f'(0) - f'(0).$

54. Complex SEL Integral Transform [61]

Emad A. Kuffi et al. presented a new complex integral transform that is called complex (Serifenur-Emad-Luay) SEL integral transform. This transform is defined for the function $f(t)$ as:

$$\eta^c\{f(t)\} = \frac{1}{\mu} \int_{-\infty}^0 f(t)e^{i\mu t} dt, \quad \frac{1}{\lambda_1} \leq \mu \leq \frac{1}{\lambda_2}.$$

Complex SEL integral transform of some elementary functions are:

1. $\eta^c\{t^n\} = \frac{(i)^{n-1}n!}{\mu^{n+2}}, n \in N.$
2. $\eta^c\{e^{\alpha t}\} = \frac{-1}{\mu} \left[\frac{-\alpha}{\alpha^2 + \mu^2} + i \frac{\mu}{\alpha^2 + \mu^2} \right], \alpha \in R.$
3. $\eta^c\{\sin \sin (\alpha t)\} = \frac{1}{\mu} \left(\frac{1}{1 - \mu^2} \right).$
4. $\eta^c\{\cos \cos (\alpha t)\} = \frac{i}{1 - \mu^2}.$

Complex SEL Integral Transform of Derivatives:

Let $\eta^c\{f(t)\}$ be the complex SEL integral transform for $f(t)$

- i. $\eta^c\{f'(t)\} = -i\mu\eta^c\{f(t)\} - \frac{1}{\mu}f(0).$
- ii. $\eta^c\{f''(t)\} = -\mu^2\eta^c\{f(t)\} - if(0) + \frac{1}{\mu}f'(0).$
- iii. $\eta^c\{f'''(t)\} = i\mu^3\eta^c\{f(t)\} - \mu f(0) - if'(0) + \frac{1}{\mu}f''(0).$

55. Generalization of Rangaig Transformation [62]

Eman A. Mansour and Emad A. Kuffi presented a generalization of Rangaig integral transform named. This transform is defined for a function $f(t)$ as:

$$\eta_g\{f(t)\} = \frac{1}{\mu^m} \int_{t=-\infty}^0 f(t)e^{p(\mu)t} dt = \Lambda_g(\mu), \quad \mu \in \left[\frac{1}{\lambda_1}, \frac{1}{\lambda_2} \right], m \in Z.$$

The generalizations of Rangaig transform of some elementary functions are:

1. $\eta_g\{t^n\} = \frac{(-1)^n n!}{\mu^m [p(\mu)]^{n+1}}, n \in N.$
2. $\eta_g\{e^{\alpha t}\} = \frac{1}{\mu^m (p(\mu) + \alpha)}, \alpha \in R.$
3. $\eta_g\{\sin \sin (t)\} = \frac{1}{\mu^m} \left(\frac{1}{[p(\mu)]^2 + 1} \right).$
4. $\eta_g\{\cos \cos (t)\} = \frac{1}{\mu^m} \left(\frac{p(\mu)}{[p(\mu)]^2 + 1} \right).$

Generalization of Rangaig Transform of Derivatives:

Let $\eta_g\{f(t)\} = \Lambda_g(\mu)$, then

- i. $\eta_g\{f'(t)\} = -p(\mu) \Lambda_g(\mu) + \frac{1}{\mu^m} f(0).$
- ii. $\eta_g\{f''(t)\} = [p(\mu)]^2 \Lambda_g(\mu) - \frac{p(\mu)}{\mu^m} f(0) + \frac{1}{\mu^m} f'(0).$
- iv. $\eta_g\{f^{(n)}(t)\} = (-1)^n [p(\mu)]^n \Lambda_g(\mu) +$

$$\frac{1}{\mu^m} \sum_{j=0}^{n-1} (-1)^j [p(\mu)]^j f^{(n-1-j)}(0), n = 1, 2, 3, \dots n = 1, 2, 3, \dots$$

56. The Generalization of Complex Al-Tememe Transform [63]

Mustafa J. Hussein et al. presented the generalization of complex Al-Tememe transform. This transform is defined for the function $f(t)$ as:

$$T_g^{c'}\{f(t)\} = \int_1^\infty t^{-iq(p)} f(t) dt = F(iq(p)), \quad t > 1.$$

The Generalization of complex Al-Tememe integral transform of some elementary functions are:

1. $T_g^{c'}\{x^n\} = \frac{-(n+1)}{[q(p)]^2 + (n+1)^2} - \frac{iq(p)}{[q(p)]^2 + (n+1)^2}, n \in N.$
2. $T_g^{c'}\{\ln \ln t\} = \frac{1 - [q(p)]^2}{([q(p)]^2 + 1)^2} + \frac{2iq(p)}{([q(p)]^2 + 1)^2}, t > 0.$
3. $T_g^{c'}\{\sin \sin (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 - 1) - \alpha^2}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4} + \frac{2iaq(p)}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4} \cdot T_g^{c'}\{\sin \sin (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 - 1) - \alpha^2 + 2iaq(p)}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4}.$
4. $T_g^{c'}\{\cos \cos (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 + 1) + \alpha^2}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4} - \frac{-iq(p)[([q(p)]^2 + 1) - \alpha^2]}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4} \cdot T_g^{c'}\{\cos \cos (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 + 1) + \alpha^2 + iq(p)[([q(p)]^2 + 1) - \alpha^2]}{([q(p)]^2 + 1)^2 - 2\alpha^2([q(p)]^2 - 1) + \alpha^4}.$
5. $T_g^{c'}\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 - 1) + \alpha^2}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4} + \frac{2iaq(p)}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4} \cdot T_g^{c'}\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{-\alpha([q(p)]^2 - 1) + \alpha^2 + 2iaq(p)}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4}.$
6. $T_g^{c'}\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{-[([q(p)]^2 + 1) + \alpha^2]}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4} - \frac{-iq(p)[([q(p)]^2 + 1) + \alpha^2]}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4} \cdot T_g^{c'}\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{-[([q(p)]^2 + 1) + \alpha^2] + iq(p)[([q(p)]^2 + 1) + \alpha^2]}{([q(p)]^2 + 1)^2 + 2\alpha^2([q(p)]^2 - 1) + \alpha^4}.$

The Generalization of complex Al-Tememe Transform of Derivatives:

Let $T_g^{c'}\{f(t)\} = F(ip)$, then

- i. $T_g^{c'}\{xf'(t)\} = (iq(p) - 1)F(iq(p)) - f(1).$

$$\begin{aligned} \text{ii. } T_g^{c'}\{x^2 f''(t)\} &= (iq(p) - 2)(iq(p) - 1)F(iq(p)) - \\ &(iq(p) - 2)f(1) - f'(1). T_g^{c'}\{x^2 f''(t)\} = \\ &(iq(p) - 2)(iq(p) - 1)F(iq(p)) - \\ &(iq(p) - 2)f(1) - f'(1). \end{aligned}$$

$$\begin{aligned} \text{iii. } T_g^{c'}\{x^n f^{(n)}(t)\} &= -f^{(n-1)}(1) - (iq(p) - \\ &n)f^{(n-2)}(1) - \\ &\dots - (iq(p) - n)(iq(p) - (n - 1))(iq(p) \\ &- (n - 2)) \dots \\ &(iq(p) - 2) f(1) + (iq(p) - n)! F(iq(p)), n \\ &= 1, 2, 3, \dots \end{aligned}$$

57. Battor-Al-Zughair Transform [64]

Ali H. Mohammed et al. presented a new integral transform called Battor-Al- Zughair integral transform. The Battor-Al- Zughair integral transform is convergent and it is defined for the function $f(t)$ as:

$$\begin{aligned} BZ\{f(t)\} &= \lambda \int_1^e \frac{(\ln \ln t)^{-\frac{1}{\lambda}}}{t} f(t) dt = F(\lambda), \quad \lambda \\ &\in \frac{R}{[-1, 0]}. \end{aligned}$$

Battor-Al-Zughair integral transform of some elementary functions are:

1. $BZ\{k\} = \frac{k\lambda^2}{1+\lambda}, k \in R.$
2. $BZ\{\ln \ln (t)\} = \frac{\lambda^2}{1+2\lambda}.$
3. $BZ\{(\ln \ln t)^n\} = \frac{\lambda^2}{1+(n+1)\lambda}, n \in R.$
4. $BZ\{\ln \ln (\ln \ln (t))\} = \frac{-\lambda^3}{(1+\lambda)^2}.$
5. $BZ\{[\ln \ln (\ln \ln (t))]^n\} = \frac{(-1)^n n! \lambda^{n+2}}{(1+\lambda)^{n+1}}, n \in N.$
6. $BZ\{\sin \sin (\alpha \ln \ln (\ln \ln t))\} = \frac{-\alpha \lambda^3}{(1+\lambda)^2 + \alpha^2 \lambda^2}.$
7. $BZ\{\cos \cos (\alpha \ln \ln (\ln \ln t))\} = \frac{\lambda^2(1+\lambda)}{(1+\lambda)^2 + \alpha^2 \lambda^2}.$
8. $BZ\{\sinh \sinh (\alpha \ln \ln (\ln \ln t))\} = \frac{-\alpha \lambda^3}{(1+\lambda)^2 - \alpha^2 \lambda^2}.$
9. $BZ\{\cosh \cosh (\alpha \ln \ln (\ln \ln t))\} = \frac{\lambda^2(1+\lambda)}{(1+\lambda)^2 - \alpha^2 \lambda^2}.$

Battor-Al-Zughair Transform of Derivatives:

Let $BZ\{f(t)\} = F(\lambda)$, then

$$\text{i. } BZ\{(\ln \ln t) f'((\ln \ln t))\} = -\left(\frac{1+\lambda}{\lambda}\right) BZ\{f(t)\} + \lambda f(1).$$

$$\begin{aligned} \text{iv. } BZ\{(\ln \ln t)^2 f''(\ln \ln t)\} &= \\ &\frac{(1+\lambda)(1+2\lambda)}{\lambda^2} BZ\{f(t)\} + \lambda f'(1) - \\ &(1 + 2\lambda)f(1). BZ\{(\ln \ln t)^2 f''(\ln \ln t)\} = \\ &\frac{(1+\lambda)(1+2\lambda)}{\lambda^2} BZ\{f(t)\} + \lambda f'(1) - \\ &(1 + 2\lambda)f(1). BZ\{(\ln \ln t)^2 f''(\ln \ln t)\} = \\ &\frac{(1+\lambda)(1+2\lambda)}{\lambda^2} BZ\{f(t)\} + \lambda f'(1) - \\ &(1 + 2\lambda)f(1). \\ \text{ii.} \\ \text{v. } BZ\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} &= \lambda f^{(n-1)}(1) + \\ &(1 + n\lambda)f^{(n-2)}(1) + \frac{(1+n\lambda)(1+(n-1)\lambda)}{\lambda} f^{(n-3)}(1) - \\ &\dots + (-1)^n \frac{(1+n\lambda)(1+(n-1)\lambda)\dots(1+\lambda)}{\lambda^n} BZ\{f(t)\}, n = \\ &1, 2, 3, \dots BZ\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} = \\ &\lambda f^{(n-1)}(1) + (1 + n\lambda)f^{(n-2)}(1) \\ &+ \frac{(1 + n\lambda)(1 + (n - 1)\lambda)}{\lambda} f^{(n-3)}(1) - \dots \\ &+ (-1)^n \frac{(1 + n\lambda)(1 + (n - 1)\lambda) \dots (1 + \lambda)}{\lambda^n} BZ\{f(t)\}, n \\ &= 1, 2, 3, \dots \end{aligned}$$

58. Kuffi-Al-Tememe Transformation [64]

Emad A. Kuffi and Ali H. Mohammed presented a new integral transform called Kuffi-Al-Tememe Transform. This transform is defined for the function $f(t)$ as:

$$KT\{f(t)\} = \frac{1 - \lambda}{\lambda} \int_1^\infty f(t) t^{-\frac{1}{\lambda}} dt = F(\lambda), \quad \lambda > 0.$$

Kuffi-Al-Tememe transform of some elementary functions are:

1. $KT\{t^n\} = \frac{1-\lambda}{1-(n+1)\lambda}, \lambda \in \left(0, \frac{1}{n+1}\right).$
2. $KT\{\ln \ln t\} = \frac{\lambda}{1-\lambda}, \lambda \in (0, 1).$
3. $KT\{(\ln \ln t)^n\} = \frac{n! \lambda^n}{(1-\lambda)^n}, \lambda \in (0, 1), n \in N.$
4. $KT\{\sin \sin \alpha \ln \ln t\} = \frac{\alpha \lambda(1-\lambda)}{(1-\lambda)^2 + \alpha^2 \lambda^2}.$
5. $KT\{\cos \cos \alpha \ln \ln t\} = \frac{(1-\lambda)^2}{(1-\lambda)^2 + \alpha^2 \lambda^2}.$
6. $KT\{\sinh \sinh \alpha \ln \ln t\} = \frac{\alpha \lambda(1-\lambda)}{(1-\lambda)^2 - \alpha^2 \lambda^2}.$
7. $KT\{\cosh \cosh \alpha \ln \ln t\} = \frac{(1-\lambda)^2}{(1-\lambda)^2 - \alpha^2 \lambda^2}.$

Kuffi-Al-Tememe of Derivatives:

Let $KT\{f(t)\} = F(\lambda)$, then

$$\text{i. } KT\{t f'(t)\} = \frac{1-\lambda}{\lambda} KT\{f(t)\} - \frac{1-\lambda}{\lambda} f(1).$$

$$\begin{aligned}
 \text{ii. } & KT\{t^2 f''(t)\} = \frac{(1-\lambda)(1-2\lambda)}{\lambda^2} KT\{f(t)\} - \\
 & \frac{1-\lambda}{\lambda} f'(1) \\
 & - \frac{(1-\lambda)(1-2\lambda)}{\lambda^2} f(1). \\
 \text{i. } & KT\{t^n f^{(n)}(t)\} = \\
 & - \frac{1-\lambda}{\lambda} f^{(n-1)}(1) - \frac{(1-n\lambda)(1-\lambda)}{\lambda} f^{(n-2)}(1) \dots \\
 & + \frac{(1-n\lambda)(1-(n-1)\lambda) \dots (1-\lambda)}{\lambda^n} f(1) - \dots + \\
 & \frac{(1-n\lambda)(1-(n-1)\lambda) \dots (1-\lambda)}{\lambda^n} KT\{f(t)\}, \\
 & n = 1, 2, 3, \dots
 \end{aligned}$$

59. Kuffi--Al-Zughair Transform [64]

Emad A. Kuffi and Ali H. Mohammed presented a new integral transform called Kuffi-Al- Zughair integral transform. The Kuffi-Al- Zughair integral transform is defined for the function $f(t)$ as:

$$\begin{aligned}
 KZ\{f(t)\} &= \frac{1+\lambda}{\lambda} \int_1^e \frac{(\ln \ln t)^{\frac{1}{\lambda}}}{t} f(t) dt = F(\lambda), \quad \lambda \\
 &\in \frac{R}{[-1, 0]}.
 \end{aligned}$$

Kuffi-Al- Zughair integral transform of some elementary functions are:

1. $KZ\{k\} = k, \quad k \in R.$
2. $KZ\{(\ln \ln t)^n\} = \frac{1+\lambda}{1+(n+1)\lambda}, \quad n \in R.$
3. $KZ\{\ln \ln (\ln \ln t)\} = \frac{-\lambda}{1+\lambda}.$
4. $KZ\{[\ln \ln (\ln \ln t)]^n\} = \frac{(-1)^n n! \lambda^n}{(1+\lambda)^n}, \quad n \in N.$
5. $KZ\{\sin \sin (\alpha \ln \ln (\ln \ln t))\} = \frac{-\alpha \lambda (1+\lambda)}{(1+\lambda)^2 + \alpha^2 \lambda^2}.$
6. $KZ\{\cos \cos (\alpha \ln \ln (\ln \ln t))\} = \frac{(1+\lambda)^2}{(1+\lambda)^2 + \alpha^2 \lambda^2}.$
7. $KZ\{\sinh \sinh (\alpha \ln \ln (\ln \ln t))\} = \frac{-\alpha \lambda (1+\lambda)}{(1+\lambda)^2 - \alpha^2 \lambda^2}.$
8. $KZ\{\cosh \cosh (\alpha \ln \ln (\ln \ln t))\} = \frac{(1+\lambda)^2}{(1+\lambda)^2 - \alpha^2 \lambda^2}.$

Kuffi-Al-Zughair Transform of Derivatives:

Let $KZ\{f(t)\} = F(\lambda)$, then

$$\begin{aligned}
 \text{i. } & KZ\{(\ln \ln t) f'((\ln \ln t))\} = \\
 & - \left(\frac{1+\lambda}{\lambda}\right) KZ\{f(\ln \ln t)\} + \\
 & \left(\frac{1+\lambda}{\lambda}\right) f(1). KZ\{(\ln \ln t) f'((\ln \ln t))\} = \\
 & - \left(\frac{1+\lambda}{\lambda}\right) KZ\{f(\ln \ln t)\} + \left(\frac{1+\lambda}{\lambda}\right) f(1).
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. } & KZ\{(\ln \ln t)^2 f''(\ln \ln t)\} = \\
 & \frac{(1+\lambda)(1+2\lambda)}{\lambda^2} KZ\{f(\ln \ln t)\} + \left(\frac{1+\lambda}{\lambda}\right) f'(1) - \\
 & \frac{(1+\lambda)(1+2\lambda)}{\lambda^2} f(1). \\
 \text{iii. } & KZ\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} = \\
 & \frac{1+\lambda}{\lambda} f^{(n-1)}(1) - \frac{(1+n\lambda)(1+\lambda)}{\lambda^2} f^{(n-2)}(1) \\
 & + \frac{(1+n\lambda)(1+(n-1)\lambda)(1+\lambda)}{\lambda^3} f^{(n-3)}(1) - \dots + \\
 & (-1)^n \frac{(1+n\lambda)(1+(n-1)\lambda) \dots (1+\lambda)}{\lambda^n} KZ\{f(\ln \ln t)\}, n \\
 & = 1, 2, 3, \dots \\
 & KZ\{(\ln \ln t)^n f^{(n)}(\ln \ln t)\} \\
 & = \frac{1+\lambda}{\lambda} f^{(n-1)}(1) - \frac{(1+n\lambda)(1+\lambda)}{\lambda^2} f^{(n-2)}(1) \\
 & + \frac{(1+n\lambda)(1+(n-1)\lambda)(1+\lambda)}{\lambda^3} f^{(n-3)}(1) - \dots \\
 & + (-1)^n \frac{(1+n\lambda)(1+(n-1)\lambda) \dots (1+\lambda)}{\lambda^n} KZ\{f(\ln \ln t)\}, \\
 & n = 1, 2, 3, \dots
 \end{aligned}$$

60. Battor-Al-Tememe Transform [65]

Ali H. Mohammed et al. presented a new integral transform called Battor-Al-Tememe integral transform. The Battor-Al-Tememe integral transform is convergent and it is defined for the function $f(t)$ as:

$$BA\{f(t)\} = \lambda \int_1^\infty t^{-\frac{1}{\lambda}} f(t) dt = F(\lambda), \quad \lambda > 0.$$

The Battor-Al-Tememe integral transform of some elementary functions are:

1. $BA\{t^n\} = \frac{\lambda^2}{1-(n+1)\lambda}, \quad n \in N.$
2. $BA\{\ln \ln t\} = \frac{\lambda^3}{(1-\lambda)^2}.$
3. $BA\{(\ln \ln t)^n\} = \frac{n! \lambda^{n+2}}{(1-\lambda)^{n+1}}, \quad n \in N.$
4. $BA\{\sin \sin (\alpha \ln \ln t)\} = \frac{\alpha \lambda^3}{(1-\lambda)^2 + \alpha^2 \lambda^2}.$
5. $BA\{\cos \cos (\alpha \ln \ln t)\} = \frac{\lambda^2 (1-\lambda)}{(1-\lambda)^2 + \alpha^2 \lambda^2}.$
6. $BA\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{\alpha \lambda^3}{(1-\lambda)^2 - \alpha^2 \lambda^2}.$
7. $BA\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{\lambda^2 (1-\lambda)}{(1-\lambda)^2 - \alpha^2 \lambda^2}.$

Battor-Al-Tememe Transforms of Derivatives:

Let $BA\{f(t)\} = F(\lambda)$, then

$$\text{i. } BA\{xf'(t)\} = \frac{1-\lambda}{\lambda} F(\lambda) - \lambda f(1).$$

ii. $BA\{x^2 f''(t)\} = \frac{(1-2\lambda)(1-\lambda)}{\lambda^2} F(\lambda) - (1-2\lambda)f(1) - \lambda f'(1).$
 iii. $BA\{x^n f^{(n)}(t)\} = -\lambda f^{(n-1)}(1) - (1-n\lambda)f^{(n-2)}(1) - \frac{(1-n\lambda)(1-(n-1)\lambda)}{\lambda} f^{(n-3)}(1) - \dots + \frac{(1-n\lambda)(1-(n-1)\lambda)\dots(1-\lambda)}{\lambda^n} F(\lambda), , n \in N.$

61. GF1 Integral Transform [66]

Nbila G. and Ali F. presented a new integral transform called GF1 integral transform. This transform is defined for the function $f(t)$ as:

$$GF1\{f(t)\} = s^3 \int_1^\infty e^{-\frac{t}{s}} f(t) dt = F(s), s > 0.$$

GF1 integral transform of some elementary functions are:

1. $GF1\{t\} = s^5, n \in N.$
2. $GF1\{e^{\alpha t}\} = \frac{s^3}{1+\alpha s}.$
3. $GF1\{\sin \sin(\alpha t)\} = \frac{\alpha s^5}{1+\alpha^2 s^2}.$
4. $GF1\{\cos \cos(\alpha t)\} = \frac{s^4}{1+\alpha^2 s^2}.$

GF1 Transforms of Derivatives:

Let $GF1\{f(t)\} = F(s)$, then

- i. $GF1\{f'(t)\} = \frac{1}{s} F(s) - s^3 f(0).$
- ii. $GF1\{f''(t)\} = \frac{1}{s^2} F(s) - s^2 f(0) - s^3 f'(0).$
- iii. $GF1\{f^{(n)}(t)\} = \frac{1}{s^n} F(s) - s^3 \sum_{j=0}^{n-1} \frac{1}{s^{n-1-j}} f^{(j)}(0), n = 1, 2, 3, \dots$

62. Abaoub-Shkheam Transform [67]

Asmaa O. Mubayrash and Huda M. Khalat presented an integral transform named Abaoub-Shkheam transform. This transform is defined for the function $f(t)$ as:

$$Q\{f(t)\} = \int_0^\infty f(ut) e^{-\frac{t}{s}} dt = T(u, s), s \in (-t_1, t_2).$$

Abaoub-Shkheam transform of some elementary functions are:

1. $Q\{t^n\} = n! u^n s^{n+1}, n \in N.$
2. $Q\{e^{\alpha t}\} = \frac{s}{1-\alpha us}, \alpha \in R.$
3. $Q\{\sin \sin(\alpha t)\} = \frac{\alpha us^2}{1+\alpha^2 u^2 s^2}.$
4. $Q\{\cos \cos(\alpha t)\} = \frac{s}{1+\alpha^2 u^2 s^2}.$

Abaoub-Shkheam Transform of Derivatives:

Let $Q\{f(t)\} = T(u, s)$, then

- i. $Q\{f'(t)\} = \frac{1}{us} T(u, s) - \frac{1}{u} f'(0).$

ii. $Q\{f''(t)\} = \frac{1}{(us)^2} T(u, s) - \frac{1}{u^2 s} f(0) - \frac{1}{u} f'(0).$

iii. $Q\{f^{(n)}(t)\} = \frac{1}{(us)^n} T(u, s) - \frac{1}{u} \sum_{j=0}^{n-1} \frac{1}{(us)^{n-j-1}} f^{(j)}(0), n = 1, 2, 3, \dots$

63. Ouideen Transformation [68]

Yasmin Ouideen and Ali Al-Aati presented an integral transform in 2022 named Ouideen transform. This transform is defined for the function $f(t)$ as:

$$O\{f(t)\} = \frac{1}{pq} \int_0^\infty f(t) e^{-\frac{p}{q}t} dt = T(p, q), p, q > 0.$$

Ouideen transform of some elementary functions are:

7. $O\{t^n\} = n! \frac{q^n}{p^{n+2}}, n \in N.$
8. $O\{e^{\alpha t}\} = \frac{1}{p(p-\alpha q)}, \alpha \in R.$
9. $O\{\sin \sin(\alpha t)\} = \frac{\alpha q}{p(p^2+\alpha^2 q^2)}.$
10. $O\{\cos \cos(\alpha t)\} = \frac{1}{p^2+\alpha^2 q^2}.$
11. $O\{\sinh \sinh(\alpha t)\} = \frac{\alpha q}{p(p^2-\alpha^2 q^2)}.$
12. $O\{\cosh \cosh(\alpha t)\} = \frac{1}{p^2-\alpha^2 q^2}.$

Ouideen Transform of Derivatives:

Let $O\{f(t)\} = T(p, q)$, then

- i. $O\{f'(t)\} = \frac{p}{q} T(p, q) - \frac{1}{pq} f(0).$
- ii. $O\{f''(t)\} = \frac{p^2}{q^2} T(p, q) - \frac{1}{q^2} f(0) - \frac{1}{pq} f'(0).$
- iii. $O\{f^{(n)}(t)\} = \frac{p^n}{q^n} T(p, q) - \sum_{j=0}^{n-1} \left(\frac{1}{p^{2-n-j}}\right) \left(\frac{1}{q^{n-j}}\right) f^{(j)}(0), n = 1, 2, 3, \dots$

64. Ali and Zafar Transform [69]

Ali Moazzam and Muhammad Zafar presented an integral transform in 2022 named . Ali and Zafar transform. This transform is defined for the function $f(t)$ as:

$$AZ\{f(\ln \ln t)\} = \int_1^\infty f(\ln \ln t) t^{-\left(\frac{1}{s^2}+1\right)} dt = F(s), \ln \ln t \geq 1.$$

where u is complex variable.

Ali and Zafar transform of some elementary functions are:

1. $AZ\{t^n\} = \frac{s^2}{1-ns^2}, n \in N.$
2. $AZ\{(\ln \ln t)^n\} = n! (s^2)^{n+1}.$
3. $AZ\{\sin \sin(\alpha(\ln \ln t))\} = \frac{\alpha(s^2)^2}{1+(\alpha s^2)^2}.$

4. $AZ\{\cos \cos (\alpha(\ln \ln t))\} = \frac{s^2}{1+(as^2)^2}$.
5. $AZ\{\sinh \sinh (\alpha(\ln \ln t))\} = \frac{\alpha(s^2)^2}{1-(as^2)^2}$.
6. $AZ\{\cosh \cosh (\alpha(\ln \ln t))\} = \frac{s^2}{1+(as^2)^2}$.

Ali and Zafar Transform of Derivatives:

Let $AZ\{f(\ln \ln t)\} = F(s)$, then

- i. $AZ\{f'(\ln \ln t)\} = \frac{1}{s^2}F(s) - f(0)$.
- ii. $AZ\{f''(\ln \ln t)\} = \frac{1}{(s^2)^2}F(s) - \frac{1}{s^2}f(0) - f'(0)$.
- iii. $AZ\{f^{(n)}(\ln \ln t)\} = \frac{1}{(s^2)^n}F(s) - \sum_{j=0}^{n-1} \frac{1}{(s^2)^{n-j}}f^{(j)}(0), n = 1, 2, 3, \dots$

65. Hunaiber Transform [70]

Mona Hunaiber presented an integral transform in 2022 named Hunaiber transform. This transform is defined for the function $f(t)$ as:

$$H\{f(t)\} = \mu^\beta \int_1^\infty f(t)e^{-t\mu^\sigma} dt = F(\mu^\sigma, \beta), \sigma \neq 0, \beta \in R.$$

Hunaiber transform of some elementary functions are:

1. $H\{t^n\} = \frac{n!\mu^\beta}{\mu^{(n\sigma+\sigma)}}$, $n \in N$.
2. $H\{e^{\alpha t}\} = \frac{\mu^\beta}{\mu^\sigma - \alpha}$.
3. $H\{\sin \sin (\alpha t)\} = \frac{\alpha\mu^\beta}{\mu^{2\sigma} + \alpha^2}$.
4. $H\{\cos \cos (\alpha t)\} = \frac{\mu^\sigma\mu^\beta}{\mu^{2\sigma} + \alpha^2}$.
5. $H\{\sinh \sinh (\alpha t)\} = \frac{\alpha\mu^\beta}{\mu^{2\sigma} - \alpha^2}$.
6. $H\{\cosh \cosh (\alpha t)\} = \frac{\mu^\sigma\mu^\beta}{\mu^{2\sigma} - \alpha^2}$.

Hunaiber Transform of Derivatives:

Let $H\{f(t)\} = F(\mu^\sigma, \beta)$, then

- i. $H\{f'(t)\} = \mu^\sigma F(\mu^\sigma, \beta) - \mu^\beta f(0)$.
- ii. $H\{f''(t)\} = \mu^{2\sigma} F(\mu^\sigma, \beta) - \mu^\sigma\mu^\beta f(0) - \mu^\beta f'(0)$.
- iii. $H\{f^{(n)}(t)\} = \mu^{n\sigma} F(\mu^\sigma, \beta) - \mu^\beta \sum_{j=0}^{n-1} \mu^{(n-1-j)\sigma} f^{(j)}(0), n = 1, 2, 3, \dots$

67. SEJI Integral Transform [71]

Sadiq A. Mehdi et al. presented a complex integral transform named (Sadiq- Emad- Jinan) SEJI integral transform. This transform is defined for the function $f(t)$ as:

$$T_g^c\{f(t)\} = F_g^c(s) = p(s) \int_{t=0}^\infty e^{-iq(s)t} f(t) dt .$$

SEJI transform of some elementary functions are:

1. $T_g^c\{t^n\} = (-i)^{n+1} \frac{n! p(s)}{[q(s)]^{n+1}}, n \in N$.
2. $T_g^c\{e^{\alpha t}\} = -p(s) \left[\frac{a}{a^2+(q(s))^2} + i \frac{q(s)}{a^2+(q(s))^2} \right], \alpha \in R$.
3. $T_g^c\{\sin \sin (\alpha t)\} = \frac{-a p(s)}{(q(s))^2 - a^2}$.
4. $T_g^c\{\cos \cos (\alpha t)\} = \frac{-i p(s) q(s)}{(q(s))^2 - a^2}$.
5. $T_g^c\{\sinh \sinh (\alpha t)\} = \frac{-a p(s)}{(q(s))^2 + a^2}$.
6. $T_g^c\{\cosh \cosh (\alpha t)\} = \frac{-i p(s) q(s)}{(q(s))^2 + a^2}$.

SEJI Transform of Derivatives:

Let $T_g^c\{f(t)\} = F_g^c(s)$, then

- i. $T_g^c\{f'(t)\} = iq(s)F_g^c(s) - f(0)p(s)$.
- ii. $T_g^c\{f''(t)\} = (iq(s))^2 F_g^c(s) - p(s)f'(0) - iq(s)p(s)f(0)$.
- iii. $T_g^c\{f^{(n)}(t)\} = (iq(s))^n F_g^c(s) - p(s) \left[\sum_{j=1}^{n-1} (iq(s))^{n-1-j} f^{(j)}(0) \right], n = 1, 2, 3, \dots$

iv.In (2023)

68. Two Parametric SEE Integral Transform [72]

Ali Moazzam et al. presented an integral transform named two parametric SEE integral transform. This transform is defined for the function $f(t)$ as:

$$SEE_{\alpha,\beta}\{f(t)\} = F_{\alpha,\beta}(v) = \frac{e^{-\beta v}}{(av)^n} \int_0^\infty e^{-(av)t} f(t) dt ,$$

$$\alpha, \beta \in R, n \in N.$$

Two parametric SEE transform of some elementary functions are:

1. $SEE_{\alpha,\beta}\{t^m\} = \frac{e^{-\beta v}}{(av)^{n+m+1}}, m \in N$.
2. $SEE_{\alpha,\beta}\{e^{\alpha t}\} = \frac{e^{-\beta v}}{(av)^n(av-\alpha)}, \alpha \in R$.
3. $SEE_{\alpha,\beta}\{\sin \sin (\alpha t)\} = \frac{e^{-\beta v}}{(av)^n} \left[\frac{a}{(av)^2 + a^2} \right]$.
4. $SEE_{\alpha,\beta}\{\cos \cos (\alpha t)\} = \frac{e^{-\beta v}}{(av)^n} \left[\frac{av}{(av)^2 + a^2} \right]$.
5. $SEE_{\alpha,\beta}\{\sinh \sinh (\alpha \ln \ln t)\} = \frac{e^{-\beta v}}{(av)^n} \left[\frac{a}{(av)^2 - a^2} \right]$.

$$6. \text{SEE}_{\alpha,\beta}\{\cosh \cosh (\alpha \ln \ln t)\} = \frac{e^{-\beta v}}{(av)^n} \left[\frac{av}{(av)^2 - \alpha^2} \right]$$

Two Parametric SEE Transform of Derivatives:

Let $\text{SEE}_{\alpha,\beta}\{f(t)\} = F_{\alpha,\beta}(v)$, then

v. $\text{SEE}_{\alpha,\beta}\{f'(t)\} =$

$$(av)F_{\alpha,\beta}(v) - \frac{e^{-\beta v}}{(av)^n} f(0).$$

vi. $\text{SEE}_{\alpha,\beta}\{f''(t)\} = (av)^2 F_{\alpha,\beta}(v) - \frac{e^{-\beta v}}{(av)^n} f'(0) -$

$$\frac{e^{-\beta v}}{(av)^{n-1}} f(0).$$

vii. $\text{SEE}_{\alpha,\beta}\{f^{(m)}(t)\} = ((av))^m F_{\alpha,\beta}(v) -$

$$e^{-\beta v} \left[\sum_{j=1}^{n-1} \left(\frac{1}{(av)} \right)^{n-(m-1)-j} f^{(j)}(0) \right], m =$$

1, 2, 3,

69. Generalization of Integral Transform [73]

Junaid Idrees Mustafa presented a generalization of integral transform. This transform is defined for the function $f(t)$ as:

$$\begin{aligned} GN\{f(t)\} &= GN(\vartheta) \\ &= h(\vartheta) \int_{t=0}^{\infty} e^{-\sigma(\vartheta)t} f(\Psi(\vartheta)t) dt, \end{aligned}$$

$$t \geq 0, h(\vartheta), \sigma(\vartheta) \neq 0$$

Generalization of integral transform of some elementary functions is:

7. $GN\{t^n\} = \frac{n! h(\vartheta)\psi^n(\vartheta)}{\sigma^{n+1}(\vartheta)}, n \in N.$

8. $GN\{e^t\} = \frac{h(\vartheta)}{\sigma(\vartheta) - \psi(\vartheta)}.$

9. $T_g^c\{\sin \sin (t)\} = \frac{h(\vartheta)\psi(\vartheta)}{\sigma^2(\vartheta) + \psi^2(\vartheta)}.$

Generalization of Integral Transform of Derivatives:

Let $GN\{f(t)\}$ be the generalization of integral transform then

i. $GN\{f'(t)\} = \frac{\sigma(\vartheta)}{\psi(\vartheta)} GN\{f(t)\} - \frac{h(\vartheta)}{\psi(\vartheta)} f(0).$

ii. $GN\{f''(t)\} = \left(\frac{\sigma(\vartheta)}{\psi(\vartheta)} \right)^2 GN\{f(t)\} - \frac{h(\vartheta)}{\psi(\vartheta)} f'(0) -$

$$\frac{\sigma(\vartheta)h(\vartheta)}{\psi^2(\vartheta)} f(0).$$

iii. $GN\{f^{(n)}(t)\} =$

$$\left(\frac{\sigma(\vartheta)}{\psi(\vartheta)} \right)^n F_g^c(s) - \sum_{j=1}^{n-1} \frac{\sigma^{n-1-j}(\vartheta)h(\vartheta)}{\psi^{n-k}(\vartheta)} f^{(j)}(0), n =$$

1, 2, 3,

1. Double Sumudu Transform [74]

In 2007 Jean M. Tchuente and Nyimvua S. Mbare introduced a double Sumudu transform. This Transform is defined for the function $f(x, t)$ as:

$$S_2\{f(x, t)\} = \frac{1}{uv} \int_0^{\infty} \int_0^{\infty} e^{-\left(\frac{t+x}{v}\right)} f(x, t) dt dx = F(u, v), x, t > 0.$$

Double Sumudu transform of some elementary functions are:

1. $S_2\{1\} = 1.$

2. $S_2\{x^m t^n\} = m! n! v^n v^m, m, n \in N.$

3. $S_2\{e^{\alpha x + \beta t}\} = \frac{1}{(1-\alpha u)(1-\beta v)}, \alpha, \beta \in R.$

4. $S_2\{\sin \sin (\alpha x + \beta t)\} = \frac{\beta v + \alpha u}{(1+\alpha^2 u^2)(1+\beta^2 v^2)}.$

5. $S_2\{\cos \cos (\alpha x + \beta t)\} = \frac{1-\alpha \beta uv}{(1+\alpha^2 u^2)(1+\beta^2 v^2)}.$

Double Sumudu Transform of Derivatives:

Let $S_2\{f(x, t)\} = F(u, v)$, then

i. $S_2\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \frac{1}{u} F(u, v) - \frac{1}{u} S\{f(0, t)\}.$

ii. $S_2\left\{\frac{\partial f(x,t)}{\partial t}\right\} = \frac{1}{v} F(u, v) - \frac{1}{v} S\{f(x, 0)\}.$

iii. $S_2\left\{\frac{\partial^n f(x,t)}{\partial x^n}\right\} = \frac{1}{u^n} F(u, v) - \sum_{j=1}^n \frac{1}{u^j} S\left\{\frac{\partial^{n-j} f(0,t)}{\partial x^{n-j}}\right\}.$

iv. $S_2\left\{\frac{\partial^n f(x,t)}{\partial t^n}\right\} = \frac{1}{v^n} F(u, v) - \sum_{j=1}^n \frac{1}{v^j} S\left\{\frac{\partial^{n-j} f(x,0)}{\partial y^{n-j}}\right\}.$

2. Double Laplace Transform [75]

In 2016 Lokenath Debnath introduced the double Laplace transform. This Transform is defined for the function $f(x, y)$ as:

$$L_2\{f(x, y)\} = \int_0^{\infty} \int_0^{\infty} e^{-(rx+sy)} f(x, y) dx dy = F(r, s), x, t > 0.$$

Double Laplace transform of some elementary functions are:

1. $L_2\{x^m y^n\} = \frac{m!n!}{r^{m+1} s^{n+1}}, m, n \in N.$

2. $L_2\{e^{\alpha x + \beta y}\} = \frac{1}{(r-\alpha)(s-\beta)}, \alpha, \beta \in R.$

3. $L_2\{e^{i(\alpha x + \beta y)}\} = \frac{(rs-\alpha\beta) + i(\alpha s + \beta r)}{(r^2 + \alpha^2)(s^2 + \beta^2)}, \alpha, \beta \in R.$

4. $L_2\{\sin \sin (\alpha x + \beta y)\} = \frac{\alpha s + \beta r}{(r^2 + \alpha^2)(s^2 + \beta^2)}$

5. $L_2\{\cos \cos (\alpha x + \beta y)\} = \frac{rs - \alpha\beta}{(r^2 + \alpha^2)(s^2 + \beta^2)}$

6. $L_2\{\sinh \sinh (\alpha x + \beta y)\} = \frac{\alpha s + \beta r}{(r^2 - \alpha^2)(s^2 - \beta^2)}$

7. $L_2\{\cosh \cosh (\alpha x + \beta y)\} = \frac{rs + \alpha\beta}{(r^2 - \alpha^2)(s^2 - \beta^2)}$

Double Laplace Transform of Derivatives:

Let $L_2\{f(x, y)\} = F(r, s)$, then

i. $L_2\left\{\frac{\partial f(x,y)}{\partial x}\right\} = rF(r, s) - L\{f(0, y)\}.$

ii. $L_2\left\{\frac{\partial f(x,y)}{\partial y}\right\} = sF(r, s) - L\{f(x, 0)\}.$

$$\text{iii. } L_2 \left\{ \frac{\partial^2 f(x,y)}{\partial x^2} \right\} = r^2 F(r,s) - rL\{f(0,y)\} - L\left\{ \frac{\partial f(0,y)}{\partial x} \right\}.$$

$$\text{iv. } L_2 \left\{ \frac{\partial^2 f(x,y)}{\partial y^2} \right\} = s^2 F(r,s) - sL\{f(x,0)\} - L\left\{ \frac{\partial f(x,0)}{\partial y} \right\}.$$

$$\text{v. } L_2 \left\{ \frac{\partial^2 f(x,y)}{\partial x \partial y} \right\} = rsF(r,s) - sL\{f(0,y)\} - rL\{f(x,0)\} + f(0,0).$$

3. Double Natural Transform [76]

In 2017 Adem K. and Maryam O. introduced new double transform called double natural integral transform. This transform is defined for the function $f(x, y)$ as:

$$N_+^2\{f(x, y)\} = \int_0^\infty \int_0^\infty e^{-(sx+py)} f(ux, vy) dx dy = R_+^2[(p, s), (u, v)].$$

Double natural transform of some elementary functions are:

1. $N_+^2\{1\} = \frac{1}{sp}$.
2. $N_+^2\{x^m y^n\} = \frac{u^m v^n m! n!}{s^{m+1} p^{n+1}}, m, n \in N$.
3. $N_+^2\{e^{ax+by}\} = \frac{1}{(s-au)(p-bv)}$.
4. $N_+^2\{e^{i(ax+by)}\} = \frac{(sp-\alpha\beta uv)+i(s\beta v+\alpha pu)}{(s^2+a^2 u^2)(p^2+\beta^2 v^2)}$.
5. $N_+^2\{\sin \sin(ax+by)\} = \frac{(s\beta v+\alpha pu)}{(s^2+a^2 u^2)(p^2+\beta^2 v^2)}$.
6. $N_+^2\{\cos \cos(ax+by)\} = \frac{(sp-\alpha\beta uv)}{(s^2+a^2 u^2)(p^2+\beta^2 v^2)}$.
7. $N_+^2\{\sinh \sinh(ax+by)\} = \frac{-(s\beta v+\alpha pu)}{(s^2+a^2 u^2)(p^2+\beta^2 v^2)}$.
8. $N_+^2\{\cosh \cosh(ax+by)\} = \frac{(sp+\alpha\beta uv)}{(s^2+a^2 u^2)(p^2+\beta^2 v^2)}$.

Double Natural Transform of Derivatives:

Let $N_+^2\{f(x, y)\} = R_+^2[(p, s), (u, v)]$, then

- i. $N_+^2\left\{ \frac{\partial f(x,y)}{\partial x} \right\} = \frac{s}{u} R_+^2[(p, s), (u, v)] - \frac{1}{u} N_+^2\{f(0, y)\}$.
- ii. $N_+^2\left\{ \frac{\partial f(x,y)}{\partial y} \right\} = \frac{p}{v} R_+^2[(p, s), (u, v)] - \frac{1}{v} N_+^2\{f(x, 0)\}$.
- iii. $N_+^2\left\{ \frac{\partial^2 f(x,y)}{\partial x^2} \right\} = \left(\frac{s}{u}\right)^2 R_+^2[(p, s), (u, v)] - \frac{s}{u^2} N_+^2\{f(0, y)\} - \frac{1}{u} N_+^2\left\{ \frac{\partial f(0,y)}{\partial x} \right\}$.
- iv. $N_+^2\left\{ \frac{\partial^2 f(x,y)}{\partial y^2} \right\} = \left(\frac{p}{v}\right)^2 R_+^2[(p, s), (u, v)] - \frac{p}{v^2} N_+^2\{f(x, 0)\} - \frac{1}{v} N_+^2\left\{ \frac{\partial f(x,0)}{\partial y} \right\}$.
- v. $N_+^2\left\{ \frac{\partial^2 f(x,y)}{\partial x \partial y} \right\} = R_+^2[(p, s), (u, v)] - \frac{s}{u} N_+^2\{f(x, 0)\} - \frac{p}{v} N_+^2\{f(0, y)\} - \frac{1}{uv} f(0, 0)$.

4. Double Aboodh Transform [77]

In 2017 K.S. Aboodh et al. introduced a new double transform called double Aboodh transform. This Transform is defined for the function $f(x, t)$ as:

$$A_2\{f(x, t)\} = \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-(ux+vt)} f(x, t) dx dt = K(u, v), \quad x, t > 0.$$

Double Aboodh transform of some elementary functions are:

1. $A_2\{1\} = \frac{1}{u^2 v^2}$.
2. $A_2\{x^m t^n\} = \frac{m! n!}{u^{m+2} v^{n+2}}, m, n \in N$.
3. $A_2\{e^{ax+\beta t}\} = \frac{1}{(u^2-au)(v^2-\beta v)}, \alpha, \beta \in R$.
4. $A_2\{\sin \sin(ax+\beta t)\} = \frac{\alpha\beta}{uv(u^2+\alpha^2)(v^2+\beta^2)}$.
5. $A_2\{\cos \cos(ax+\beta t)\} = \frac{1}{(u^2+\alpha^2)(v^2+\beta^2)}$.
6. $A_2\{\sinh \sinh(ax+\beta t)\} = \frac{\alpha\beta}{uv(u^2-\alpha^2)(v^2-\beta^2)}$.
7. $A_2\{\cosh \cosh(ax+\beta t)\} = \frac{u^2 v^2 (1+\alpha\beta uv)}{(u^2-\alpha^2)(v^2-\beta^2)}$.

Double Aboodh Transform of Derivatives:

Let $A_2\{f(x, t)\} = K(u, v)$, then

- i. $A_2\left\{ \frac{\partial f(x,t)}{\partial x} \right\} = uK(u, v) - \frac{1}{u} A\{f(0, t)\}$.
- ii. $A_2\left\{ \frac{\partial f(x,t)}{\partial t} \right\} = vK(u, v) - \frac{1}{v} A\{f(x, 0)\}$.
- iii. $A_2\left\{ \frac{\partial^2 f(x,t)}{\partial x^2} \right\} = u^2 K(u, v) - A\{f(0, t)\} - \frac{1}{u} \frac{\partial A\{f(0,t)\}}{\partial x}$.
- iv. $A_2\left\{ \frac{\partial^2 f(x,t)}{\partial t^2} \right\} = v^2 K(u, v) - A\{f(x, 0)\} - \frac{1}{v} \frac{\partial A\{f(x,0)\}}{\partial t}$.
- v. $A_2\left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = uvK(u, v) - \frac{u}{v} A\{f(x, 0)\} - \frac{v}{u} A\{f(0, t)\} + \frac{1}{uv} f(0, 0)$.

5. Double Kamal Transform [78]

In 2019 Sandip M. Sonawane and S. B. KIWNE introduced the double Kamal transform. This Transform is defined for the function $f(x, y)$ as:

$$K_2\{f(x, y)\} = pq \int_0^\infty \int_0^\infty e^{-(x+y)} f(px, qy) dx dy = F(p, q), \quad p, q \in R.$$

Double Laplace transform of some elementary functions are:

1. $K_2\{x^m y^n\} = m! n! p^{m+1} q^{n+1}, m, n \in N$.
2. $K_2\{e^{ax+\beta y}\} = \frac{pq}{(1-ap)(1-\beta q)}, \alpha, \beta \in R$.
3. $K_2\{\sin \sin(ax+\beta y)\} = \frac{pq(\alpha p+\beta q)}{(1+\alpha^2 p^2)(1+\beta^2 q^2)}$.

$$4. K_2\{\cos \cos (\alpha x + \beta y)\} = \frac{pq(1-\alpha\beta pq)}{(1+\alpha^2 p^2)(1+\beta^2 q^2)}.$$

$$5. K_2\{\sinh \sinh (\alpha x + \beta y)\} = \frac{pq(\alpha p+\beta q)}{(1-\alpha^2 p^2)(1-\beta^2 q^2)}.$$

$$6. K_2\{\cosh \cosh (\alpha x + \beta y)\} = \frac{pq(1-\alpha\beta pq)}{(1-\alpha^2 p^2)(1-\beta^2 q^2)}.$$

Double Kamal Transform of Derivatives:

Let $K_2\{f(x, y)\} = F(p, q)$, then

$$i. K_2\left\{\frac{\partial f(x,y)}{\partial x}\right\} = \frac{1}{p}F(p, q) - K\{f(0, y)\}.$$

$$ii. K_2\left\{\frac{\partial f(x,y)}{\partial y}\right\} = \frac{1}{q}F(p, q) - K\{f(x, 0)\}.$$

$$iii. K_2\left\{\frac{\partial^2 f(x,y)}{\partial x^2}\right\} = \frac{1}{p^2}F(p, q) - \frac{1}{p}K\{f(0, y)\} - K\left\{\frac{\partial f(0,y)}{\partial x}\right\}.$$

$$iv. K_2\left\{\frac{\partial^2 f(x,y)}{\partial y^2}\right\} = \frac{1}{q^2}F(p, q) - \frac{1}{q}K\{f(x, 0)\} - K\left\{\frac{\partial f(x,0)}{\partial y}\right\}.$$

$$v. K_2\left\{\frac{\partial^2 f(x,y)}{\partial x\partial y}\right\} = \frac{1}{pq}F(p, q) - \frac{1}{q}K\{f(0, y)\} - \frac{1}{p}K\{f(x, 0)\} - f(0, 0).$$

6. Double Elzaki Transform [79]

In 2020 Moh A. Hassan and Tarig M. Elzaki introduced a new double transform called double Elzaki transform. This Transform is defined for the function $f(x, t)$ as:

$$E_2\{f(x, t)\} = uv \int_0^\infty \int_0^\infty e^{-\left(\frac{x}{u} + \frac{t}{v}\right)} f(x, t) dx dt = T(u, v), \quad x, t > 0.$$

Double Elzaki transform of some elementary functions are:

$$1. E_2\{1\} = u^2 v^2.$$

$$2. E_2\{x^m t^n\} = m! n! u^{m+2} v^{n+2}, \quad m, n \in N.$$

$$3. E_2\{e^{\alpha x + \beta t}\} = \frac{u^2 v^2}{(1-\alpha u)(1-\beta v)}, \quad \alpha, \beta \in R.$$

$$4. E_2\{\sin \sin (\alpha x + \beta t)\} = \frac{u^2 v^2 (\alpha u + \beta v)}{(1+\alpha^2 u^2)(1+\beta^2 v^2)}.$$

$$5. E_2\{\cos \cos (\alpha x + \beta t)\} = \frac{u^2 v^2 (1-\alpha\beta uv)}{(1+\alpha^2 u^2)(1+\beta^2 v^2)}.$$

$$6. E_2\{\sinh \sinh (\alpha x + \beta y)\} = \frac{-u^2 v^2 (\alpha u + \beta v)}{(1-\alpha^2 u^2)(1-\beta^2 v^2)}.$$

$$7. E_2\{\cosh \cosh (\alpha x + \beta y)\} = \frac{u^2 v^2 (1+\alpha\beta uv)}{(1-\alpha^2 u^2)(1-\beta^2 v^2)}.$$

Double Elzaki Transform of Derivatives:

Let $E_2\{f(x, t)\} = T(u, v)$, then

$$i. E_2\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \frac{1}{u}T(u, v) - uE\{f(0, t)\}.$$

$$ii. E_2\left\{\frac{\partial f(x,t)}{\partial t}\right\} = \frac{1}{v}T(u, v) - vE\{f(x, 0)\}.$$

$$iii. E_2\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\} = \frac{1}{u^2}T(u, v) - E\{f(0, t)\} - uE\left\{\frac{\partial f(0,t)}{\partial x}\right\}.$$

$$iv. E_2\left\{\frac{\partial^2 f(x,t)}{\partial t^2}\right\} = \frac{1}{v^2}T(u, v) - E\{f(x, 0)\} - vE\left\{\frac{\partial f(x,0)}{\partial y}\right\}.$$

$$v. E_2\left\{\frac{\partial^2 f(x,t)}{\partial x\partial t}\right\} = \frac{1}{uv}T(u, v) - \frac{u}{v}E\{f(0, t)\} - \frac{v}{u}E\{f(x, 0)\} + uvf(0, 0).$$

7. Double Shehu Transform [80]

In 2020 Suliman A. and Emine M. generalized the concept of Shehu transform to a new double Shehu integral transform. This Transform is defined for the function $f(x, t)$ as:

$$H_{xt}^2\{f(x, t)\} = \int_0^\infty \int_0^\infty e^{-\left(\frac{p}{u}x + \frac{q}{v}t\right)} f(x, t) dx dt = G[(p, q), (u, v)].$$

Double Shehu transform of some elementary functions are:

$$1. H_{xt}^2\{1\} = \frac{uv}{pq}.$$

$$2. H_{xt}^2\{x^m t^n\} = m! n! \left(\frac{u}{p}\right)^{m+1} \left(\frac{v}{q}\right)^{n+1}, \quad m, n \in N.$$

$$3. H_{xt}^2\{e^{\alpha x + \beta t}\} = \frac{uv}{(p-\alpha u)(q-\beta v)}, \quad \alpha, \beta \in R.$$

$$4. H_{xt}^2\{e^{i(\alpha x + \beta t)}\} = \frac{uv(pq + \alpha\beta uv) + iuv(\alpha qu + \beta pv)}{(p^2 + \alpha^2 u^2)(q^2 + \beta^2 v^2)}, \quad \alpha, \beta \in R.$$

$$5. H_{xt}^2\{\sin \sin (\alpha x + \beta t)\} = \frac{uv(\beta pv + \alpha qu)}{(p^2 + \alpha^2 u^2)}.$$

$$6. H_{xt}^2\{\cos \cos (\alpha x + \beta t)\} = \frac{uv(pq + \alpha\beta uv)}{(q^2 + \beta^2 v^2)}.$$

$$7. H_{xt}^2\{\sinh \sinh (\alpha x + \beta t)\} = \frac{1}{2} \left[\frac{uv}{(p-\alpha u)(q-\beta v)} - \frac{uv}{(p+\alpha u)(q+\beta v)} \right].$$

$$8. H_{xt}^2\{\cosh \cosh (\alpha x + \beta t)\} = \frac{1}{2} \left[\frac{uv}{(p-\alpha u)(q-\beta v)} + \frac{uv}{(p+\alpha u)(q+\beta v)} \right].$$

Double Shehu Transform of Derivatives:

Let $H_{xt}^2\{f(x, t)\} = G[(p, q), (u, v)]$, then

$$i. H_{xt}^2\left\{\frac{\partial f(x,t)}{\partial t}\right\} = \frac{q}{v}G[(p, q), (u, v)] - H\{f(x, 0)\}.$$

$$ii. H_{xt}^2\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \frac{p}{u}G[(p, q), (u, v)] - H\{f(0, t)\}.$$

$$iii. H_{xt}^2\left\{\frac{\partial^2 f(x,t)}{\partial t^2}\right\} = \left(\frac{q}{v}\right)^n G[(p, q), (u, v)] - \sum_{j=0}^{n-1} \left(\frac{q}{v}\right)^{n-1-j} H\left\{\frac{\partial^j f(x,0)}{\partial t^j}\right\}.$$

$$\begin{aligned} \text{iv. } H_{xt}^2 \left\{ \frac{\partial^n f(x,t)}{\partial x^n} \right\} &= \left(\frac{p}{u} \right)^n G[(p, q), (u, v)] - \\ &\sum_{j=0}^{n-1} \left(\frac{p}{u} \right)^{n-1-j} H \left\{ \frac{\partial^j f(0,t)}{\partial x^j} \right\}. \\ \text{v. } H_{xt}^2 \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} &= \\ \frac{pq}{uv} G[(p, q), (u, v)] - \frac{p}{u} H\{f(x, 0)\} - \frac{q}{v} H\{f(0, t)\} - \\ &f(0, 0). \end{aligned}$$

8. Double Mahgoub Transform [81]

In 2020 D. P. Patil introduced a new double transform called double Mahgoub transform. This Transform is defined for the function $f(x, t)$ as:

$$M_2\{f(x, t)\} = uv \int_0^\infty \int_0^\infty e^{-(ux+vt)} f(x, t) dx dt = H(u, v), \quad x, t \geq 0.$$

Double Mahgoub Transform of Derivatives:

Let $M_2\{f(x, t)\} = H(u, v)$, then

$$\begin{aligned} \text{i. } M_2 \left\{ \frac{\partial f(x,t)}{\partial x} \right\} &= uH(u, v) - uM\{f(0, t)\}. \\ \text{ii. } M_2 \left\{ \frac{\partial f(x,t)}{\partial t} \right\} &= vH(u, v) - vM\{f(x, 0)\}. \\ \text{iii. } M_2 \left\{ \frac{\partial^2 f(x,t)}{\partial x^2} \right\} &= u^2H(u, v) - u^2M\{f(0, t)\} - \\ &uM \left\{ \frac{\partial f(0,t)}{\partial x} \right\}. \\ \text{iv. } M_2 \left\{ \frac{\partial^2 f(x,t)}{\partial t^2} \right\} &= v^2H(u, v) - v^2M\{f(x, 0)\} - \\ &vM \left\{ \frac{\partial f(x,0)}{\partial t} \right\}. \\ \text{v. } M_2 \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} &= uvH(u, v) - uvM\{f(0, t)\} - \\ &uvM\{f(x, 0)\} + uvf(0, 0). \end{aligned}$$

9. The New General Double Integral Transform [82]

In 2021 Hossein Jafari et al. introduced new double transform called the new general double integral transform. This transform is defined for the function $f(x, y)$ as:

$$\begin{aligned} T_2\{f(x, y)\} &= \\ p(s)q(r) \int_0^\infty \int_0^\infty e^{-(\varphi(s)x+\psi(r)y)} f(x, y) dx dy &= \\ F_D(s, r), \quad p(s), q(r) \neq 0. \end{aligned}$$

The new general double integral transform of some elementary functions are:

$$\begin{aligned} 1. T_2\{1\} &= \frac{p(s)q(r)}{\varphi(s)\psi(r)}, \quad m, n \geq 0. \\ 2. T_2\{x^m y^n\} &= \frac{n!m!p(s)q(r)}{[\varphi(s)]^{m+1}[\psi(r)]^{n+1}}, \quad m, n \geq 0. \\ 3. T_2\{e^{\alpha x + \beta y}\} &= \frac{p(s)q(r)}{(\varphi(s)-\alpha)(\psi(r)-\beta)}, \quad \alpha, \beta \in R. \end{aligned}$$

$$\begin{aligned} 4. T_2\{\sin \sin (\alpha x + \beta y)\} &= \\ p(s)q(r) \frac{[\alpha\psi(r)+\beta\varphi(s)]}{([\varphi(s)]^2+\alpha^2)([\psi(r)]^2+\beta^2)}. \\ 5. T_2\{\cos \cos (\alpha x + \beta y)\} &= \\ p(s)q(r) \frac{[\varphi(s)\psi(r)-\alpha\beta]}{([\varphi(s)]^2+\alpha^2)([\psi(r)]^2+\beta^2)}. \\ 6. T_2\{\sinh \sinh (\alpha x + \beta y)\} &= \\ p(s)q(r) \frac{[\alpha\psi(r)+\beta\varphi(s)]}{([\varphi(s)]^2-\alpha^2)([\psi(r)]^2-\beta^2)}. \\ 7. T_2\{\cosh \cosh (\alpha x + \beta y)\} &= \\ p(s)q(r) \frac{[\varphi(s)\psi(r)+\alpha\beta]}{([\varphi(s)]^2-\alpha^2)([\psi(r)]^2-\beta^2)}. \end{aligned}$$

The New General Double Integral Transform of Derivatives:

Let $T_2\{f(x, y)\}$ be the double general integral transform for $f(x, y)$, then

$$\begin{aligned} \text{i. } T_2 \left\{ \frac{\partial f(x,y)}{\partial x} \right\} &= \varphi(s)F_D(s, r) - p(s)F_G(0, r). \\ \text{ii. } T_2 \left\{ \frac{\partial f(x,y)}{\partial y} \right\} &= \psi(r)F_D(s, r) - q(r)F_G(s, 0). \\ \text{iii. } T_2 \left\{ \frac{\partial^n f(x,y)}{\partial x^n} \right\} &= \\ &[\varphi(s)]^n F_D(s, r) - \\ &p(s) \sum_{j=0}^{n-1} [\varphi(s)]^{n-j-1} \frac{\partial^j F_G(0, r)}{\partial x^j}. \\ \text{iv. } T_2 \left\{ \frac{\partial^2 f(x,y)}{\partial y^2} \right\} &= \\ &[\psi(r)]^n F_D(s, r) - \\ &q(r) \sum_{j=0}^{n-1} [\psi(r)]^{n-j-1} \frac{\partial^j F_G(s, 0)}{\partial y^j}. \end{aligned}$$

10. Double ARA Transform [83]

In 2022 Rania Saadeh introduced a new double integral transform named double ARA transform. This Transform is defined for the function $f(x, t)$ as:

$$\begin{aligned} G_x G_t \{f(x, t)\} &= \\ vs \int_0^\infty \int_0^\infty e^{-(vx+st)} f(x, t) dx dt = Q(v, s), \quad v, s > \\ &0. \end{aligned}$$

Double ARA transform of some elementary functions are:

$$\begin{aligned} 1. G_x G_t \{x^n t^m\} &= \frac{n!m!}{v^n s^m}, \quad n \geq 0. \\ 2. G_x G_t \{e^{\alpha x + \beta t}\} &= \frac{vs}{(v-\alpha)(s-\beta)}, \quad m, n \in N. \\ 3. G_x G_t \{e^{i(\alpha x + \beta t)}\} &= \frac{vs}{(v-i\alpha)(s-i\beta)}, \quad \alpha, \beta \in R. \\ 4. G_x G_t \{\sin \sin (\alpha x + \beta t)\} &= \frac{vs(v\beta + s\alpha)}{(v^2 + \alpha^2)(s^2 + \beta^2)} \\ 5. G_x G_t \{\cos \cos (\alpha x + \beta t)\} &= \frac{vs(sv - \alpha\beta)}{(v^2 + \alpha^2)(s^2 + \beta^2)}. \\ 6. G_x G_t \{\sinh \sinh (\alpha x + \beta t)\} &= \frac{vs(v\beta + s\alpha)}{(v^2 - \alpha^2)(s^2 - \beta^2)}. \end{aligned}$$

$$7. G_x G_t \{ \cosh \cosh (\alpha x + \beta t) \} = \frac{vs(sv+\alpha\beta)}{(v^2-\alpha^2)(s^2-\beta^2)}.$$

Double ARA Transform of Derivatives:

Let $G_x G_t \{ f(x, t) \} = Q(v, s)$, then

- i. $G_x G_t \left\{ \frac{\partial f(x,t)}{\partial t} \right\} = sQ(v, s) - sG \{ f(x, 0) \}.$
- ii. $G_x G_t \left\{ \frac{\partial f(x,t)}{\partial x} \right\} = sQ(v, s) - vG \{ f(0, t) \}.$
- iii. $G_x G_t \left\{ \frac{\partial^2 f(x,t)}{\partial t^2} \right\} = s^2 Q(v, s) - s^2 G \{ f(x, 0) \} - sG \left\{ \frac{\partial f(x,0)}{\partial t} \right\}.$
- iv. $G_x G_t \left\{ \frac{\partial^2 f(x,t)}{\partial x^2} \right\} = v^2 Q(v, s) - v^2 G \{ f(0, t) \} - vG \left\{ \frac{\partial f(0,t)}{\partial x} \right\}.$
- v. $G_x G_t \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = vsQ(v, s) - vsG \{ f(x, 0) \} - vsG \{ f(0, t) \} - vsf(0, 0).$

11. Double Emad-Falih Transform [84]

In 2022 Emad A. Kuffi and Saed M. Turq introduced a new double integral transform named double Emad-Falih integral transform. This Transform is defined for the function $f(x, t)$ as:

$$D_{EF} \{ f(x, t) \} = \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-(u^2x+v^2t)} f(x, t) dx dt = T(u, v), \quad v, s > 0.$$

Double Emad-Falih integral transform of some elementary functions are:

- 1. $D_{EF} \{ x^n t^m \} = \frac{n!m!}{u^{2n+3}v^{2m+3}}, \quad m, n \in N.$
- 2. $D_{EF} \{ e^{\alpha x + \beta t} \} = \frac{1}{uv(u^2-\alpha)(v^2-\beta)}, \quad \alpha, \beta \in R.$
- 3. $D_{EF} \{ e^{i(\alpha x + \beta t)} \} = \frac{1}{uv(u^2-i\alpha)(v^2-i\beta)}, \quad \alpha, \beta \in R.$
- 4. $D_{EF} \{ \sin \sin (\alpha x + \beta t) \} = \frac{\beta u^2 + \alpha v^2}{uv(u^4 + \alpha^2)(v^4 + \beta^2)}$
- 5. $D_{EF} \{ \cos \cos (\alpha x + \beta t) \} = \frac{u^2 v^2 - \alpha \beta}{uv(u^4 + \alpha^2)(v^4 + \beta^2)}.$
- 6. $D_{EF} \{ \sinh \sinh (\alpha x + \beta t) \} = \frac{\beta u^2 + \alpha v^2}{uv(u^4 - \alpha^2)(v^4 - \beta^2)}.$
- 7. $D_{EF} \{ \cosh \cosh (\alpha x + \beta t) \} = \frac{u^2 v^2 + \alpha \beta}{uv(u^4 - \alpha^2)(v^4 - \beta^2)}.$

Double Emad-Falih Integral Transform of Derivatives:

Let $D_{EF} \{ f(x, t) \} = T(u, v)$, then

- i. $D_{EF} \left\{ \frac{\partial f(x,t)}{\partial t} \right\} = v^2 T(u, v) - \frac{1}{v} T(u, 0).$
- ii. $D_{EF} \left\{ \frac{\partial f(x,t)}{\partial x} \right\} = u^2 T(u, v) - \frac{1}{u} T(0, v).$
- iii. $D_{EF} \left\{ \frac{\partial^n f(x,t)}{\partial t^n} \right\} = v^{2n} T(u, v) - \sum_{j=1}^n v^{2j-3} \frac{\partial^{n-j} T(u, 0)}{\partial x^{n-j}}.$

- iv. $D_{EF} \left\{ \frac{\partial^n f(x,t)}{\partial x^n} \right\} = u^{2n} T(u, v) - \sum_{j=1}^n u^{2j-3} \frac{\partial^{n-j} T(0, v)}{\partial x^{n-j}}.$
- v. $D_{EF} \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = u^2 v^2 T(u, v) - \frac{u^2}{v} T(0, v) - \frac{1}{u} \frac{\partial T(0, v)}{\partial t}.$
- vi. $D_{EF} \left\{ \frac{\partial^2 f(x,t)}{\partial t \partial x} \right\} = u^2 v^2 T(u, v) - \frac{v^2}{u} T(0, v) - \frac{1}{v} \frac{\partial T(u, 0)}{\partial x}.$

12. Double General Integral Transform [85]

In 2022 Dinkar P. Patil et al. introduced a new double integral transform named double general integral transform. This Transform is defined for the function $f(x, y)$ as:

$$T_2 \{ f(x, y) \} = p_1(s) p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x + q_2(s)t)} f(x, y) dx dy, \quad p_1(s), p_2(s) \neq 0, q_1(s), q_2(s) > 0.$$

Double general integral transform of some elementary functions are:

- 1. $T_2 \{ x^n y^m \} = \frac{n!m! p_1(s) p_2(s)}{[q_1(s)]^{m+1} [q_2(s)]^{n+1}}, \quad m, n \in N.$
- 2. $T_2 \{ e^{\alpha x + \beta t} \} = \frac{p_1(s) p_2(s)}{(q_1(s) - \alpha)(q_2(s) - \beta)}, \quad \alpha, \beta \in R.$
- 3. $T_2 \{ e^{i(\alpha x + \beta t)} \} = \frac{p_1(s) p_2(s)}{(q_1(s) - i\alpha)(q_2(s) - i\beta)}, \quad \alpha, \beta \in R.$
- 4. $T_2 \{ \sin \sin (\alpha x + \beta t) \} = \frac{1}{2} \left[\frac{p_1(s) p_2(s)}{(q_1(s) - i\alpha)(q_2(s) - i\beta)} - \frac{p_1(s) p_2(s)}{(q_1(s) + i\alpha)(q_2(s) + i\beta)} \right].$
- 5. $T_2 \{ \cos \cos (\alpha x + \beta t) \} = \frac{1}{2} \left[\frac{p_1(s) p_2(s)}{(q_1(s) - i\alpha)(q_2(s) - i\beta)} + \frac{p_1(s) p_2(s)}{(q_1(s) + i\alpha)(q_2(s) + i\beta)} \right].$
- 6. $T_2 \{ \sinh \sinh (\alpha x + \beta t) \} = \frac{1}{2} \left[\frac{p_1(s) p_2(s)}{(q_1(s) - \alpha)(q_2(s) - \beta)} - \frac{p_1(s) p_2(s)}{(q_1(s) + \alpha)(q_2(s) + \beta)} \right].$
- 7. $T_2 \{ \cosh \cosh (\alpha x + \beta t) \} = \frac{1}{2} \left[\frac{p_1(s) p_2(s)}{(q_1(s) - \alpha)(q_2(s) - \beta)} + \frac{p_1(s) p_2(s)}{(q_1(s) + \alpha)(q_2(s) + \beta)} \right].$

Double General Integral Transform of Derivatives:

Let $T_2 \{ f(x, y) \}$ be the double general integral transform for $f(x, y)$, then

- i. $T_2 \left\{ \frac{\partial f(x,y)}{\partial x} \right\} = q_1(s) T_2 \{ f(x, y) \} - p_1(s) T \{ f(0, y) \}.$
- ii. $T_2 \left\{ \frac{\partial f(x,y)}{\partial y} \right\} = q_2(s) T_2 \{ f(x, y) \} - p_2(s) T \{ f(x, 0) \}.$

$$\text{iii. } T_2 \left\{ \frac{\partial^2 f(x,y)}{\partial x^2} \right\} = [q_1(s)]^2 T_2 \{ f(x,y) \} - p_1(s) \left[q_1(s) T \{ f(0,y) \} + T \left\{ \frac{\partial f(0,y)}{\partial x} \right\} \right].$$

$$\text{iv. } T_2 \left\{ \frac{\partial^2 f(x,y)}{\partial y^2} \right\} = [q_2(s)]^2 T_2 \{ f(x,y) \} - p_2(s) \left[q_2(s) T \{ f(x,0) \} + T \left\{ \frac{\partial f(x,0)}{\partial y} \right\} \right].$$

13. Generalized Double Rangaig Integral Transform [86]

In 2022 M. S. Derle et al. developed a generalized double Rangaig integral transform. This Transform is defined for the function $f(x, t)$ as:

$$\eta_{2g} \{ f(x, t) \} = \frac{1}{\mu_1^{n_1} \mu_2^{n_2}} \int_{-\infty}^0 \int_{-\infty}^0 e^{p(\mu_1)x + q(\mu_2)t} f(x, t) dx dt =$$

$$\Lambda_D(\mu_1, \mu_2), \frac{1}{\lambda_1} \leq \mu_1 \leq \frac{1}{\lambda_2}, \frac{1}{\psi_1} \leq \mu_2 \leq \frac{1}{\psi_2}, \mu_1, \mu_2, \psi_1, \psi_2 \in R,$$

$$n_1, n_2 \in Z.$$

Generalized double Rangaig integral transform of some elementary functions are:

$$1. \eta_{2g} \{ x^n t^m \} = \frac{1}{\mu_1^{n_1} \mu_2^{n_2}} \left(\frac{(-1)^{n+m} n! m!}{[p(\mu_1)]^{n+1} [q(\mu_2)]^{m+1}} \right), m, n \in N.$$

$$2. \eta_{2g} \{ e^{\alpha x + \beta t} \} = \frac{1}{\mu_1^{n_1} \mu_2^{n_2}} \left(\frac{1}{(p(\mu_1) + \alpha)(q(\mu_2) + \beta)} \right), \alpha, \beta \in R.$$

$$3. \eta_{2g} \{ \cos \cos(\alpha x + \beta t) \} = \frac{1}{\mu_1^{n_1} \mu_2^{n_2}} \left[\frac{-\alpha q(\mu_2) - \beta p(\mu_1)}{([p(\mu_1)]^2 + \alpha^2)([q(\mu_2)]^2 + \beta^2)} \right].$$

$$4. \eta_{2g} \{ \cosh \cosh(\alpha x + \beta t) \} = \frac{1}{\mu_1^{n_1} \mu_2^{n_2}} \left[\frac{p(\mu_1)q(\mu_2) + \alpha\beta}{([p(\mu_1)]^2 - \alpha^2)([q(\mu_2)]^2 - \beta^2)} \right].$$

Generalized Double Rangaig Integral Transform for Partial Derivatives:

Let $\eta_{2g} \{ f(x, t) \} = \Lambda_D(\mu_1, \mu_2)$, then

$$\text{i. } \eta_{2g} \left\{ \frac{\partial f(x,t)}{\partial t} \right\} = \frac{1}{\mu_2^{n_2}} \Lambda_g(\mu_1, 0) - q(\mu_2) \Lambda_D(\mu_1, \mu_2).$$

$$\text{ii. } \eta_{2g} \left\{ \frac{\partial f(x,t)}{\partial x} \right\} = \frac{1}{\mu_1^{n_1}} \Lambda_g(0, \mu_2) - p(\mu_1) \Lambda_D(\mu_1, \mu_2).$$

$$\text{iii. } \eta_{2g} \left\{ \frac{\partial^n f(x,t)}{\partial t^n} \right\} = (-1)^n (q(\mu_2))^n \Lambda_D(\mu_1, \mu_2) - \frac{1}{\mu_2^{n_2}} \sum_{j=0}^{n-1} (-1)^j (q(\mu_2))^j \frac{\partial^{n-1-j} \Lambda_g(\mu_1, 0)}{\partial t^{n-1-j}}.$$

$$\text{iv. } \eta_{2g} \left\{ \frac{\partial^n f(x,t)}{\partial x^n} \right\} = (-1)^n [p(\mu_1)]^n \Lambda_D(\mu_1, \mu_2) - \frac{1}{\mu_1^{n_1}} \sum_{j=0}^{n-1} (-1)^j [p(\mu_1)]^j \frac{\partial^{n-1-j} \Lambda_g(0, \mu_2)}{\partial x^{n-1-j}}.$$

14. Double Kushare Transform [87]

In 2022 Dinkar P. Patil et al. introduced a new double integral transform named double Kushare integral transform. This Transform is defined for the function $f(x, y)$ as:

$$K_2 \{ f(x, y) \} = v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\delta x + v_2^\delta y)} f(x, y) dx dy, x, y > 0.$$

Double Kushare integral transform of some elementary functions are:

$$1. K_2 \{ x^n y^m \} = \frac{n! m!}{v_1^{\delta((n+1)-1)} v_2^{\delta((m+1)-1)}}, m, n \in N.$$

$$2. K_2 \{ e^{\alpha x + \beta y} \} = \frac{v_1 v_2}{(v_1^\delta - \alpha)(v_2^\delta - \beta)}, \alpha, \beta \in R.$$

$$3. K_2 \{ e^{i(\alpha x + \beta y)} \} = \frac{v_1 v_2}{(v_1^\delta - i\alpha)(v_2^\delta - i\beta)}, \alpha, \beta \in R.$$

$$4. K_2 \{ \sin \sin(\alpha x + \beta y) \} = \frac{1}{2i} \left[\frac{v_1 v_2}{(v_1^\delta - i\alpha)(v_2^\delta - i\beta)} - \frac{v_1 v_2}{(v_1^\delta + i\alpha)(v_2^\delta + i\beta)} \right].$$

$$5. K_2 \{ \cos \cos(\alpha x + \beta y) \} = \frac{1}{2} \left[\frac{v_1 v_2}{(v_1^\delta - i\alpha)(v_2^\delta - i\beta)} + \frac{v_1 v_2}{(v_1^\delta + i\alpha)(v_2^\delta + i\beta)} \right].$$

$$6. K_2 \{ \sinh \sinh(\alpha x + \beta y) \} = \frac{1}{2} \left[\frac{v_1 v_2}{(v_1^\delta - \alpha)(v_2^\delta - \beta)} - \frac{v_1 v_2}{(v_1^\delta + \alpha)(v_2^\delta + \beta)} \right].$$

$$7. K_2 \{ \cosh \cosh(\alpha x + \beta y) \} = \frac{1}{2} \left[\frac{v_1 v_2}{(v_1^\delta - \alpha)(v_2^\delta - \beta)} + \frac{v_1 v_2}{(v_1^\delta + \alpha)(v_2^\delta + \beta)} \right].$$

Double Kushare Integral Transform of Derivatives:

Let $K_2 \{ f(x, y) \}$ be the double Kushare integral transform for $f(x, y)$, then

$$\text{i. } K_2 \left\{ \frac{\partial f(x,y)}{\partial x} \right\} = v_1^\delta K_2 \{ f(x, y) \} - v_1 K \{ f(0, y) \}.$$

$$\text{ii. } K_2 \left\{ \frac{\partial f(x,y)}{\partial y} \right\} = v_2^\delta K_2 \{ f(x, y) \} - v_2 K \{ f(x, 0) \}.$$

15. Double Laplace-ARA Transform [88]

In 2022 Abdelilah K. Sedeeg et al. introduced a new double integral transform named double Laplace-ARA transform. This Transform is defined for the function $f(x, t)$ as:

$$L_x G_t \{ f(x, t) \} = s \int_0^\infty \int_0^\infty e^{-(vx+st)} f(x, t) dx dt = Q(v, s), v, s > 0.$$

Double Laplace-ARA transform of some elementary functions are:

1. $L_x G_t \{ x^n t^m \} = \frac{n!m!}{v^{n+1}s^m}, m, n \in N.$
2. $L_x G_t \{ e^{\alpha x + \beta t} \} = \frac{s}{(v-\alpha)(s-\beta)}, \alpha, \beta \in R.$
3. $L_x G_t \{ \sin \sin (\alpha x + \beta t) \} = \frac{s(v\beta + s\alpha)}{(v^2 + \alpha^2)(s^2 + \beta^2)}$
4. $L_x G_t \{ \cos \cos (\alpha x + \beta t) \} = \frac{s(sv - \alpha\beta)}{(v^2 + \alpha^2)(s^2 + \beta^2)}$
5. $L_x G_t \{ \sinh \sinh (\alpha x + \beta t) \} = \frac{s(v\beta + s\alpha)}{(v^2 - \alpha^2)(s^2 - \beta^2)}$
6. $L_x G_t \{ \cosh \cosh (\alpha x + \beta t) \} = \frac{s(sv + \alpha\beta)}{(v^2 - \alpha^2)(s^2 - \beta^2)}$

Double Laplace-ARA Transform of Derivatives:

Let $L_x G_t \{ f(x, t) \} = Q(v, s)$, then

- i. $L_x G_t \left\{ \frac{\partial f(x, t)}{\partial t} \right\} = sQ(v, s) - sL\{ f(x, 0) \}.$
- ii. $L_x G_t \left\{ \frac{\partial f(x, t)}{\partial x} \right\} = sQ(v, s) - G\{ f(0, t) \}.$
- iii. $L_x G_t \left\{ \frac{\partial^2 f(x, t)}{\partial t^2} \right\} = s^2 Q(v, s) - s^2 L\{ f(x, 0) \} - sL \left\{ \frac{\partial f(x, 0)}{\partial t} \right\}.$
- iv. $L_x G_t \left\{ \frac{\partial^2 f(x, t)}{\partial x^2} \right\} = v^2 Q(v, s) - vG\{ f(0, t) \} - G \left\{ \frac{\partial f(0, t)}{\partial x} \right\}.$
- v. $L_x G_t \left\{ \frac{\partial^2 f(x, t)}{\partial x \partial t} \right\} = vsQ(v, s) - vsL\{ f(x, 0) \} - sG\{ f(0, t) \} - sf(0, 0).$

16. Double ARA-Sumudu Transform [89]

In 2022 Rania Saadeh et al. introduced a new double integral transform named double ARA-Sumudu transform. This Transform is defined for the function $f(x, y)$ as:

$$G_x S_y \{ f(x, y) \} = \frac{s}{u} \int_0^\infty \int_0^\infty e^{-(sx + \frac{y}{u})} f(x, y) dx dy = G(s, u), s, u > 0.$$

Double ARA-Sumudu transform of some elementary functions are:

1. $G_x S_y \{ x^n y^m \} = \frac{n!u^m m!}{s^n}, m, n \in N.$
2. $G_x S_y \{ e^{\alpha x + \beta y} \} = \frac{s}{(s-\alpha)(1-\beta u)}, \alpha, \beta \in R.$
3. $G_x S_y \{ e^{i(\alpha x + \beta y)} \} = \frac{is}{(s-i\alpha)(\beta u + i)}, \alpha, \beta \in R.$
4. $G_x S_y \{ \sin \sin (\alpha x + \beta y) \} = \frac{s(\alpha + \beta su)}{(\alpha^2 + s^2)(\beta^2 u^2 + 1)}$
5. $G_x S_y \{ \cos \cos (\alpha x + \beta y) \} = \frac{s(s - \alpha\beta u)}{(\alpha^2 + s^2)(\beta^2 u^2 + 1)}$
6. $G_x S_y \{ \sinh \sinh (\alpha x + \beta y) \} = \frac{s(\alpha + \beta su)}{(\alpha^2 - s^2)(\beta^2 u^2 - 1)}$
7. $G_x S_y \{ \cosh \cosh (\alpha x + \beta y) \} = \frac{s(s + \alpha\beta u)}{(\alpha^2 - s^2)(\beta^2 u^2 - 1)}$

Double ARA-Sumudu Transform of Derivatives:

Let $G_x S_y \{ f(x, y) \} = G(s, u)$, then

- i. $G_x S_y \left\{ \frac{\partial f(x, y)}{\partial y} \right\} = \frac{1}{u} G(s, u) - \frac{1}{u} G\{ f(x, 0) \}.$
- ii. $G_x S_y \left\{ \frac{\partial f(x, y)}{\partial x} \right\} = sG(s, u) - sS\{ f(0, y) \}.$
- iii. $G_x S_y \left\{ \frac{\partial^2 f(x, y)}{\partial y^2} \right\} = \frac{1}{u^2} G(s, u) - \frac{1}{u^2} G\{ f(x, 0) \} - \frac{1}{u} G \left\{ \frac{\partial f(x, 0)}{\partial t} \right\}.$
- iv. $G_x S_y \left\{ \frac{\partial^2 f(x, y)}{\partial x^2} \right\} = s^2 G(s, u) - s^2 S\{ f(0, y) \} - sS \left\{ \frac{\partial f(0, y)}{\partial x} \right\}.$
- v. $G_x S_y \left\{ \frac{\partial^2 f(x, y)}{\partial x \partial y} \right\} = \frac{s}{u} [G(s, u) - S\{ f(0, y) \}] - G\{ f(x, 0) \} - f(0, 0).$

17. Double Laplace-Sumudu Transform [90]

In 2022 Shams A. Ahmed et al. introduced a new double integral transform named double Laplace-Sumudu transform. This Transform is defined for the function $f(x, t)$ as:

$$L_x S_t \{ f(x, t) \} = \frac{1}{\sigma} \int_0^\infty \int_0^\infty e^{-(\rho s + \frac{t}{v})} f(x, t) dx dt = H(s, v), x, t > 0.$$

Double Laplace- Sumudu transform of some elementary functions are:

1. $L_x S_t \{ x^m t^n \} = \frac{m!n!v^n}{s^{m+1}}, m, n \in N.$
2. $L_x S_t \{ e^{\alpha x + \beta t} \} = \frac{s}{(s-\alpha)(1-\beta v)}, \alpha, \beta \in R.$
3. $L_x S_t \{ e^{i(\alpha x + \beta t)} \} = \frac{1}{(s-i\alpha)(1-i\beta v)}, \alpha, \beta \in R.$
4. $L_x S_t \{ \sin \sin (\alpha x + \beta t) \} = \frac{\alpha + \beta sv}{(s^2 + \alpha^2)(1 + \beta^2 v^2)}$
5. $L_x S_t \{ \cos \cos (\alpha x + \beta t) \} = \frac{s - \alpha\beta v}{(s^2 + \alpha^2)(1 + \beta^2 v^2)}$
6. $L_x S_t \{ \sinh \sinh (\alpha x + \beta t) \} = \frac{\alpha + \beta sv}{(s^2 - \alpha^2)(1 - \beta^2 v^2)}$
7. $L_x S_t \{ \cosh \cosh (\alpha x + \beta t) \} = \frac{s - \alpha\beta v}{(s^2 - \alpha^2)(1 - \beta^2 v^2)}$

Double Laplace-Sumudu Transform of Derivatives:

Let $L_x S_t \{ f(x, t) \} = H(s, v)$, then

- i. $L_x S_t \left\{ \frac{\partial f(x, t)}{\partial x} \right\} = sH(s, v) - S\{ f(0, t) \}.$
- ii. $L_x S_t \left\{ \frac{\partial f(x, t)}{\partial t} \right\} = \frac{1}{v} Q(v, s) - \frac{1}{v} L\{ f(x, 0) \}.$
- iii. $L_x S_t \left\{ \frac{\partial^2 f(x, t)}{\partial x^2} \right\} = s^2 Q(v, s) - sS\{ f(0, t) \} - S \left\{ \frac{\partial f(0, t)}{\partial x} \right\}.$
- iv. $L_x S_t \left\{ \frac{\partial^2 f(x, t)}{\partial t^2} \right\} = \frac{1}{v^2} Q(v, s) - \frac{1}{v^2} L\{ f(x, 0) \} - \frac{1}{v} L \left\{ \frac{\partial f(x, 0)}{\partial t} \right\}.$

$$v. L_x S_t \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = \frac{s}{v} Q(v, s) - \frac{s}{v} L\{f(x, 0)\} - S \left\{ \frac{\partial f(0,t)}{\partial t} \right\}.$$

18. Double Formable Transform [91]

In 2022 Bayan Ghazal et al. introduced new double transform called double formable integral transform. This Transform is defined for the function $f(x, t)$ as:

$$R_x R_t \{f(x, t)\} = \frac{s v}{u r} \int_0^\infty \int_0^\infty e^{-\left(\frac{v}{r}x + \frac{s}{u}t\right)} f(x, t) dx dt, \quad x, t > 0.$$

Double formable transform of some elementary functions are:

1. $R_x R_t \{1\} = 1.$
2. $R_x R_t \{x^m y^n\} = m! n! \left(\frac{u}{s}\right)^m \left(\frac{r}{v}\right)^n, \quad m, n \in N.$
3. $R_x R_t \{e^{\alpha x + \beta y}\} = \frac{sv}{(s-\alpha u)(v-\beta r)}, \quad \alpha, \beta \in R.$
4. $R_x R_t \{e^{i(\alpha x + \beta y)}\} = \frac{sv(sv-\alpha\beta ur) + isv(sr\beta + uva)}{(s^2 + a^2 u^2)(v^2 + \beta^2 r^2)}, \quad \alpha, \beta \in R.$
5. $R_x R_t \{\sin \sin (\alpha x + \beta y)\} = \frac{sv(sr\beta + uva)}{(s^2 + a^2 u^2)(v^2 + \beta^2 r^2)}.$
6. $R_x R_t \{\cos \cos (\alpha x + \beta y)\} = \frac{sv(sv-\alpha\beta ur)}{(s^2 + a^2 u^2)(v^2 + \beta^2 r^2)}.$
7. $R_x R_t \{\sinh \sinh (\alpha x + \beta y)\} = \frac{sv(sr\beta + uva)}{(s^2 - a^2 u^2)(v^2 - \beta^2 r^2)}.$
8. $R_x R_t \{\cosh \cosh (\alpha x + \beta y)\} = \frac{sv(sv+\alpha\beta ur)}{(s^2 - a^2 u^2)(v^2 - \beta^2 r^2)}.$

Double Formable Transform of Derivatives:

Let $R_x R_t \{f(x, t)\}$ be the double formable transform of $f(x, t)$, then

- i. $R_x R_t \left\{ \frac{\partial f(x,t)}{\partial x} \right\} = \frac{v}{r} R_x R_t \{f(x, t)\} - \frac{v}{r} R_t \{f(0, t)\}.$
- ii. $R_x R_t \left\{ \frac{\partial f(x,t)}{\partial t} \right\} = \frac{s}{u} R_x R_t \{f(x, t)\} - \frac{s}{u} R_x \{f(x, 0)\}.$
- iii. $R_x R_t \left\{ \frac{\partial^2 f(x,t)}{\partial x^2} \right\} = \left(\frac{v}{r}\right)^2 R_x R_t \{f(x, t)\} - \left(\frac{v}{r}\right)^2 R_t \{f(0, t)\} - \frac{v}{r} R_t \left\{ \frac{\partial f(0,t)}{\partial x} \right\}.$
- iv. $R_x R_t \left\{ \frac{\partial^2 f(x,t)}{\partial t^2} \right\} = \left(\frac{s}{u}\right)^2 R_+^2 [(p, s), (u, v)] - \left(\frac{s}{u}\right)^2 R_x \{f(x, 0)\} - \frac{s}{u} R_x \left\{ \frac{\partial f(x,0)}{\partial t} \right\}.$
- v. $R_x R_t \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = \frac{vs}{ru} R_x R_t \{f(x, t)\} - R_x \{f(x, 0)\} - R_t \{f(0, t)\} + f(0, 0).$

19. Double Laplace-Aboodh Transform [92]

In 2022 Mona Hunaiber and Ali Al-Aati introduced a new double integral transform named double Laplace-Aboodh transform. This Transform is defined for the function $f(x, t)$ as:

$$L_x A_t \{f(x, t)\} = \frac{1}{\lambda} \int_0^\infty \int_0^\infty e^{-(\rho x + \lambda t)} f(x, t) dx dt = F(\rho, \lambda), \quad x, t > 0.$$

Double Laplace- Aboodh transform of some elementary functions are:

1. $L_x A_t \{1\} = \frac{1}{\rho \lambda^2}.$
2. $L_x A_t \{x^n t^m\} = \frac{n! m!}{\rho^{n+1} \lambda^{m+2}}, \quad n, m \in N.$
3. $L_x A_t \{e^{\alpha x + \beta t}\} = \frac{1}{(\rho - \alpha)(\lambda^2 - \beta \lambda)}, \quad \alpha, \beta \in R.$
4. $L_x A_t \{e^{i(\alpha x + \beta t)}\} = \frac{(\rho \lambda - \alpha \beta) + i(\beta \rho + \alpha \lambda)}{(\rho^2 + \alpha^2)(\lambda^3 + \beta^2 \lambda)}, \quad \alpha, \beta \in R.$
5. $L_x A_t \{\sin \sin (\alpha x + \beta t)\} = \frac{\beta \rho + \alpha \lambda}{(\rho^2 + \alpha^2)(\lambda^3 + \beta^2 \lambda)}$
6. $L_x A_t \{\cos \cos (\alpha x + \beta t)\} = \frac{\rho \lambda - \alpha \beta}{(\rho^2 + \alpha^2)(\lambda^3 + \beta^2 \lambda)}$
7. $L_x A_t \{\sinh \sinh (\alpha x + \beta t)\} = \frac{\beta \rho + \alpha \lambda}{(\rho^2 - \alpha^2)(\lambda^3 - \beta^2 \lambda)}$
8. $L_x A_t \{\cosh \cosh (\alpha x + \beta t)\} = \frac{\rho \lambda + \alpha \beta}{(\rho^2 - \alpha^2)(\lambda^3 - \beta^2 \lambda)}$

Double Laplace-Aboodh Transform of Derivatives:

Let $L_x A_t \{f(x, t)\} = F(\rho, \lambda)$, then

- i. $L_x A_t \left\{ \frac{\partial f(x,t)}{\partial t} \right\} = \lambda F(\rho, \lambda) - \frac{1}{\lambda} L\{f(x, 0)\}.$
- ii. $L_x A_t \left\{ \frac{\partial f(x,t)}{\partial x} \right\} = \rho F(\rho, \lambda) - A\{f(0, t)\}.$
- iii. $L_x A_t \left\{ \frac{\partial^2 f(x,t)}{\partial t^2} \right\} = \lambda^2 F(\rho, \lambda) - L\{f(x, 0)\} - \frac{1}{\lambda} L\left\{ \frac{\partial f(x,0)}{\partial t} \right\}.$
- iv. $L_x A_t \left\{ \frac{\partial^2 f(x,t)}{\partial x^2} \right\} = \rho^2 F(\rho, \lambda) - \rho A\{f(0, t)\} - A\left\{ \frac{\partial f(0,t)}{\partial x} \right\}.$
- v. $L_x A_t \left\{ \frac{\partial^2 f(x,t)}{\partial x \partial t} \right\} = \rho \lambda F(\rho, \lambda) - \frac{\rho}{\lambda} L\{f(x, 0)\} - A\left\{ \frac{\partial f(0,t)}{\partial t} \right\}.$

20. Double Sumudu-Kamal Transform [93]

In 2022 Mona Hunaiber and Ali Al-Aati introduced a new double integral transform named double Sumudu-Kamal transform. This Transform is defined for the function $f(x, t)$ as:

$$S_x K_t \{f(x, t)\} = \frac{1}{\mu} \int_0^\infty \int_0^\infty e^{-\left(\frac{x}{\mu} + \frac{t}{\vartheta}\right)} f(x, t) dx dt = F(\mu, \vartheta), \quad x, t > 0.$$

Double Sumudu-Kamal transform of some elementary functions are:

1. $S_x K_t \{1\} = \vartheta.$
2. $S_x K_t \{x^n t^m\} = n! m! \mu^n \vartheta^{m+1}, \quad n, m \in N.$
3. $S_x K_t \{e^{\alpha x + \beta t}\} = \frac{\vartheta}{(1 - \alpha \mu)(1 - \beta \vartheta)}, \quad \alpha, \beta \in R.$
4. $S_x K_t \{\sin \sin (\alpha x + \beta t)\} = \frac{\vartheta(\alpha \mu + \beta \vartheta)}{(1 + \alpha^2 \mu^2)(1 + \beta^2 \vartheta^2)}$

$$5. S_x K_t \{ \cos \cos (ax + \beta t) \} = \frac{\vartheta - \alpha \beta \mu \vartheta^2}{(1 + \alpha^2 \mu^2)(1 + \beta^2 \vartheta^2)},$$

$$6. S_x K_t \{ \sinh \sinh (ax + \beta t) \} = \frac{\vartheta(\alpha \mu + \beta \vartheta)}{(1 - \alpha^2 \mu^2)(1 - \beta^2 \vartheta^2)},$$

$$7. S_x K_t \{ \cosh \cosh (ax + \beta t) \} = \frac{\vartheta + \alpha \beta \mu \vartheta^2}{(1 - \alpha^2 \mu^2)(1 - \beta^2 \vartheta^2)}.$$

Double Sumudu-Kamal Transform of Derivatives:

Let $S_x K_t \{ f(x, t) \} = F(\mu, \vartheta)$, then

$$i. S_x K_t \left\{ \frac{\partial f(x, t)}{\partial x} \right\} = \frac{1}{\mu} F(\mu, \vartheta) - \frac{1}{\mu} K \{ f(0, t) \}.$$

$$ii. S_x K_t \left\{ \frac{\partial f(x, t)}{\partial t} \right\} = \frac{1}{\vartheta} F(\mu, \vartheta) - S \{ f(x, 0) \}.$$

$$iii. S_x K_t \left\{ \frac{\partial^2 f(x, t)}{\partial t^2} \right\} = \frac{1}{\vartheta^2} F(\mu, \vartheta) - \frac{1}{\vartheta} S \{ f(x, 0) \} - S \left\{ \frac{\partial f(x, 0)}{\partial t} \right\}.$$

$$iv. S_x K_t \left\{ \frac{\partial^2 f(x, t)}{\partial x^2} \right\} = \frac{1}{\mu^2} \left[F(\mu, \vartheta) - K \{ f(0, t) \} - \mu K \left\{ \frac{\partial f(0, t)}{\partial x} \right\} \right].$$

21. Double Gupta Transform [94]

In 2022 Rahul Gupta et al. introduced a new double transform called double Gupta transform. This Transform is defined for the function $f(x, t)$ as:

$$\hat{R}_x \hat{R}_t \{ f(x, t) \} = \frac{1}{q^3} \frac{1}{r^3} \int_0^\infty \int_0^\infty e^{-(qx+rt)} f(x, t) dx dt = G(q, r), \quad x, t > 0.$$

Double Gupta Transform of Derivatives:

Let $\hat{R}_x \hat{R}_t \{ f(x, t) \} = G(q, r)$, then

$$i. \hat{R}_x \hat{R}_t \left\{ \frac{\partial f(x, t)}{\partial x} \right\} = qG(q, r) - \frac{1}{q^3} \hat{R} \{ f(0, t) \}.$$

$$ii. \hat{R}_x \hat{R}_t \left\{ \frac{\partial f(x, t)}{\partial t} \right\} = rG(q, r) - \frac{1}{r^3} \hat{R} \{ f(x, 0) \}.$$

$$iii. \hat{R}_x \hat{R}_t \left\{ \frac{\partial^2 f(x, t)}{\partial x^2} \right\} = q^2 G(q, r) - \frac{1}{q^2} \hat{R} \{ f(0, t) \} - \frac{1}{q^3} \hat{R} \left\{ \frac{\partial f(0, t)}{\partial x} \right\}.$$

$$iv. \hat{R}_x \hat{R}_t \left\{ \frac{\partial^2 f(x, t)}{\partial t^2} \right\} = r^2 G(q, r) - \frac{1}{r^2} \hat{R} \{ f(x, 0) \} - \frac{1}{r^3} \hat{R} \left\{ \frac{\partial f(x, 0)}{\partial t} \right\}.$$

22. Complex Double Sadik Transform [95]

In 2023 Saed M. Turq and Emad A. Kuffi introduced a new double transform called complex double Sadik transform. This Transform is defined for the function $f(x, t)$ as:

$$D^c \{ f(x, t) \} = \frac{1}{(uv)^\beta} \int_0^\infty \int_0^\infty e^{-i(u^\alpha x + v^\alpha t)} f(x, t) dt dx = F^c(u, v), \quad \alpha \neq 0.$$

Complex double Sadik transform of some elementary functions are:

$$1. D^c \{ 1 \} = \frac{-1}{(uv)^{\alpha+\beta}}$$

$$2. D^c \{ x^n t^m \} = \frac{(-i)^{n+m+2} n! m!}{u^{n\alpha + (\alpha+\beta)v^{m\alpha + (\alpha+\beta)}}, \quad n, m \in N.$$

$$3. D^c \{ e^{ax+bt} \} = \frac{1}{(uv)^\beta} \left[\frac{(a+iu^\alpha)(b+iv^\alpha)}{(u^{2\alpha} + a^2)(v^{2\alpha} + b^2)} \right], \quad a, b \in R.$$

$$4. D^c \{ e^{i(ax+bt)} \} = \frac{-1}{(uv)^\beta (u^\alpha - a)(v^\alpha - b)}, \quad a, b \in R.$$

$$5. D^c \{ \sin \sin (ax + bt) \} = i \left[\frac{av^\alpha + bu^\alpha}{(uv)^\beta (u^{2\alpha} - a^2)(v^{2\alpha} - b^2)} \right]$$

$$6. D^c \{ \cos \cos (ax + bt) \} = \frac{-(u^\alpha v^\alpha + ab)}{(uv)^\beta (u^{2\alpha} - a^2)(v^{2\alpha} - b^2)}.$$

$$7. D^c \{ \sinh \sinh (ax + bt) \} = i \left[\frac{av^\alpha + bu^\alpha}{(uv)^\beta (u^{2\alpha} + a^2)(v^{2\alpha} + b^2)} \right].$$

$$8. D^c \{ \cosh \cosh (ax + bt) \} = \frac{ab - u^\alpha v^\alpha}{(uv)^\beta (u^{2\alpha} + a^2)(v^{2\alpha} + b^2)}.$$

Complex Double Sadik Transform of Derivatives:

Let $D^c \{ f(x, t) \} = F^c(u, v)$, then

$$i. D^c \left\{ \frac{\partial f(x, t)}{\partial x} \right\} = iu^\alpha F^c(u, v) - \frac{1}{u^\beta} S_a^c \{ f(0, t) \}.$$

$$ii. D^c \left\{ \frac{\partial f(x, t)}{\partial t} \right\} = iv^\alpha F^c(u, v) - \frac{1}{v^\beta} S_a^c \{ f(x, 0) \}.$$

$$iii. D^c \left\{ \frac{\partial^2 f(x, t)}{\partial x^2} \right\} = (iu^\alpha)^2 F^c(u, v) - \frac{iu^\alpha}{u^\beta} S_a^c \{ f(0, t) \} - \frac{1}{u^\beta} \frac{\partial S_a^c \{ f(0, t) \}}{\partial x}.$$

$$iv. D^c \left\{ \frac{\partial^2 f(x, t)}{\partial t^2} \right\} = (iv^\alpha)^2 F^c(u, v) - \frac{iv^\alpha}{v^\beta} S_a^c \{ f(x, 0) \} - \frac{1}{v^\beta} \frac{\partial S_a^c \{ f(x, 0) \}}{\partial t}.$$

$$v. D^c \left\{ \frac{\partial^n f(x, t)}{\partial x^n} \right\} = (iu^\alpha)^n F^c(u, v) - \frac{1}{u^\beta} \sum_{j=1}^n (iu^\alpha)^{j-1} \frac{\partial^{n-j} S_a^c \{ f(0, t) \}}{\partial x^{n-j}}.$$

$$vi. D^c \left\{ \frac{\partial^n f(x, t)}{\partial t^n} \right\} = (iv^\alpha)^n F^c(u, v) - \frac{1}{v^\beta} \sum_{j=1}^n (iv^\alpha)^{j-1} \frac{\partial^{n-j} S_a^c \{ f(x, 0) \}}{\partial t^{n-j}}.$$

$$vii. D^c \left\{ \frac{\partial^2 f(x, t)}{\partial x \partial t} \right\} = i^2 (uv)^\alpha F^c(u, v) - \frac{iu^\alpha}{v^\beta} S_a^c \{ f(x, 0) \} - \frac{1}{u^\beta} \frac{\partial S_a^c \{ f(0, t) \}}{\partial t}.$$

$$viii. D^c \left\{ \frac{\partial f^2(x, t)}{\partial t \partial x} \right\} = i^2 (uv)^\alpha F^c(u, v) - \frac{iv^\alpha}{u^\beta} S_a^c \{ f(0, t) \} - \frac{1}{v^\beta} \frac{\partial S_a^c \{ f(0, t) \}}{\partial x}.$$

23. Double ARA-Formable Transform [96]

In 2023 Rania Saadeh and Motasem M. Mustafa introduced a new double integral transform named double ARA-Formable transform. This Transform is defined for the function $\phi(z, t)$ as:

$$G_{n,z}R_t\{\phi(z,t)\} = \frac{sv}{u} \int_0^\infty \int_0^\infty z^{n-1} e^{-\left(vz+\frac{st}{u}\right)} \phi(z,t) dz dt = \Phi_n(v,s,u),$$

$z, t > 0.$

Double ARA-Formable transform of some elementary functions are:

1. $G_{n,z}R_t\{1\} = \frac{\Gamma(n)}{v^{n-1}}, \operatorname{Re}(v) > 0.$
2. $G_{n,z}R_t\{z^\alpha t^\beta\} = \frac{u^\beta \Gamma(\beta+n) \Gamma(\alpha+n)}{s^\beta v^{\alpha+n-1}}, \alpha > -1, \beta > -1.$
3. $G_{n,z}R_t\{e^{\alpha z + \beta t}\} = \frac{sv \Gamma(n)}{(v-\alpha)^n (s-u\beta)}, v > \alpha, \frac{s}{u} > \beta.$
4. $G_{n,z}R_t\{\sin \sin(\alpha z + \beta t)\} = \frac{sv \Gamma(n)}{2i(v-i\alpha)^n (s-iu\beta)} - \frac{sv \Gamma(n)}{2i(v+i\alpha)^n (s+iu\beta)}.$
5. $G_{n,z}R_t\{\cos \cos(\alpha z + \beta t)\} = \frac{sv \Gamma(n)}{2i(v-i\alpha)^n (s-iu\beta)} + \frac{sv \Gamma(n)}{2i(v+i\alpha)^n (s+iu\beta)}.$
6. $G_{n,z}R_t\{\sinh \sinh(\alpha z + \beta t)\} = \frac{sv \Gamma(n)}{2(v-\alpha)^n (s-u\beta)} - \frac{sv \Gamma(n)}{2(v+\alpha)^n (s+u\beta)}.$
7. $G_{n,z}R_t\{\cosh \cosh(\alpha z + \beta t)\} = \frac{sv \Gamma(n)}{2(v-\alpha)^n (s-u\beta)} + \frac{sv \Gamma(n)}{2(v+\alpha)^n (s+u\beta)}.$

Double ARA-Formable Transform of Derivatives:

Let $G_{n,z}R_t\{\phi(z,t)\} = \Phi_n(v,s,u)$, then

- i. $G_{n,z}R_t\left\{\frac{\partial \phi(z,t)}{\partial z}\right\} = (-1)^{n-1} \frac{d^{n-1}}{dv^{n-1}} [v\Phi_1(v,s,u) - vR_t\{\phi(0,t)\}].$
- ii. $G_{n,z}R_t\left\{\frac{\partial \phi(z,t)}{\partial t}\right\} = \frac{s}{u} \Phi_n(v,s,u) - \frac{s}{u} G_{n,z}\{\phi(z,0)\}.$
- iii. $G_{n,z}R_t\left\{\frac{\partial^2 \phi(z,t)}{\partial t^2}\right\} = \frac{s^2}{u^2} \Phi_n(v,s,u) - \frac{s^2}{u^2} G_{n,z}\{\phi(z,0)\} - \frac{s}{u} G_{n,z}\left\{\frac{\partial \phi(z,0)}{\partial t}\right\}.$
- iv. $G_{n,z}R_t\left\{\frac{\partial^2 \phi(z,t)}{\partial z^2}\right\} = (-1)^{n-1} \frac{d^{n-1}}{dv^{n-1}} [v^2 \Phi_1(v,s,u) - v^2 R_t\{\phi(0,t)\} - v R_t\left\{\frac{\partial \phi(0,t)}{\partial z}\right\}].$
- v. $G_{n,z}R_t\left\{\frac{\partial^n \phi(z,t)}{\partial z^n}\right\} = v^n \Phi(v,s,u) - \sum_{j=0}^{n-1} v^{n-j-1} R_t\left\{\frac{\partial^j \phi(0,t)}{\partial z^j}\right\}.$
- vi. $G_{n,z}R_t\left\{\frac{\partial^2 \phi(z,t)}{\partial z \partial t}\right\} = (-1)^{n-1} \frac{d^{n-1}}{dv^{n-1}} \left[\frac{vs}{u} \Phi_n(v,s,u) - \frac{vs}{u} R_t\{\phi(0,t)\}\right] - \frac{v^2 s}{u} G_{n,z}\{\phi(x,0)\} + \frac{v^2 s}{u} \phi(0,0).$
- vii. $G_{n,z}R_t\left\{\frac{\partial^n \phi(z,t)}{\partial t^n}\right\} = \frac{s^n}{u^n} \Phi(v,s,u) - \frac{s}{u} \sum_{j=0}^{n-1} \left(\frac{s}{u}\right)^{n-j-1} G_{n,z}\left\{\frac{\partial^j \phi(z,0)}{\partial t^j}\right\}.$

Acknowledgment

1. The purpose of using the integral transform is to get the exact solutions for scientific problems.
2. The Laplace integral transform is the basis for all integral transforms, i.e., these transforms are derived from Laplace transform.
3. All integral transforms that mentioned in this paper are continuous transforms.
4. No integral transform is better than another because each transform has its fit applications.

Through our study for the types of integral transforms we notice some important points:

- ❖ Alenezi transform which introduced in 2021 by Ahmed M. Alenezi is equivalent to Jafari transform which introduced in 2020 by H. Jafari.
- ❖ g –transform which introduced in 2021 by Yusra Al-Ameri and Methaq Hamza is equivalent to Sadik Transform which introduced in 2018 by S.L. Shaikh.
- ❖ Polynomial Transform that proposed in 2016 by Benedict Barnes is equivalent to Al-Tememe Transformation that presented by Ali Hassan Mohammed et al.
- ❖ Rangaig Transform that introduced in 2017 by Norodin A. Rangaig et al. is equivalent to Aboodh Transformation which introduced in 2013 by Khalid Suliman Aboodh .
- ❖ AMK Transformation (with kernel $\sin \sin t$) that introduced in 2021 by M. Kashif et al. is equivalent to Shaban Transform (with kernel $\cos \cos t$) which introduced in the same year by Rehab A. Khudair et al. .
- ❖ In 2022 many integral transforms appear that have a great similarity with ZZ integral transformation that presented by Zain UIAbadin Zafar in 2016, these transforms are [Formable transformation which introduced by Rania Zohair and Bayan Fu'ad; KKAT transformation which introduced by Karry Iqbal et al.; Rishi transform which introduced by Kumar et al.
- ❖ ZZ integral transform is the generalization of which introduced by Shehu transform Shehu Maitama and Weidong Zhao.
- ❖ In the double integral transforms, the Double General Integral Transform that introduced in 2022 by Dinkar P. Patil et al. is equivalent to The New

General Double Integral Transform which introduced in 2021 by Hossein Jafari et al.

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مراجعة للتحويلات المتكاملة

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الخلاصة:

تم اقتراح العديد من التحويلات المتكاملة وتجربتها وإثبات قدرتها على حل العديد من التطبيقات في مختلف المجالات العلمية. يأتي تنوع التحويلات المتكاملة من قدرتها الفريدة على حل المشكلات عن طريق تحويلها من مجال واحد حيث يتم تقديم الحل من خلال إجراء رياضي معقد إلى مجال آخر حيث يمكن للطرق الجبرية البسيطة حلها. وعلى هذا الأساس ظهر العديد من هذه التحويلات والمعرفة في مجال الأعداد الحقيقية ومجال الأعداد المركبة. سبب تنوع هذه التحويلات هو الحاجة إليها في حل بعض المعادلات الرياضية والمعادلات الفيزيائية أو التي تتعلق بأي فرع من فروع العلوم التطبيقية، لذلك أثبتوا قوتهم في حل المعادلات التفاضلية والتكاملية والأنظمة التفاضلية بالإضافة إلى ذلك التطبيقات العلمية. كان تحويل لابلاس أحد التحويلات الأولى التي ساهمت في حل المعادلات الرياضية، ثم ظهرت العشرات من التحويلات، والتي اعتمدت بشكل أساسي على تحويل لابلاس. يوضح هذا العمل التنوع الزمني للتحويلات المتكاملة من خلال تقديم مجموعة من التحويلات المتكاملة بخصائصها الأساسية. تعاملنا مع تحويلات تكاملية فردية ومزدوجة.

الكلمات المفتاحية: تحويلات تكاملية، تحويلات تكاملية ثنائية، مشتقة التحويلات التكاملية، مشتقة التحويلات تكاملية ثنائية.