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Solving higher orders linear complex partial differential equations via two dimensional differential transform method

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ABSTRACT

Complex partial differential equation (CPDEs) appeared around the year 1900. D. Pompeiu was a famous mathematician who left a large impact in this field through introducing the Pompeiu integral operator, which forms a basis in the subject CDEs. The complexity of some real-world problems has been conquered via the methods of solution for CDEs . Two-dimensional differential transform was proposed by Chen and Ho as a powerful tool for solving PDEs and used to solve linear and nonlinear complex partial differential equations. This paper presents two-dimensional differential transform for the complex partial derivatives of higher orders for a complex functions of two complex independent variables, and then use these complex partial derivatives to find an exact solution to a complex partial differential equation of the fourth order using two-dimensional differential transform method.

Introduction

Complex differential equations appeared at the end of the last century and became today are of great interest by the researchers because it has many applications in science and engineering such as quantum systems and neural networks. Two-dimensional differential transform was introduced by Chen and Ho [3] and it is regarded an effective method among the methods that used for solving partial differential equations [1], [2] and [9]. Two-dimensional differential transform method are used for solving linear complex partial differential equations such as [4], [5], [6] and [7] and for nonlinear complex partial differential equations [8]. In this paper, we are interested in solving complex partial differential equations of higher orders using two-dimensional differential transform method analytically. Our review begins with.

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Our paper begins with the basic concepts for our work. Then, it includes finding two dimensional differential transform for some complex partial derivatives of higher orders. Also, it introduces two dimensional differential transform method for solving fourth order complex partial differential equation. Finally, the conclusions are given.

Basic Concepts

Definition 2.1. [3] Let $f(x,y)$ be a function of two variables which is analytic and continuously differentiable on the nonnegative integer. Then two dimensional differential transform of $f(x,y)$ is defined as follows:

$$F(p,q)=\frac{1}{p!q!}\left.\frac{\partial^{p+q}f(x,y)}{\partial x^{p}\partial y^{q}}\right|_{x=0,y=0}, \qquad (2.1)$$

Where $f(x,y)$ is the original function and $F(p,q)$ is the transformed function. The differential inverse transform of function $F(p,q)$ is defined as follows:

$$f(x,y)=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}F(p,q)x^p y^q. \qquad (2.2)$$

Theorem 2.1. [3] If $w(x,y)=u(x,y)\pm v(x,y)$ then $W(p,q)=U(p,q)\pm V(p,q)$.

Theorem 2.2. [3] If $w(x,y)=\frac{\partial^{r+s}u(x,y)}{\partial x^r \partial y^s}$ then

$$W(p,q)=(p+1)(p+2)\cdots(p+r)(q+1)(q+2)\cdots(q+s)\,U(p+r,q+s).$$

Consider the complex function $w(z,\bar{z})=u(x,y)+iv(x,y)$ where $z=x+iy$, and $\bar{z}=x-iy$. The first, second and third orders partial derivatives of $w(z,\bar{z})$ are given as follows:

Theorem 2.3. [5] The partial derivatives of the 1st order of $w(z,\bar{z})$ are shown as:

$$\frac{\partial w}{\partial \bar{z}}=\frac{1}{2}(\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y})$$

$$\frac{\partial w}{\partial z}=\frac{1}{2}(\frac{\partial w}{\partial x}-i\frac{\partial w}{\partial y})$$

where

$$\frac{\partial w}{\partial x}=u_x+iv_x,\quad \frac{\partial w}{\partial y}=u_y+iv_y.$$

Theorem 2.4. [6] The partial derivatives of the 2 nd order of $w(z,\bar{z})$ are shown as:

$$1-\ \frac{\partial^2 w(z,\bar{z})}{\partial \bar{z}^2}=\frac{1}{4}(\frac{\partial^2 w}{\partial x^2}+2i\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial^2 w}{\partial y^2}).$$

$$2-\ \frac{\partial^2 w(z,\bar{z})}{\partial z\partial \bar{z}}=\frac{1}{4}(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})=\frac{1}{4}\Delta w.$$

$$3-\ \frac{\partial^2 w(z,\bar{z})}{\partial z^2}=\frac{1}{4}(\frac{\partial^2 w}{\partial x^2}-2i\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial^2 w}{\partial y^2}).$$

Theorem 2.5. [7] The partial derivatives of the 3 nd order of $w(z,\bar{z})$ are shown as:

$$1-\ \frac{\partial^3 w(z,\bar{z})}{\partial \bar{z}^3}=\frac{1}{8}(\frac{\partial^3 w}{\partial x^3}-3\frac{\partial^3 w}{\partial x\partial y^2}+3i\frac{\partial^3 w}{\partial x^2\partial y}-i\frac{\partial^3 w}{\partial y^3})$$

$$2-\ \frac{\partial^3 w(z,\bar{z})}{\partial z\partial \bar{z}^2}=\frac{1}{8}(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}+i\frac{\partial^3 w}{\partial x^2\partial y}+i\frac{\partial^3 w}{\partial y^3})$$

$$3-\ \frac{\partial^3 w(z,\bar{z})}{\partial z^2\partial \bar{z}}=\frac{1}{8}(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}-i\frac{\partial^3 w}{\partial x^2\partial y}-i\frac{\partial^3 w}{\partial y^3})$$

$$4-\ \frac{\partial^3 w(z,\bar{z})}{\partial z^3}=\frac{1}{8}(\frac{\partial^3 w}{\partial x^3}-3\frac{\partial^3 w}{\partial x\partial y^2}-3i\frac{\partial^3 w}{\partial x^2\partial y}+i\frac{\partial^3 w}{\partial y^3})$$

The Complex Partial Derivatives of Higher Orders of the Complex Functions

This section introduces the fourth and fifth order partial derivatives of the complex function $w(z,\bar{z})$ in terms of two real variables x and y where $z=x+iy$, $\bar{z}=x-iy$ and $w(z,\bar{z})=u(x,y)+iv(x,y)$.

Theorem 3.1. The fourth order partial derivatives of complex a function $w(z,\bar{z})$ are given as follows:

$$1-\ \frac{\partial^4 w(z,\bar{z})}{\partial \bar{z}^4}=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}-6\frac{\partial^4 w}{\partial x^2\partial y^2}-4i\frac{\partial^4 w}{\partial x\partial y^3}+4i\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 w}{\partial y^4}).$$

$$2-\ \frac{\partial^4 w(z,\bar{z})}{\partial z\partial \bar{z}^3}=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}+2i\frac{\partial^4 w}{\partial x\partial y^3}+2i\frac{\partial^4 w}{\partial x^3\partial y}-\frac{\partial^4 w}{\partial y^4}).$$

$$3-\ \frac{\partial^4 w(z,\bar{z})}{\partial z^2\partial \bar{z}^2}=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}).$$

$$4-\ \frac{\partial^4 w(z,\bar{z})}{\partial z^3\partial \bar{z}}=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}-2i\frac{\partial^4 w}{\partial x\partial y^3}-2i\frac{\partial^4 w}{\partial x^3\partial y}-\frac{\partial^4 w}{\partial y^4}).$$

$$5-\ \frac{\partial^4 w(z,\bar{z})}{\partial z^4}=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}-6\frac{\partial^4 w}{\partial x^2\partial y^2}+4i\frac{\partial^4 w}{\partial x\partial y^3}-4i\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 w}{\partial y^4}).$$

Proof. To prove 2, we have

$$\frac{\partial^4 w(z,\bar{z})}{\partial z\partial \bar{z}^3}=\frac{1}{8}\frac{\partial}{\partial z}(\frac{\partial^3 w}{\partial x^3}-3\frac{\partial^3 w}{\partial x\partial y^2}+3i\frac{\partial^3 w}{\partial x^2\partial y}-i\frac{\partial^3 w}{\partial y^3})$$

$$=\frac{1}{8}\left[\frac{\partial}{\partial x}(\frac{\partial^3 w}{\partial x^3}-3\frac{\partial^3 w}{\partial x\partial y^2}+3i\frac{\partial^3 w}{\partial x^2\partial y}-i\frac{\partial^3 w}{\partial y^3})\frac{\partial x}{\partial z}+\frac{\partial}{\partial y}(\frac{\partial^3 w}{\partial x^3}-3\frac{\partial^3 w}{\partial x\partial y^2}+3i\frac{\partial^3 w}{\partial x^2\partial y}-i\frac{\partial^3 w}{\partial y^3})\frac{\partial y}{\partial z}\right]$$

$$=\frac{1}{16}(\frac{\partial^4 w}{\partial x^4}+2i\frac{\partial^4 w}{\partial x\partial y^3}+2i\frac{\partial^4 w}{\partial x^3\partial y}-\frac{\partial^4 w}{\partial y^4})$$

The other proofs are similar.

Theorem 3.2. The fifth order partial derivatives of a complex function $w(z,\bar{z})$ are given as follows:

$$1-\ \frac{\partial^5 w(z,\bar{z})}{\partial \bar{z}^5}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}+5\frac{\partial^5 w}{\partial x\,\partial y^4}-10\frac{\partial^5 w}{\partial x^3\partial y^2}-10i\,\frac{\partial^5 w}{\partial x^2\partial y^3}+5i\,\frac{\partial^5 w}{\partial x^4\partial y}+i\,\frac{\partial^5 w}{\partial y^5}\right).$$

$$2-\ \frac{\partial^5 w(z,\bar{z})}{\partial z\,\partial \bar{z}^4}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}-3\frac{\partial^5 w}{\partial x\,\partial y^4}-2\frac{\partial^5 w}{\partial x^3\partial y^2}+2i\,\frac{\partial^5 w}{\partial x^2\partial y^3}+3i\,\frac{\partial^5 w}{\partial x^4\partial y}-i\,\frac{\partial^5 w}{\partial y^5}\right).$$

$$3-\ \frac{\partial^5 w(z,\bar{z})}{\partial z^2\partial \bar{z}^3}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}+\frac{\partial^5 w}{\partial x\,\partial y^4}+2\frac{\partial^5 w}{\partial x^3\partial y^2}+2i\,\frac{\partial^5 w}{\partial x^2\partial y^3}+i\,\frac{\partial^5 w}{\partial x^4\partial y}+i\,\frac{\partial^5 w}{\partial y^5}\right).$$

$$4-\ \frac{\partial^5 w(z,\bar{z})}{\partial z^3\partial \bar{z}^2}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}+\frac{\partial^5 w}{\partial x\,\partial y^4}+2\frac{\partial^5 w}{\partial x^3\partial y^2}-2i\,\frac{\partial^5 w}{\partial x^2\partial y^3}-i\,\frac{\partial^5 w}{\partial x^4\partial y}-i\,\frac{\partial^5 w}{\partial y^5}\right).$$

$$5-\ \frac{\partial^5 w(z,\bar{z})}{\partial z^4\partial \bar{z}}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}-3\frac{\partial^5 w}{\partial x\,\partial y^4}-2\frac{\partial^5 w}{\partial x^3\partial y^2}-2i\,\frac{\partial^5 w}{\partial x^2\partial y^3}-3i\,\frac{\partial^5 w}{\partial x^4\partial y}+i\,\frac{\partial^5 w}{\partial y^5}\right).$$

$$6-\ \frac{\partial^5 w(z,\bar{z})}{\partial z^5}=\frac{1}{32}\left(\frac{\partial^5 w}{\partial x^5}+5\frac{\partial^5 w}{\partial x\,\partial y^4}-10\frac{\partial^5 w}{\partial x^3\partial y^2}+10i\,\frac{\partial^5 w}{\partial x^2\partial y^3}-5i\,\frac{\partial^5 w}{\partial x^4\partial y}-i\,\frac{\partial^5 w}{\partial y^5}\right).$$

The proofs can be made as in Theorem 3.1.

Solution of Higher Order Complex Partial Differential Equation by Two- Dimensional Differential Transform Method

In this section, we introduce an exact solution to a complex partial differential equation of the fourth order by using two dimensional differential transform method.

Example 4.1 Consider the fourth order complex initial value problem:

$$\frac{\partial^4 w}{\partial \bar{z}^4}+\frac{\partial^2 w}{\partial z^2}-w=0, \qquad (4.1)$$

$$w(x,0)=e^x+2\cosh x \qquad (4.2)$$

$$\frac{\partial w}{\partial y}(x,0)=-ie^x+2i\sinh x \qquad (4.3)$$

$$\frac{\partial^2 w}{\partial y^2}(x,0)=-e^x-2\cosh x \qquad (4.4)$$

$$\frac{\partial^3 w}{\partial y^3}(x,0)=ie^x-2i\sinh x. \qquad (4.5)$$

Let us utilize two-dimensional differential transform on both sides of the equation (4.1). By Theorem 3.1, Theorem 2.4 and Theorem 2.2 we obtain:

$$U(p,q+4)=\frac{-1}{(q+1)(q+2)(q+3)(q+4)}[4(p+1)(q+1)(q+2)(q+3)V(p+1,q+3)-6(p+1)$$
$$(p+2)(q+1)(q+2)U(p+2,q+2)-4(p+1)(p+2)(p+3)(q+1)V(p+3,q+1)$$
$$+(p+1)(p+2)(p+3)(p+4)U(p+4,q)-4(q+1)(q+2)U(p,q+2)+8(p+1)$$
$$(q+1)V(p+1,q+1)+4(p+1)(p+2)U(p+2,q)-16U(p,q)] \qquad (4.6)$$

$$V(p,q+4)=\frac{-1}{(q+1)(q+2)(q+3)(q+4)}[-4(p+1)(q+1)(q+2)(q+3)U(p+1,q+3)-6(p+1)$$
$$(p+2)(q+1)(q+2)V(p+2,q+2)+4(p+1)(p+2)(p+3)(q+1)U(p+3,q+1)+$$
$$(p+1)(p+2)(p+3)(p+4)V(p+4,q)-4(q+1)(q+2)V(p,q+2)-8(p+1)(q+1)$$
$$U(p+1,q+1)+4(p+1)(p+2)V(p+2,q)-16V(p,q)] \qquad (4.7)$$

Since $w(x,0)=\sum_{p=0}^{\infty}W(p,0)x^p=\sum_{p=0}^{\infty}(U(p,0)+iV(p,0))x^p$,

and according to condition (4.2) we have:

$$w(x,0)=e^x+2\cosh x$$
$$=3+\frac{x}{1!}+\frac{3x^2}{2!}+\frac{x^3}{3!}+\frac{3x^4}{4!}+\frac{x^5}{5!}+\frac{3x^6}{6!}+\cdots+\frac{2+(-1)^n}{n!}x^n+\cdots.$$

As a result, we obtain:

$$U(p,0)=\frac{2+(-1)^p}{p!},\ p=0,1,2,\ldots \qquad (4.8)$$
$$V(p,0)=0,\ p=0,1,2,\ldots\ .$$

As well as , since

$\frac{\partial w(x,y)}{\partial y}=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}qW(p,q)x^p y^{q-1}$, that means:

$$\frac{\partial w(x,0)}{\partial y}=\sum_{p=0}^{\infty}W(p,1)x^p=\sum_{p=0}^{\infty}(U(p,1)+iV(p,1))x^p,$$

and according to condition (4.3), we have:

$$\frac{\partial w(x,0)}{\partial y}=-ie^x+2i\sinh x$$
$$=i[-1+\frac{x}{1!}-\frac{x^2}{2!}+\frac{x^3}{3!}-\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots+\frac{(-1)^{n+1}}{n!}x^n+\cdots].$$

As a result , we obtain:

$$U(p,1)=0,\ p=0,1,2,\ldots$$
$$V(p,1)=\frac{(-1)^{p+1}}{p!},\ p=0,1,2,\ldots \qquad (4.9)$$

Similarly, since

$\frac{\partial^2 w(x,y)}{\partial y^2}=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}q(q-1)W(p,q)x^p y^{q-2}$, that means:

$$\frac{\partial^2 w(x,0)}{\partial y^2}=2\sum_{p=0}^{\infty}W(p,2)x^p=2\sum_{p=0}^{\infty}(U(p,2)+iV(p,2))x^p,$$

and according to condition (4.4) , we have:

$$\frac{\partial^2 w(x,0)}{\partial y^2}=-(e^x+2\cosh x)$$

$$=-3-\frac{x}{1!}-\frac{3x^2}{2!}-\frac{x^3}{3!}-\frac{3x^4}{4!}-\frac{x^5}{5!}-\frac{3x^6}{6!}-\cdots+\frac{-2-(-1)^n}{n!}x^n-\cdots].$$

As a result , we obtain:

$$U(p,2)=\frac{-2-(-1)^p}{2p!},\ p=0,1,2,\ldots \qquad (4.10)$$

$$V(p,2)=0,\ p=0,1,2,\ldots$$

Finally, since

$$\frac{\partial^3 w(x,y)}{\partial y^3}=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}q(q-1)(q-2)W(p,q)x^p y^{q-3},$$

which means:

$$\frac{\partial^3 w(x,0)}{\partial y^3}=6\sum_{p=0}^{\infty}W(p,3)x^p=6\sum_{p=0}^{\infty}(U(p,3)+iV(p,3))x^p,$$

and according to condition (4.5) , we have:

$$\frac{\partial^3 w(x,0)}{\partial y^3}=-i(-e^x+2\sinh x)$$

$$=i[1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}-\cdots+\frac{(-1)^n}{n!}x^n-\cdots].$$

As a result , we obtain:

$$U(p,3)=0,\ p=0,1,2,\ldots$$

$$V(p,3)=\frac{(-1)^p}{6p!},\ p=0,1,2,\ldots \qquad (4.11)$$

Now, we shall find the values of $U(p,q)$ and $V(p,q)$ for $q=4,5,\ldots,$ and $p=0,1,2,\ldots$. Therefore, it is clear that:

$$U(p,2r+1)=0$$
$$V(p,2r)=0, \qquad (4.12) \quad \text{where} \quad r=0,1,2,\ldots ,$$

respectively.

If $q=0$ is placed into equation (4.6), the result is:

$$U(p,4)=\frac{-1}{24}[24(p+1)V(p+1,3)-12(p+1)(p+2)U(p+2,2)-4(p+1)(p+2)(p+3)V(p+3,1)+(p+1)(p+2)(p+3)(p+4)U(p+4,0)-8U(p,2)+8(p+1)V(p+1,1)+4(p+1)(p+2)U(p+2,0)-16U(p,0)]$$

$$=\frac{-2-(-1)^p}{6!p!},p=0,1,2,\ldots$$

$$=\frac{2+(-1)^p}{24p!},\ p=0,1,2,\ldots.$$

If $q=1$ is placed into equation (4.7), the result is:

$$V(p,5)=\frac{-1}{120}[-96(p+1)U(p+1,4)-36(p+1)(p+2)V(p+2,3)+8(p+1)(p+2)(p+3)U(p+3,2)+(p+1)(p+2)(p+3)(p+4)V(p+4,1)-24V(p,3)-16(p+1)U(p+1,2)+4(p+1)(p+2)V(p+2,1)-16V(p,1)]$$

$$=\frac{(-1)^{p+1}}{120p!},\ p=0,1,2,\ldots.$$

Similarly, if $q=2$ is placed into equation (4.6), the result is:

$$U(p,6)=\frac{-1}{360}[240(p+1)V(p+1,5)-72(p+1)(p+2)U(p+2,4)-12(p+1)(p+2)(p+3)V(p+3,3)+(p+1)(p+2)(p+3)(p+4)U(p+4,2)-48U(p,4)+24(p+1)V(p+1,3)+4(p+1)(p+2)U(p+2,2)-16U(p,2)]$$

$$=\frac{-2-(-1)^p}{6!p!},p=0,1,2,\ldots$$

and if $q=3$ is placed into equation (4.7), the result is:

$$V(p,7)=\frac{-1}{840}[-480(p+1)U(p+1,6)-120(p+1)(p+2)V(p+2,5)+16(p+1)(p+2)(p+3)U(p+3,4)+(p+1)(p+2)(p+3)(p+4)V(p+4,3)-80V(p,5)-32(p+1)U(p+1,4)+4(p+1)(p+2)V(p+2,3)-16V(p,3)]$$

$$=\frac{-2-(-1)^p}{7!p!},\ p=0,1,2,\ldots..$$

By applying equations (4.6) and (4.7) as above manner repeatedly, we can obtain that:

$$U(p,2r)=\frac{(-1)^r(2+(-1)^p)}{(2r)!p!} \qquad (4.13)$$

and

$$V(p,2r+1)=\frac{(-1)^{p+r+1}}{(2r+1)!p!}, \qquad (4.14)$$

where $r=2,3,\ldots$, respectively.

Now, using the above values of $U(p,q)$ and $V(p,q)$, we obtain:

$$w(z,\bar{z})=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}W(p,q)x^p y^q$$

$$=U(0,0)+U(1,0)x+U(2,0)x^2+U(3,0)x^3+U(4,0)x^4+\cdots+U(p,0)x^p+\cdots$$
$$+i[V(0,1)+V(1,1)x+V(2,1)x^2+V(3,1)x^3+V(4,1)x^4+\cdots+V(p,1)x^p+\cdots]y$$
$$+[U(0,2)+U(1,2)x+U(2,2)x^2+U(3,2)x^3+U(4,2)x^4+\cdots+U(p,2)x^p+\cdots]y^2$$

$$+i[V(0,3)+V(1,3)x+V(2,3)x^2+V(3,3)x^3+V(4,3)x^4+\cdots+V(p,3)x^p+\cdots]y^3$$

$$+[U(0,4)+U(1,4)x+U(2,4)x^2+U(3,4)x^3+U(4,4)x^4+\cdots+U(p,4)x^p+\cdots]y^4$$

$$\vdots$$

$$=3+x-iy+\frac{3}{2}x^2+ixy-\frac{3}{2}y^2+\frac{1}{3!}x^3-\frac{i}{2!}x^2y-\frac{1}{2}xy^2+\frac{i}{6}y^3+\frac{3}{4!}x^4+\frac{i}{3!}x^3y$$

$$-\frac{3}{2.2!}x^2y^2$$

$$=-\frac{i}{6}xy^3+\frac{3}{24}y^4+\cdots$$

By putting $z=x+iy$ and $\bar{z}=x-iy$, we obtain:

$$w(z,\bar{z})=3+\bar{z}+\frac{3}{8}(z^2+2z\bar{z}+\bar{z}^2)+\frac{1}{4}(z^2-\bar{z}^2)+\frac{3}{8}(z^2-2z\bar{z}+\bar{z}^2)+\frac{1}{48}(z^3+3z^2\bar{z}$$

$$+3z\bar{z}^2+\bar{z}^3)-\frac{1}{16}(z^3+z^2\bar{z}-z\bar{z}^2-\bar{z}^3)+\frac{1}{16}(z^3-z^2\bar{z}-z\bar{z}^2+\bar{z}^3)-\frac{1}{48}(z^3$$

$$-3z^2\bar{z}+3z\bar{z}^2-\bar{z}^3)+\frac{1}{128}(z^4+4z\bar{z}^3+6z^2\bar{z}^2+4z^3\bar{z}+\bar{z}^4)+\frac{1}{96}(z^4+2z^3\bar{z}$$

$$-2z\bar{z}^3+\bar{z}^4)+\frac{3}{64}(z^4-2z^2\bar{z}^2+\bar{z}^4)+\frac{1}{96}(z^4-2z^3\bar{z}+2z\bar{z}^3-\bar{z}^4)+\frac{1}{128}(z^4$$

$$-4z\bar{z}^3+6z^2\bar{z}^2-4z^3\bar{z}+\bar{z}^4)+\cdots$$

As a result, the solution of the CIVP (4.1)-(4.5) is:

$$w(z,\bar{z})=3+\bar{z}+z^2+\frac{\bar{z}^2}{2}+\frac{\bar{z}^3}{6}+\frac{z^4}{12}+\frac{\bar{z}^4}{24}+\cdots$$

$$=(1+\bar{z}+\frac{\bar{z}^2}{2!}+\frac{\bar{z}^3}{3!}+\frac{\bar{z}^4}{4!}+\cdots)+(2+z^2+\frac{z^4}{12}+\cdots)$$

$$=(1+\bar{z}+\frac{\bar{z}^2}{2!}+\frac{\bar{z}^3}{3!}+\frac{\bar{z}^4}{4!}+\cdots)+2(1+\frac{z^2}{2!}+\frac{z^4}{4!}+\cdots)$$

$$=e^{\bar{z}}+2\cosh z.$$

Conclusion

Two-dimensional differential transform method can be used for solving complex partial differential equations of higher orders by transforming the complex partial derivatives to real derivatives. Two-dimensional differential transform method is regarded an effective tool in finding exact solutions to the complex partial differential equations.

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حل معادلات تفاضلية جزئية عقدية خطية من الرتب العليا باستخدام طريقة التحويل التفاضلي ثنائي الابعاد

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الخلاصة:

ظهرت المعادلات التفاضلية الجزئية العقدية بحدود عام 1900، و كان عالم الرياضيات الشهير بومبيو قد ترك أثرا كبيرا في هذا المجال من خلال تقديمه مؤثر يسمى مؤثر تكامل بومبيو، الذي يشكل حجر الاساس في موضوع المعادلات التفاضلية العقدية. تكمن اهمية موضوع المعادلات التفاضلية العقدية في انه يمكن التغلب على التعقيد المصاحب لحل بعض المسائل في المستوي الحقيقي بواسطة تحويلها الى المستوي العقدي ثم حلها باستخدام طرق حل المعادلات التفاضلية العقدية. ان احدى طرق حل المعادلات التفاضلية العقدية والتي عمل عليها العديد من الباحثين هي طريقة التحويل التفاضلي ثنائي الابعاد والتي تعتبر ذات كفاءه عالية في حل المعادلات التفاضلية العقدية الخطية و غير الخطية. في هذا البحث نقدم التحويل التفاضلي ثنائي الابعاد للمشتقات الجزئية العقدية من الرتب العليا وتحديدا الرتبة الرابعة والرتبة الخامسة لدالة عقدية بمتغيرين عقديين ، ثم بعد ذلك نستخدم هذه المشتقات الجزئية العقدية لإيجاد حل مضبوط لمعادلة تفاضلية جزئية عقدية من الرتبة الرابعة باستخدام طريقة التحويل التفاضلي ثنائي الابعاد.

الكلمات المفتاحية: معادلات تفاضلية عقدية، التحويل التفاضلي.