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| D:\مجلة\last\شعار المجلة.jpg **Bipolar Valued Fuzzy SA-subalgebras and Fuzzy SA-ideals of SA- algebra.** **Alaa Salih Abed1\*, Areej Tawfeeq Hameed 2**1,2Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq |
| **ARTICLE INFO** |  | **ABSTRACT** |
| Received: 08/ 08 /2023Accepted: 27 / 08 / 2023Available online: 05 / 12 / 2023

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| DOI: 000000000000000000000 |

 |  |  In this paper, the notions of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals on SA-algebras with their properties are familiarized. Several theorems are stated and proved with their examples. After that we introduced new notion which is negative anti-fuzzy SA-subalgebra(SA-ideal) of SA-algebra . The image and inverse image of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals are defined and how the homomorphic images and inverse images of bipolar valued become bipolar valued fuzzy on SA-algebras is studied as well.. |
| **Keywords:**SA-algebras, fuzzy SA-subalgebra, bipolar valued fuzzy SA-subalgebra, fuzzy SA-ideal, bipolar valued fuzzy SA-ideal. Copyright Authors, 2022, College of Sciences, University of Anbar. This is an open-access article under the CC BY 4.0 license ([http://creativecommons.org/licens es/by/4.0/](http://creativecommons.org/licens%20es/by/4.0/)). |  |

**Introduction:**

Areej Tawfeeq Hameed and et al ([2]) presented a different algebraic building, named SA-algebra, they have calculated a few belongings of these algebras, the conception of SA-ideals on SA-algebras was conveyed and some of its properties are scrutinized. The conception of a fuzzy set, was familiarized by L.A. Zadeh [10]. In [9], S.M. Mostafa and A.T. Hameed made an extension of the conception of fuzzy set by an interval-valued fuzzy set (i.e., a fuzzy set with an interval-valued membership function).

This interval- valued fuzzy KUS-ideals on KUS-algebras is referred to as an i-v fuzzy KUS-ideals on KUS-algebras. they created a way of estimated inference using his i-v fuzzy KUS-ideals on KUS-algebras. In this paper, using the conception of [bipolar valued fuzzy subset](http://academic.research.microsoft.com/Keyword/20792/interval-valued-fuzzy-set), we familiarize the conception of a bipolar valued fuzzy SA-ideals (briefly**,** BVFSAI) of a SA-algebra, and reading some of their properties.

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 Using a bipolar valued [level set](http://academic.research.microsoft.com/Keyword/22633/level-set) of a bipolar valued fuzzy set, we public a characterization of a bipolar valued fuzzy SA-ideals. We evidence that every SA-ideals of a SA-algebra Ѥ can be appreciated as a bipolar valued level SA-ideals of a bipolar valued fuzzy SA ideals of Ѥ. In connection with the idea of homomorphism, we educat.

how the images and inverse images of bipolar valued fuzzy SA-ideals develop bipolar valued fuzzy SA-ideals.

"**2. PRELIMINARIES**

Now, we offer some definitions and preliminary results wanted in the later sections.

**Definition (2.1)[2].**  Let Ѥ be an algebra with two binary operations and and constant . Ѥ is named an -algebra if it fulfills the next identities: for any ,

() ,

() ,

 () ,

 () .

 In Ѥ we can describe a binary relation ( ≤ ) by :

 if and only if , Ѥ**.** And we will symbolize it by

**Example (2.2)[2].**  Let Ѥ ={ 0 , 1 , 2 , 3 } be a set with the following tables:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** |  |  | **0** | **1** | **2** | **3** |
| **0** | **0** | **1** | **2** | **3** | **0** | **0** | **3** | **2** | **1** |
| **1** | **1** | **2** | **3** | **0** | **1** | **1** | **0** | **3** | **2** |
| **2** | **2** | **3** | **0** | **1** | **2** | **2** | **1** | **0** | **3** |
| **3** | **3** | **0** | **1** | **2** | **3** | **3** | **2** | **1** | **0** |

ThenѤ is

**Lemma (2.3)[2].**  In .For any , ∈ Ѥ,

 () .

 () ,

 () .

 **Proposition (2.4)[2].**  In .The next holds: for any , , ∈ Ѥ,

,

 ,

 imply ,

  imply

and,

 and imply

**Definition (2.5)[2].** In , let S be a nonempty set of Ѥ. S is named a *SA*-subalgebra of Ѥ if whenever ∈ S.And we will symboliz it by SAS-

**Definition (2.6)[2].** A nonempty subset I of a is named a *SA*-ideal of Ѥ if it fulfills: for , , ∈Ѥ,

(1) ,

(2) . And we will symboliz it by SAI-

**Proposition (2.7)[2].** Every *SA-* of is a *SA*S- of Ѥ and the converse is not true .

**Lemma (2.8)[2].** An *SA*I of has the following property:

1. If for any ∈ Ѥ, for all ∈ I, ≤ implies ∈ I.
2. If for any ∈ I implies .

**Definition (2.9)[10].** Suppose Ѥ be a nonempty set, a fuzzy subset of is a function

**Definition (2.10)[2].** Suppose Ѥ be a nonempty set and be a fuzzy subset of Ѥ, for t ∈ [0,1], the set

 is named a level subset of .

**Definition (2.11)[2].** IN , a fuzzy subset of Ѥ is called a **fuzzy *SA*-subalgebra of Ѥ** if for all

 , ∈ Ѥ, and

.And we will symbliz it by FSAS-

**Theorem (2.12)[2].**  Suppose be a fuzzy subset of , then

1- If is a FSAS- of **Ѥ** , then for any , is a SAS- of , when

2- If for all , is a SAS- of Ѥ, then is a FSAS- of .

**Definition (2.13)[4].**  In , a fuzzy subset of Ѥ is called a **fuzzy *SA*-ideal of Ѥ** if

for all , , ∈Ѥ, and.

And we will symbliz it by FSAI-

**Theorem (2.14)[4].**  suppose be a fuzzy subset of , then

1- If is a FSAS- of , then for any , is a SAS- of , when

2- If for all , is a SAS- of Ѥ, then is a FSAS- of .

**Theorem (2.15)[4].**  Suppose be a fuzzy subset of . is a FSAI- of Ѥ if and only if,

for every t ∈ [0,1], is a SAI- of Ѥ, when .

**Proposition (2.16)[4].** FSAI- of is a FSAS- of and the converse is not true.

**Definition (2.17)[2].** Supposeand be *SA*-algebras, the mapping

 is named **a homomorphism** if it fulfills:

 , , for all , ∈ Ѥ.

**Definition (2.18)[7,8].** Suppose be a mapping nonempty sets Ѥ

 and respectively. If ρ is a fuzzy subset of Ѥ, then the fuzzy subset β of defined by:

is known as the image of under **.**

 Similarly if is a fuzzy subset of , then the fuzzy subset ) of

 ( i.e the fuzzy subset defined by for all) is named the pre-image of β under .

**Theorem (2.19)[2].**  1- An onto homomorphic pre-image of a FSAS- is also a FSAS-.

2- An onto homomorphic pre-image of a FSAI- is also a FSAI-.

**Definition (2.20)[7,8].** A fuzzy subset of a set Ѥ **has sup property** if for any subset T of Ѥ,

there exist t0 ∈T such that t0

**Theorem (2.21)[1].**  Let be a homomorphism between  *SA*-algebras Ѥ and respectively.

1- For every FSAS- of and with sup property, is a FSAS- of .

2- For every FSAI- of Ѥ and with sup property, is a FSAI- of .

**Definition (2.22)[9].**  Assume be , a fuzzy subset of Ѥ is named **an anti-fuzzy**

 **SA-subalgebra of** Ѥ if for all ,

AFSAS1)

AFSAS2) And we will symbliz it by AFSAS-

**Proposition (2.23)[9].** Suppose be an AFSAS- of .

1- If is an AFSAS- of Ѥ , then it satisfies for any t∈[0, 1],

 implies is a FSAS- of Ѥ .

2**-** If is a FSAS- of Ѥ, for all t∈[0, 1], ,

then is an AFSAS- of Ѥ .

**Definition (2.24)[9].**  Let be, is named **an anti-fuzzy SA-ideal of** Ѥ

if it fulfills the following conditions, for all , , ,

(AFSAI1) ,

(AFSAI2) . And we will symbliz it by AFSAI-

**Proposition (2.25)[9].** Let be an anti-fuzzy subset of.

1- If is an AFSAI- of Ѥ , then it fulfills for any t∈[0, 1],

implies is a FSAI- of Ѥ.

2- If is a FSAI- of Ѥ, for all t∈[0, 1], ,

then is an AFSAS- of Ѥ .

**Definition (2.26)[9].** Assume be a mapping nonempty SA-algebras

and respectively. If is anti-fuzzy subset of Ѥ, then the anti-fuzzy subset of defined by:

is known as the image of under .

 Similarly if is anti-fuzzy subset of , then the fuzzy subset of

( i.e the anti-fuzzy subset defined by ,

for all ) is named the pre-image of under .

**Theorem (2.27)[9].** 1- An onto homomorphic pre-image of AFSAS- is also AFSAS-.

2- An onto homomorphic pre-image of an AFSAI- is also AFSAI-.

**Definition (2.28)[9].** A fuzzy subset of a set **has inf property** if for any subset T of ,

there exist t0 ∈T such that

**Theorem (2.29)[9].**  Let be a homomorphism between  *SA*-algebras Ѥ and separately.

1- For every AFSAS- of Ѥ and with inf property, is AFSAS- of .

2- For every AFSAI- of Ѥ and with inf property, is AFSAI- of .

**Remark (2.30)[3].** An interval number is , where 0 ≤ ≤ ≤ 1.

 Let I be a closed unit interval, (i.e., I = [0, 1]). Let

D[0, 1] denote the family of all closed subintervals of

 I = [0, 1] , that is,

 D[0, 1] = { | ≤ , for ,∈ I} .

 Now, we describe what is known as cultured minimum (briefly, rmin) of two element in D[0,1].

**Definition (2.31)[3].** We also define the symbols (≽), (≼) , (=) ,"rmin " and "rmax "

in situation of two elements in D[0, 1] .

Consider two interval numbers (elements numbers)"

 , in D[0, 1] : Then

(1) ≽ if and only if, ≥ and ≥,

(2) ≼ if and only if, ≤ and ≤,

(3) = if and only if, = and =,

(4) rmin {, }= [min {,}, min {,}],

(5) rmax {, }= [max {,}, max {,}],

**Remark (2.32)[3].** It is obvious that (D[0, 1] , ≼ , ∨ , ∧ ) is a complete lattice with

 = [0, 0] as its least element and = [1, 1] as its greatest element.

Let ∈D[0, 1] where i ∈Λ. We define

 = [ , ], = [ , ]".

**Definition (2.33)[7,8].**  An **interval-valued fuzzy subset** on is defined as

 = {< , [ () , () ]>| ∈Ѥ} .

Where () ≤ (), for all ∈ Ѥ. Then the fuzzy subsets : Ѥ → [-1, 0] and

 : Ѥ → [0, 1] are named a **lower fuzzy subset and an upper fuzzy subset** of separately".

 Let () = [ () , () ] ,:Ѥ → D[0, 1], then A = {< , () >| ∈ Ѥ} .

**Remark (2.34)[1].**  Let Ѥ be the universe of discourse. A **bipolar fuzzy subset**  **of *Ѥ***

is an object ensuring the form ,

where : Ѥ → [−1, 0] and : Ѥ → [0, 1] are mappings.

"The positive membership degree () denoted the satisfaction degree

 of an element Ѥ to the property corresponding to a bipolar-valued fuzzy

 , and the negative membership degree

 () means the satisfaction degree of Ѥ to some implicit counter-property"of

 . For the sake of plainness, we shall use the symbol

 , for the bipolar fuzzy set

 , and use the conception

 of bipolar fuzzy sets instead of the conception of bipolar-valued fuzzy sets.

**Definition (2.35)[5].** A bipolar fuzzy subset  of is named **a**

**bipolar fuzzy -subalgebra of**  if itfulfills the next properties: for any ,

1. ,
2. ,
3. and
4. . And we will symbliz it by BFSAS-

**Definition (2.36)[5].** A bipolar fuzzy subset

 of is named

**a bipolar fuzzy *SA*-ideal of *Ѥ*** if it fulfillsthe following: for any , , ,

1. (0) *≤*  (),
2. (0) *≥* (),
3. and

.And we will symbliz it by BFSAI-**.**

**Definition (2.37)[7,8].** Assume be a mapping from set Ѥ

 into a set . let B be a bipolar valued fuzzy subset of . Then the inverse image of B,

 denoted by , is a bipolar valued fuzzy subset of Ѥ, with the membership function given by for all ∈ Ѥ .

**Proposition (2.38)[6].** Assume be a mapping

 from set Ѥ into set , let

 be bipolar valued fuzzy subsets

 of sets Ѥ and separately. Then

(1) ,

(2) .

**3. BIPOLAR VALUED FUZZY SA- SUBALGEBRAS OF SA-ALGEBRA**

 In the part, the conception of the bipolar valued fuzzy SA-subalgebras of SA-algebra is presented. Some theorems and properties are itemized and ascertained.

**Definition (3.1):**A interval valued fuzzy subset Ɣ = {< , () >| ∈ Ѥ} =

{< , [ (), () ]>| ∈ Ѥ} of is called

**a bipolar valued fuzzy SA-subalgebra** denoted by  **(BVFSAS- ) of**

 = {< , (),() >| ∈ Ѥ}

= {< , (), () >| ∈ Ѥ } ,

, if for all.

1-

 2-

3- and

4-

i.e.,

1-

 2-

3- and

 4-

i.e.,

1- and

 2- and

3- and

 4- and

**Remark (3.2):** A bipolar valued fuzzy subset

of , for all, thus,

Since , ,

*(* and ,

 then and

.

**Example (3.3):** Let **Ѥ** = {0, a, b, c} in which the operations be define by the following tables:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **a** | **b** | **c** |  |  | **0** | **a** | **b** | **c** |
| **0** | **0** | **a** | **b** | **c** | **0** | **0** | **c** | **b** | **a** |
| **a** | **a** | **b** | **c** | **0** | **a** | **a** | **0** | **c** | **b** |
| **b** | **b** | **c** | **0** | **a** | **b** | **b** | **a** | **0** | **c** |
| **c** | **c** | **0** | **a** | **b** | **c** | **c** | **b** | **a** | **0** |

Then is an SA-algebra.

of where I ={0,b} is a SAS- of Ѥ,such that:

The fuzzy subset: Ѥ→ [0,1] and : Ѥ→ [-1,0] by:

, () = , () is BVFSAS**-** of Ѥ.

**Proposition (3.4):** If is a **BVFSAS-**, then ≤ and for all ∈ Ѥ .

**Proof:** For all ∈ Ѥ and = , we have

 and

 , then

 and

 and for all ∈ Ѥ .⌂

**Proposition (3.5):** Let be aBVFSAS-,

 if there exist a sequence of Ѥ such that

and then and .

**Proof:**

By Proposition (3.4), we have and

 for all ∈ Ѥ, then and

 for every positive integer n.

 Consider the inequality

 and .

Hence and . ⌂

**Theorem (3.6):** A bipolar valued fuzzy subset of

is a **BVFSAS-** of Ѥ if and only if, and are **AFSAS-** of Ѥ

and and are **FSAS-** of Ѥ.

**Proof:**

 Suppose that is a **BVFSAS-** of Ѥ, then for all ∈ Ѥ, we have

 =

 .Therefore ,

and . Also,

 Therefore,

and .

Hence, we get that and are **AFSAS-** of Ѥ and and are **AFSAS-** of Ѥ.

Conversely, if and are **AFSAS-** of Ѥ

 and and are **FSAS-** of Ѥ,

 for all , ∈ Ѥ. Observe :

 and

 Also,

 = r min

 And

Thus, we can conclude that is a **BVFSAS-** of Ѥ.⌂

**Definition (3.7):** In . A bipolar valued fuzzy subset of Ѥ ,

 for all ∈ D[0, 1], the set is **a level set** of Ѥ such that

 .

**Proposition (3.8):** Assume be . A bipolar valued fuzzy subset

 of If is a **BVFSAS-** of Ѥ,

then for any ∈ D[0, 1], the set is a of .

**Proof.**

 Assume that is a **BVFSAS-** of and let ∈ D[0, 1]

 such that ≠∅, and suppose such that

 , then ,

and . Since is a **BVFSAS-** of Ѥ, we get

1-

 2-

3-

4-

 Therefor,

 Hence the set is aof . ⌂

**Proposition (3.9):** In. A bipolar valued fuzzy subset of .

 If for all ∈ D[0, 1], the set is **SAS-** of , then is **BVFSAS-** of Ѥ.

**Proof.**

 Suppose that is a **SAS-** of Ѥ and be such that

Consider

and

 We have , () >>

 max { (), () },

and () >> max { (), ()}.

 It follows that and ( +) ∉ *.* This is a contradiction.

Hence, Summarily,

2-

3- and

4-

 Therefore is a **BVFSAS-** of . ⌂

**Theorem (3.10):** Any **SAS-** ofcan be realized as the upper [t1,t2]-Level

of some **BVFSAS-** of .

**Proof.**

Suppose I be a **SAS-** of and be

 bipolar valued fuzzy subset on defined by

 ()= .

 For all ],

we deliberate the following cases

**Case 1)** If , then

 , , thus

2-

3- and

4-

**Case 2)** If and then

 , , thus

1. ,

2-

3- and

4-

**Case 3)** If and ythen ,

 , thus

 ,

 2-

3- and

4-

C**ase 4)** If and ythen , and

 , thus

 ,

3- and

4-

Therefore, is a **BVFSAS-** of .⌂

**Corollary (3.11):** In, be a subset of Ѥ and let

 be an bipolar valued fuzzy subset on Ѥ defined by :

 .

Where α1 , α2 ∈ (0, 1] with . If is a **BVFSAS-** of Ѥ,

 then is a SA-subalgebra of Ѥ.

**Proof:**

Since that is a **BVFSAS-** of Ѥ. Let , ∈, then by Definition(3.1)

1. ,

2-

3- and

4-

 This implies that Hence is a **SAS-** of Ѥ. ⌂

**Proposition (3.12):** Assume be homomorphism of SA-algebras. If B is a **BVFSAS-** of ,

then the inverse image of B is a **BVFSAS-** of Ѥ.

**Proof:**

 Since is a **BVFSAS-** of ,

it follows from Theorem (3.6), that and

are **AFSAS-** of and and are **FSAS-** of .

Using Theorem (2.19) and Theorem (2.27), we discern

 and are **AFSAS-** of Ѥ and and are **FSAS-** of Ѥ.

 Hence is a **BVFSAS-** of Ѥ. ⌂

**Definition (3.13):** Assume be a mapping from a set

 into a set .

is a bipolar valued subset of **has sup and inf properties** if for any subset T of ,

there exist t, s ∈ T such that and

**Proposition (3.14):** Let be an epimorphism of SA-algebras.

 If is a **BVFSAS-** of Ѥ with inf-sup property, then is a **BVFSAS-** of .

**Proof:**

 Assume that is a **BVFSAS-** of Ѥ.

It follows from Theorem (3.6), that and are **AFSAS-**

of Ѥ and and are **FSAS-** of Ѥ. of Ѥ.

 Using (2.21), Theorem (2.29), the images

and are **AFSAS-** of and and

 are **BVFSAS-** of . Hence is a **BVFSAS-** of .⌂

**4. BIPOLAR VALUED FUZZY SA-IDEALS OF SA-ALGEBRA**

 In the part, the conception of the bipolar valued fuzzy SA-ideals of SA-algebra is introduced. Some theorems and properties are detailed and evidenced.

**Definition (4.1):** A interval valued fuzzy subset

 = {< , () >| ∈ Ѥ}

= {< , [ (), () ]>| ∈ } of SA-algebra

is named **a bipolar valued fuzzy SA-ideal (BVFSAI- ) of** Ѥ signified by

 = {< , (),() >| ∈ }

= {< , (), () >| ∈ } ,

, if for all, , .

1-

2- and

3- i.e.,

1- and

2-

 3- i.e.,

1- and ,

2-

 and

3-

**Example (4.2):** Let = {0, 1, 2, 3} in which the operations be define by the following tables:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** |  |  | **0** | **1** | **2** | **3** |
| **0** | **0** | **1** | **2** | **3** | **0** | **0** | **3** | **2** | **1** |
| **1** | **1** | **2** | **3** | **0** | **1** | **1** | **0** | **3** | **2** |
| **2** | **2** | **3** | **0** | **1** | **2** | **2** | **1** | **0** | **3** |
| **3** | **3** | **0** | **1** | **2** | **3** | **3** | **2** | **1** | **0** |

 Then is . Define

of where I ={0,1} is a SA-ideal of Ѥ,such that:

 The fuzzy subsets : → [0,1] and by:

 () =

Then () is is a **BVFSAI-** of Ѥ.

**Theorem (4.3):** A bipolar valued fuzzy subset

of is a **BVFSAI-** of Ѥ if and only if,

 and are **AFSAI-** of Ѥ and

and are **FSAI-** of Ѥ.

**Proof:**

 Suppose that is a **BVFSAI-** of Ѥ, then for all ∈ Ѥ,

 then

 and . For all ∈ Ѥ, we have

Therefore , and

 . Also,

Therefore, and

 .

Hence, we become that and are **AFSAI-** of Ѥ and and are **FSAI-** of Ѥ.

Conversely, if and are **AFSAI-** of Ѥ and and are **FSAI-** of Ѥ,

for all , , ∈ Ѥ. Observe :

 Also

 Thus, we can settle that is a **BVFSAI-** of Ѥ.⌂

**Proposition (4.4):** In. A bipolar valued fuzzy subset

 of If is a **BVFSAI-**

of Ѥ, then for any ∈ D[0, 1], the set is a **SAI-** of .

**Proof.**

 Assume that is a **BVFSAI-** of and let ∈ D[0, 1] be such that ≠∅,

 and assume such that

 , then , *,*

 and .

Since is a **BVFSAI-** of Ѥ, we get

1- and implies that :

 ,

2-

 3-

 Therefore, , Hence the set is a **SAI-** of . ⌂

**Proposition (4.5):** In. A bipolar valued fuzzy subset

 of . If for all ∈ D[0, 1],

the set is an SAI- of , then is a **BVFSAI-** of Ѥ.

**roof.**

 Assume that is a **SAI-** of Ѥ ,for any ,

and And assume

 be such that

Consider: and

 We have ,

.

It follows that

 and  *.* This is a contradiction.

Therefore is a **BVFSAI-** of . ⌂

**Theorem (4.6):** Any **SAI-** of can be realized as the upper [t1,t2]-Level of some **BVFSAI-** of .

**Proof.**

Assume I be a **SAI-** of and

be bipolar valued fuzzy subset on defined by

 ()= .

For all ], we contemplate the following cases:

**Case 1)** If, then

 , ,

 ,

 , , and

 , thus

1- and

1. ,

3-

**Case 2)** If and then

 ,

and , thus

1- ,

 and

 ,

3-

**Case 3)** If and then

 ,

and , thus

1-

 and

2-

C**ase 4)** If then

 , , thus

 1-

 and

 ,

 3-

 Therefore, is a **BVFSAI-** of .⌂

**Corollary (4.7):** Let be a **SAS-**, be a subset of Ѥ

and let be an bipolar valued fuzzy subset on Ѥ

 defined by : .

Where α1 , α2 ∈ (0, 1] with . If is a **BVFSAI-** of Ѥ, then is a SAI- of Ѥ.

**Proof:**

Since that is a **BVFSAI-** of Ѥ. Assume , , ∈,

 then so we have by Theorem (4.6),

1-

 and

1. ,

3-

this implies that Hence is a **SAI-** of Ѥ.⌂

**Proposition (4.8):** Every **BVFSAI-** of is **BVFSAS-** of .

**Proof:**

Since is **BVFSAI-** of , then by Theorem (4.6) ,

 is a **SAI-** of Ѥ. By Proposition (2.7),

 is a **SAS-** of Ѥ. Hence is **BVFSAS-** of Ѥ by Proposition (3.9) . ⌂

**Remark (4.9):** The convers of Proposition (4.8) is not true as shows in the example (3.3),

 it is easy to show that is .And the fuzzy subsets : → [0,1] and by:

 Define a bipolar valued subset of is a BVFSAS- of as:

 () =

It is easy to check that is a **BVFSAS-**, but not **BVFSAI-**.

 **Proposition (4.10):** Let be homomorphism of SA-algebras.

 If B is a **BVFSAI-** of , then the inverse image of B is a **BVFSAI-** of Ѥ.

**Proof:**

 Since is a **BVFSAI-** of , it follows from Theorem (4.3), that and are **AFSAI-** of

and and are **FSAI-** of .

Using Theorem (2.19) and Theorem (2.27),

we know and are **AFSAI-** of Ѥ

 and and are **FSAI-** of Ѥ.

 Hence is a **BVFSAI-** of Ѥ. ⌂

**Proposition (4.11):**  Assume be an epimorphism

of SA-algebras. If is **BVFSAI-** of Ѥ with inf-sup property, then is a **BVFSAI-** of .

**Proof:**

 Assume that is a **BVFSAI-** of Ѥ, it follows from Theorem (4.3), that and are **AFSAI-** of Ѥ and and are **FSAI-**. of Ѥ. Using Theorem (2.21) andTheorem (2.29), the images and are **AFSAI-** of and and are **FSAI-** of . Hence is a **BVFSAI-** of .⌂

**CONCLLUSION**

The idea of this study avails as abasis for of new readings in the SA-algebra. We started by explaining of bipolar valued fuzzy SA-subalgebras and bipolar valued fuzzy SA-ideals on SA-algebras with their properties and substantial examples and theorems and The image and inverse image of them are defined.

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**فترات ثنائي القطب للجبر الضبابي الجزئيSA- والمثالي الضبابي SA- في جبر SA-**

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**الملخص:**

في هذا البحث قدمنا تعريف لفترات ثنائي القطب للجبر الجزئي- SA وفترات ثنائي القطب للمثالي-SA في جبر-SA مع ذكر الخواص لكل منهم كما قمنا باعطاء وبرهان مجموعة من المبرهنات مع ذكر مجموعة من الامثلة الخاصة بهم. ايضا قمنا بتعريف الصورة والصورة العكسية ضمن تعريف التشاكل من جبر SA- الى جبرSA- واثبتنا ان الصورة لفترة ثنائي القطب للجبر الجزئيSA- هي ايضا فترة ثنائي القطب للجبر الجزئي SA- وكذلك الصورة العكسية لفترة ثنائي القطب للجبر الجزئيSA- هي ايضا فترة ثنائي القطب للجبر الجزئي SA- .كما قدمنا تعريف جديد وهو القيم السالبة للجبر الجزئي الضد ضبابي مع ذكر الامثلة والمبرهنات الخاصة به والمرتبطة بفترات ثنائي القطب للجبر الجزئي SA- .ثم بعد ذلك انتقلنا الى اثبات ان الصورة والصورة العكسية لفترات ثنائي القطب للمثالي SA- انها فترات ثنائي القطب للمثالي SA- وقمنا بتعريف جديد هو القيم السالبة للمثالي الضد العكسي مع ذكر الامثلة والمبرهنات المتعلقة بربط هذا التعريف بفترات ثنائي القطب للمثالي SA- من خلال مجموعة من المبرهنات.

**الكلمات المفتاحية:** جبرSA- وفترات ثنائي القطب للجبر الجزئي-SA وفترات ثنائي القطب للمثالي SA-.